Screening Corrections in Problems Involving Bound Electrons.

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The screening of the Coulomb field of the nucleus by atomic electrons is involved in the calculation of many different processes. Some examples which come to mind in which screening corrections have been considered are β-decay spectra [1-5] internal conversion coefficients [3,6-8], pair conversion [9], and the decay of bound muons [10]. In all these cases, the inclusion of screening has added considerably to the complexity of the problem under consideration. It is the purpose of this letter to show how the effect of screening of a bound electron may be taken into account very simply, in a manner which will have immediate application to many different types of problems.

CHU and PERLMAN [8] have considered empirical screening corrections for M-subshell internal conversion coefficients. They deduced that screening may be taken into account, for $Z \geq 30$, by replacing Z by $Z_{\text{eff}} = Z - \sigma_i$, where σ_i is a constant for each subshell. As we have already remarked [11], this method suffers from the defect of being based on a comparison of experimental results for particular elements with theoretical results obtained by extrapolation of the unscreened M shell results of ROSE [6]. In fact, one would at least expect that Z_{eff} for a particular shell should also be a function of Z itself.

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[1] M. E. ROSE: *Phys. Rev.*, **49**, 727 (1936).

[2] C. LONGMIRE and H. BROWN: *Phys. Rev.*, **75**, 264, E 1102 (1949).

[3] J. R. REITZ: *Phys. Rev.*, **77**, 10 (1950).

[4] L. DURAND III: *Phys. Rev.*, **135**, B 310 (1964).

[5] L. S. BROWN: *Phys. Rev.*, **135**, B 314 (1964).

[6] M. E. ROSE: *Internal Conversion Coefficients* (Amsterdam, New York, 1958).

[7] L. A. SLIV and I. M. BAND: *Coefficients of Internal Conversion of Gamma Radiation*, part I: *K shell* and part II: *L shell* Leningrad, 1956; issued in U. S. as Report 57ICC K1 and Report 58ICC L1, Physics Dept., University of Illinois, Urbana, Ill.).

[8] Y. Y. CHU and L. L. PERLMAN: *Phys. Rev.*, **135**, B 319 (1964).

[9] V. G. GORSHKOV and K. RIDEL: *Sov. Phys. JETP*, **18**, 1103 (1964).

[10] R. F. O'CONNELL: *Ph. D. Thesis* in the Department of Physics, University of Notre Dame, Notre Dame, Ind. (unpublished).

[11] R. F. O'CONNELL and C. O. CARROLL: *Internal conversion coefficients: general formulation for all shells and application to low-energy transitions*: *Phys. Rev.*, **138**, B 1042 (1965).

TABLE I. – *Values of Z_b and $\sigma_b = Z - Z_b$ for the L_I subshell for elements with atomic number in the range $8 \leq Z \leq 23$.*

Z	Z_b	σ_b	Z	Z_b	σ_b
8	2.64	5.36	16	8.13	7.87
9	3.16	5.84	17	8.89	8.11
10	3.64	6.36	18	9.63	8.37
11	4.37	6.63	19	10.38	8.62
12	5.12	6.88	20	11.14	8.86
13	5.87	7.13	21	11.89	9.11
14	6.62	7.38	22	12.63	9.37
15	7.37	7.63	23	13.38	9.62

Our approach is to consider not just one but two different types of Z_{eff}. The bound and continuum-state electrons will see effective atomic numbers Z_b and Z_c, respectively, where $Z_b \neq Z_c$. We will concern ourselves here with the calculation of Z_b.

The energy of an electron bound in the Coulomb field of a point nucleus is given by [12]

$$W_B = \left\{1 + \frac{\alpha^2 Z^2}{(\gamma' + n')^2}\right\}^{-\frac{1}{2}}, \tag{1}$$

where [13]

$$n' = n - |\varkappa'| \quad (n = 1, 2, 3, \ldots),$$

$$\varkappa' = \mp (j' + \tfrac{1}{2}) \quad \text{for } j' = l' \pm \tfrac{1}{2}$$

and

$$\gamma' = (\varkappa'^2 - \alpha^2 Z^2)^{\frac{1}{2}}. \tag{2}$$

The binding energy of the electron is given by [13]

$$B = 1 - W_B. \tag{3}$$

Solving eq. (1) for γ', and using eq. (3) we find

$$\gamma' = -b - (1 - B)(\varkappa'^2 - \varkappa' b)^{\frac{1}{2}}, \tag{4}$$

where [14]

$$b \equiv \varkappa' B(2 - B). \tag{5}$$

Thus, from eqs. (2), (4) and (5), we see that we may solve for Z in terms of the quantum numbers of a particular shell and the appropriate binding energy.

If we now consider the case of an electron bound in the field of a screened nucleus we see that Z and B become replaced by Z_b and B_{expt}, where B_{expt} is the experimentally determined binding energy. Thus, knowing B_{expt}, we may obtain Z_b. In general, we see that Z_b will depend both on the shell designation and on Z itself. FAHLMAN *et al.* [15], have recently published a revised table of L_I subshell electron binding energies in light elements; we have calculated the corresponding values of Z_b and $\sigma_b \equiv Z - Z_b$ and present the results in Table I. Elsewhere, we will publish a complete set of tables giving Z_b for every element and for every shell.

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[12] M. E. ROSE: *Relativistic Electron Theory* (New York, 1960).

[13] In our units $\hbar = c = m = 1$, where m refers to the mass of the electron.

[14] A sign ambiguity in our results is resolved by remembering that $\gamma' > 0$.

[15] A. FAHLMAN, K. HAMRIN, R. NORDBERG, C. NORDLING and K. SIEGBAHN: *Phys. Rev. Lett.*, **14**, 127 (1965).