

## ONE-DIMENSIONAL HYDROGENIC ATOM IN AN ELECTRIC FIELD WITH SOLID-STATE APPLICATIONS

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A Hamiltonian describing a one-dimensional Coulomb field with an electric field in the same direction is useful for the discussion of electrons outside a free surface of liquid helium [1, 2] and also for the study of far-infrared emission from Si inversion layers [3]. We present both a semi-classical and a WKB solution to the problem, which exhibits many of the features found experimentally.

Electrons outside a free surface of liquid helium are trapped in an image potential, which is essentially one-dimensional Coulombic [1, 2]. Recent experiments have measured Stark-shifted energies for transitions from the ground state to excited states in this potential. Thus, the approximate Hamiltonian is (in atomic units) the same as that for a one-dimensional hydrogenic atom in an electric field viz.

$$H = \frac{P^2}{2} - \frac{Z}{x} + \mathcal{E}x. \quad (1)$$

Here  $P$ ,  $\mathcal{E}$  and  $Z$  refer to the momentum in the  $x$  direction, the electric field (applied in the  $x$  direction), and the effective strength of the trapping potential. For liquid helium [1, 2],  $Z = 7 \times 10^{-3}$ . This Hamiltonian is also appropriate to the study of far-infrared emission from Si inversion layers [3] — one simply uses different numerical values for  $Z$  and the effective electronic mass  $m_{\text{eff}}$ .

The Schrödinger equation for this problem cannot be solved exactly. Furthermore, the use of perturbation theory is limited to weak  $\mathcal{E}$  fields. Thus we are led to consider the WKB approach (used in ref. [4] for the case of  $Z = 0$ ) or the semi-classical approach of the present author [5] (which was used successfully to analyze the measurements on highly excited states in a magnetic field [6]). It turns out that both of these approaches lead to similar results and so we will present here an analysis based on the latter approach.

In the case where  $\mathcal{E} = 0$ , the energy levels  $E$  are given by [7]

$$E \equiv E_c = -Z^2/2n^2 \quad (n = 1, 2, 3, \dots), \quad (2)$$

and, in the case where  $Z = 0$ , the energy levels are given by [4]

$$E \equiv E_F = \frac{1}{2}(3\pi n \mathcal{E})^{2/3} \quad (n = 1, 2, 3, \dots). \quad (3)$$

In the latter case we replaced  $(n - \frac{1}{4})$  by  $n$ , which is the appropriate quantum number when we deal with the combined fields (due to the elimination of a  $(\pi/2)$  phase factor characteristic of such problems).

For the combined fields problem, we see from an examination of eqs. (2) and (3) that the electric field dominates over the Coulomb field for  $\mathcal{E}$  values greater than about<sup>‡</sup>

$$\mathcal{E}^* \equiv Z^3/2n^4. \quad (4)$$

In this regime ( $\mathcal{E} \gtrsim \mathcal{E}^*$ ), we calculate the effect of the Coulomb field by assuming (see ref. [5]) that the magnitude of  $x$  appearing in the Coulomb potential is determined primarily by the  $\mathcal{E}$  field forces. As a result, we find that the total energy in this strong  $\mathcal{E}$  field region ( $E \equiv E_s$ ) is given by

$$E_s = E_F \{1 - 0.4(\mathcal{E}^*/\mathcal{E})^{1/3}\} \quad (\mathcal{E} > \mathcal{E}^*). \quad (5)$$

Using similar techniques, we find that the total energy in the strong Coulomb field or weak electric field region ( $E \equiv E_w$ ) is given by

$$E_w = E_c \{1 - (\mathcal{E}/\mathcal{E}^*)\} \quad (\mathcal{E} < \mathcal{E}^*). \quad (6)$$

Since experimental observations were carried out on transitions from the ground state  $n = 1$ , we calculate the transition energies  $\omega$  between levels  $n$  and 1

<sup>‡</sup> In general  $\mathcal{E}^* \sim (m_{\text{eff}}^*/m)^2$ . For the helium problem, we have  $m_{\text{eff}}^* = m$ , which is equal to unity in our units.

and find (with the subscripts s and w again denoting strong and weak electric field regimes) that

$$\omega_s = \frac{1}{2}(3\pi\mathcal{E})^{2/3} (n^{2/3} - 1)(1 + bn^{-2/3}), \quad (7)$$

and

$$\omega_w = \frac{Z^2}{2} \left(1 - \frac{1}{n^2}\right) (1 + an^2), \quad (8)$$

where

$$b \equiv \{6Z/(3\pi)^{4/3} \mathcal{E}^{1/3}\}, \quad (9)$$

and

$$a \equiv (2 \mathcal{E}/Z^3). \quad (10)$$

Thus, a plot of transition frequency versus electric field  $\mathcal{E}$  will go linearly as  $\mathcal{E}$  initially (since  $a \sim \mathcal{E}$ ) but for stronger fields it will go as  $\mathcal{E}^{2/3}$  (the dominant term in eq. (7)). As a result, the curve will display a decrease in slope as we approach  $\mathcal{E}^*$ . In the case of liquid helium, we find (recalling that  $\mathcal{E}$  (atomic) =  $5.142 \times 10^9$  V/cm) that

$$\mathcal{E}^* = 882 n^{-4} \text{ V/cm}. \quad (11)$$

Thus, for  $n = 1, 2, 3$ , we have that  $\mathcal{E}^*$  equals 882, 55, and 11 V/cm, respectively. In other words, the present experiments (which used fields as high as 60 V/cm) are already in the strong field regime. In fact, the existing data (see fig. 2 of ref. [1]) display the decrease in slope discussed above, as well as the  $n$  dependence of the transition frequency (slope of  $\omega_w \sim n^2$  and slope of  $\omega_s \sim n^{2/3}$ ) given in eqs. (7) and (8).

We have also carried out a WKB calculation (extension of the calculation of ref. [4] to include the Coulomb potential). The result obtained is valid for all values of  $\mathcal{E}$  but unfortunately the energy must be obtained from the transcendental equation

$$n\pi = \frac{4}{3}(\mathcal{E}\lambda)^{1/2} \left\{ (\lambda - \beta)F\left(\frac{\pi}{2}, \alpha\right) + 2\beta E\left(\frac{\pi}{2}, \alpha\right) \right\}, \quad (12)$$

where

$$\beta = \frac{E}{2\mathcal{E}}, \quad \lambda = \left(\frac{Z}{\mathcal{E}} + \beta^2\right)^{1/2}, \quad \alpha = \frac{\lambda + \beta}{2\lambda},$$

and where  $F((\pi/2), \alpha)$  and  $E((\pi/2), \alpha)$  are the complete elliptic integrals [8] of the first and second kind, respectively. In the strong and weak field limits, the results for the energy exhibit the same behavior discussed above. For values of  $\mathcal{E} \approx \mathcal{E}^*$ , a numerical evaluation of eq. (12) will be necessary, the details of which will be given elsewhere, along with a detailed comparison with experimental data.

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