Perihelion Precession for the Charged Two-Body Problem in General Relativity.

B. M. BARKER

Department of Physics and Astronomy, The University of Alabama University, Ala., 35486

R. F. O'CONNELL

Department of Physics and Astronomy, Louisiana State University Baton Rouge, La., 70803

(ricevuto il 14 Marzo 1977)

The possible existence of charged black holes (1) has motivated us to carry out a detailed study of the two-body problem with electric charge in general relativity. The problem is also of theoretical interest (2). Of particular interest is the calculation of the perihelion precession Ω^*. This may be derived by starting with the Lagrangian (3), to order c^{-2}, in the center-of-mass system (4). Denoting the mass, charge, position and velocity of each of the two bodies by m_i, e_i, $\boldsymbol{r}_i$, and $\boldsymbol{v}_i$, respectively ($i = 1, 2$); relative positions and velocities by $\boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2$ and $\boldsymbol{v} = \boldsymbol{v}_1 - \boldsymbol{v}_2$, respectively; and the gravitational coupling constant by G, we find that the use of a co-ordinate transformation enables us to write the Lagrangian $\mathscr{L}$ in the simple form (4)

$$\mathscr{L} = \tfrac{1}{2}\mu v^2 + G'\mu M/r + \tfrac{1}{8}k_1 \mu v^4/c^2 + \tfrac{3}{2}k_2 G'\mu M v^2/c^2 r - \tfrac{1}{2}k_3 G'^2 \mu M^2/c^2 r^2 , \qquad (1)$$

where

$$G' = G - e_1 e_2/m_1 m_2 , \qquad (2)$$

$$k_1 = 1 - 3\mu/M , \qquad (3)$$

$$k_2 = 1 + \tfrac{2}{3}\mu/M + Z_a , \qquad (4)$$

$$k_3 = 1 + \mu/M + Z_b + Z_a Z_b - Z_a^2 , \qquad (5)$$

$$M = m_1 + m_2 , \qquad (6)$$

$$\mu = m_1 m_2/(m_1 + m_2) \qquad (7)$$

(1) E. R. HARRISON: *Nature*, **264**, 525 (1976), and references therein.
(2) A. R. KHAN and R. F. O'CONNELL: *Nature*, **261**, 480 (1976), and references therein.
(3) S. BAŻAŃSKI: *Acta Phys. Pol.*, **15**, 363 (1956); **16**, 423 (1957).
(4) B. M. BARKER and R. F. O'CONNELL: to be published.

and

$$Z_a = \frac{e_1 e_2}{G' m_1 m_2}, \tag{8}$$

$$Z_b = \frac{e_1^2 m_2 + e_2^2 m_1}{G' m_1 m_2 M}. \tag{9}$$

Now using the same techniques we employed for the uncharged two-body problem ([5]), we obtain

$$\boldsymbol{\Omega}^* = \frac{(\frac{1}{2}k_1 + 3k_2 - \frac{1}{2}k_3)G' M \bar{\omega}}{c^2 a(1-e^2)} \boldsymbol{n}, \tag{10}$$

where a is the semimajor axis, e is the eccentricity, $\bar{\omega}$ is the average orbital angular velocity, and $\boldsymbol{n}$ is a unit vector in the direction of the angular momentum $\boldsymbol{L}$. To ensure a bound orbit we must have $G' > 0$. The result for $\boldsymbol{\Omega}^*$ can be cast into different forms by using the relations

$$\frac{L/\mu}{a^2(1-e^2)^{\frac{1}{2}}} = \left(\frac{G' M}{a^3}\right)^{\frac{1}{2}} = \frac{2\pi}{T} = \bar{\omega}, \tag{11}$$

where T is the orbital period.

In the particular case when $e_1 = e_2 = 0$, we obtain

$$\boldsymbol{\Omega}^* = \frac{3GM\bar{\omega}}{c^2 a(1-e^2)} \boldsymbol{n}, \tag{12}$$

which is the result of Robertson ([6]) and ourselves ([5]).

When $G = 0$, we obtain

$$\boldsymbol{\Omega}^* = \frac{|e_1 e_2| \bar{\omega}/\mu}{2c^2 a(1-e^2)} \boldsymbol{n}, \tag{13}$$

where $e_1 e_2 < 0$ because the orbit must be bound. Equation (13) is the two-body generalization of the «one-body» Sommerfeld ([7]) result and reduces to the Sommerfeld result under the large-mass approximation $m_2 \gg m_1$. A more detailed discussion of this work will appear elsewhere ([4]).

([5]) B. M. BARKER and R. F. O'CONNELL: *Phys. Rev. D*, **12**, 329 (1975).
([6]) H. P. ROBERTSON: *Ann. Math.*, **39**, 101 (1938).
([7]) A. SOMMERFELD: *Atomic Structure and Spectral Lines* (New York, N. Y., 1934), p. 251.