

Atomic ionization potentials in a plane electromagnetic wave

G. W. Ford

Department of Physics, University of Michigan, Ann Arbor, Michigan 48104

R. F. O'Connell*

Dublin Institute for Advanced Studies, Dublin, Ireland

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We point out that the well-known second-order time-dependent perturbation theory result for the level shift of an atomic system in a plane electromagnetic wave is appropriate for calculating the change in ionization potential due to the presence of the wave. The calculation is valid when the electric field strength of the wave is small compared with atomic field strengths (intensity $\ll 10^{16}$ W/cm²). We use this result to show that corresponding calculations carried out using the Kramers-Henneberger transformation yield a result which is valid only when the frequency of the plane wave is high compared with the atomic frequencies.

The range of validity of the Kramers-Henneberger transformation¹ for calculating the change in atomic ionization potentials in the presence of a plane electromagnetic wave^{2,3} is presently being debated.^{4,5} Here we point out that the appropriate way to calculate this change is by means of second-order time-dependent perturbation theory, which in turn is equivalent to the well-known Kramers-Heisenberg dispersion formula.⁶ The resulting expression for the shift in energy of the n th atomic level, using the notation and units (atomic) of Ref. 3, takes the form⁷

$$(\Delta E_b)_n = \frac{1}{2} \sum_m \frac{\omega_{nm}}{\omega_{nm}^2 - \omega^2} |\vec{E}_0 \cdot \vec{r}_{mn}|^2, \quad (1)$$

where $\omega_{nm} = E_n - E_m$. In the limit of high frequencies, i.e., $\omega \gg \omega_{nm}$, this becomes

$$(\Delta E_b)_n \rightarrow -\frac{1}{2\omega^2} \sum_m \omega_{nm} |\vec{E}_0 \cdot \vec{r}_{mn}|^2 = \frac{|\vec{E}_0|^2}{4\omega^2} \equiv \Delta E_c, \quad (2)$$

where we have used the familiar Thomas-Reiche-Kuhn sum rule,⁸

$$\sum_m \omega_{nm} |\vec{E}_0 \cdot \vec{r}_{mn}|^2 = -\frac{1}{2} |\vec{E}_0|^2. \quad (3)$$

Here ΔE_c is the shift in the energy of a free electron (continuum electron) in the plane-wave field. The change in the ionization potential is identified as $\Delta E_I = \Delta E_c - \Delta E_b$, and the cancellation in the high-frequency limit is what was referred to as "renormalization" in Ref. 3. A similar renormalization is implicit in Welton's calculation of Bethe's nonrelativistic formula for the Lamb shift.⁹ Using these results, we obtain the following expression for the shift in the ionization potential for the n th level:

$$(\Delta E_I)_n = -\frac{1}{2\omega^2} \sum_m \frac{\omega_{nm}^3}{\omega_{nm}^2 - \omega^2} |\vec{E}_0 \cdot \vec{r}_{nm}|^2. \quad (4)$$

This result is valid for all nonresonant frequencies (i.e., $\omega \neq \omega_{nm}$) and for electric field strengths which are small compared with atomic electric-field strengths (i.e., $|\vec{E}_0| \ll 5 \times 10^9$ V/cm; intensity $\ll 7 \times 10^{16}$ W/cm²).

Again in the limit of high frequencies ($\omega \gg \omega_{nm}$), the expression (4) becomes

$$\begin{aligned} (\Delta E_I)_n &\rightarrow \frac{1}{2\omega^4} \sum_m \omega_{nm}^3 |\vec{E}_0 \cdot \vec{r}_{nm}|^2 \\ &= -\frac{1}{4\omega^4} \langle (\vec{E}_0 \cdot \vec{\nabla})^2 V \rangle_n, \end{aligned} \quad (5)$$

where again we have used a familiar sum rule.⁸ But the limit (5) is exactly the formula obtained in Ref. 3 using the Kramers-Henneberger transformation. Thus, we see explicitly that it is valid only at high frequencies. This should not be too surprising since in Ref. 3 this result was derived under the assumption that the classical displacement R_0 of the electron is small compared with the atomic radius. In atomic units this condition becomes $R_0 = E_0/\omega^2 \ll 1$, which if E_0 is smaller than atomic fields [the condition of validity of the general result (4)] requires that the frequency be large, compared with atomic frequencies.

In the low-frequency regime ($\omega \ll \omega_{nm}$), the general result (4) becomes, again using the sum rule (3),

$$(\Delta E_I)_n \rightarrow \frac{E_0^2}{4\omega^2} - \frac{1}{2} \sum_{m=n} \frac{|\vec{E}_0 \cdot \vec{r}_{nm}|^2}{\omega_{nm}} + \dots \quad (6)$$

This again should not be surprising since the first term is just the shift in the free-electron energy while the second term is the well-known formula for the second-order Stark shift¹⁰ of the energy of

an atom in a static field (the factor of $\frac{1}{2}$ comes from the time average of the square of a slowly varying field).

In summary, we conclude that the use of the Kramers-Henneberger transformation for the calculation of the shift in ionization potentials is valid only for frequencies high compared with typical atomic frequencies. At such frequencies other processes, such as direct photoionization, become important, so there is some question as to whether the result is useful even where it is valid.

The result (4), however, is valid for any nonresonant frequency, provided only that the electric-field strength be small compared with typical atomic-field strengths.

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*Permanent address: Department of Physics, Louisiana State University, Baton Rouge, La. 70803.

¹W. C. Henneberger, *Phys. Rev. Lett.* **21**, 838 (1968).

²C. C. Choi, W. C. Henneberger, and F. C. Sanders, *Phys. Rev. A* **9**, 1895 (1974).

³R. F. O'Connell, *Phys. Rev. A* **12**, 1134 (1975).

⁴S. K. Vermani and B. L. Beers, *Phys. Rev. A* **12**, 715 (1975).

⁵C. K. Choi, W. C. Henneberger, S. N. Mian, and F. C. Sanders, *Phys. Rev. A* **12**, 719 (1975).

⁶H. A. Kramers and W. Heisenberg, *Z. Phys.* **31**, 681

(1925); E. Schrödinger, *Ann. Phys. (Leipz.)* **81**, 109 (1926); M. Born, W. Heisenberg, and P. Jordan, *Z. Phys. (Leipz.)* **35**, 557 (1926).

⁷For a more modern derivation of this result see, e.g., I. I. Sobel'man, *Introduction to the Theory of Atomic Spectra* (Pergamon, Oxford, 1972), pp. 272-275.

⁸H. A. Bethe and E. E. Salpeter, *Quantum Mechanics of One- and Two-Electron Atoms* (Springer, Berlin, 1957).

⁹T. Welton, *Phys. Rev.* **74**, 1157 (1948).

¹⁰See, e.g., Ref. 7, p. 263.