THE ASTROPHYSICAL JOURNAL, 199:L25–L26, 1975 July 1

RELATIVISTIC EFFECTS IN THE BINARY PULSAR PSR 1913+16

B. M. BARKER AND R. F. O'CONNELL
Department of Physics and Astronomy, Louisiana State University, Baton Rouge
Received 1975 February 24

ABSTRACT

The relativistic precession of both the spin and the periastron of the pulsar 1913+16, in the gravitational field of its companion, are analyzed.

Subject headings: binaries — black holes — neutron stars — pulsars — relativity

The discovery of the pulsar PSR 1913+16 in a binary system (Hulse and Taylor 1975) provides a unique opportunity for studying various gravitational and relativistic effects. Up to now such studies were invariably carried out on "test bodies" in the gravitational field of a "large mass" (essentially a one-body problem). We have now carried out a detailed analysis of the gravitational two-body problem with arbitrary masses, spins, and quadrupole moments (Barker and O'Connell 1975). Here, we apply these results to find the precession of the spin and precession of the orbit of PSR 1913+16 in the gravitational field of its companion.

One notable result which emerges is that, in the case of arbitrary masses m_1 and m_2, the spin-orbit contribution to the *spin* precession of body 1 is a factor $(m_2 + \mu/3)/(m_1 + m_2)$ times what it would be for a test body moving in the field of a fixed central mass $(m_1 + m_2)$. Here μ denotes the reduced mass $m_1m_2/(m_1 + m_2)$. This contrasts with the result of Robertson (1938) for the *periastron* precession where the corresponding factor is unity.

Let m_1 and m_2 denote the masses of the pulsar and its companion, respectively. Using the notation of Barker and O'Connell (1975), we write $\mathbf{S}^{(1)} \equiv S^{(1)}\mathbf{n}^{(1)}$, where $S^{(1)}$ is the spin angular momentum of the pulsar. Then the dominant contribution to the secular spin precession of the pulsar is given by

$$\dot{\mathbf{n}}_{\mathrm{av}}^{(1)} = \mathbf{\Omega}_{\mathrm{DS\ av}}^{(1)} \times \mathbf{n}^{(1)}\,, \qquad (1)$$

where $\mathbf{\Omega}_{\mathrm{DS}}^{(1)}$ denotes the so-called de Sitter geodetic (spin-orbit) general-relativistic contribution. Explicitly, we have (Barker and O'Connell 1975)

$$\mathbf{\Omega}_{\mathrm{DS\ av}}^{(1)} = \frac{3G\bar{\omega}(m_2 + \mu/3)}{2c^2a(1 - e^2)}\,\mathbf{n}\,, \qquad (2)$$

where a is the semimajor axis, $\mathbf{n}$ is a unit vector in the direction of the orbital angular momentum, e is the eccentricity, $\bar{\omega} = 2\pi/T$ is the average orbital angular velocity, and T is the orbital period. Since the *observed* elements of the orbit (Hulse and Taylor 1975) are $T = 27908$ s and $e = 0.615$, we eliminate a from equation (2) using the relation

$$\bar{\omega}^2a^3 = G(m_1 + m_2)\,. \qquad (3)$$

Thus,

$$\mathbf{\Omega}_{\mathrm{DS\ av}}^{(1)} = \frac{3G^{2/3}\bar{\omega}^{5/3}}{2c^2(1 - e^2)}\,f_{\mathrm{DS}}(m_1, m_2)\,\mathbf{n}\,, \qquad (4)$$

where

$$f_{\mathrm{DS}}(m_1, m_2) = \frac{m_2 + \mu/3}{(m_1 + m_2)^{1/3}}\,. \qquad (5)$$

Thus, for PSR 1913+16, we have

$$\mathbf{\Omega}_{\mathrm{DS\ av}}^{(1)} = (0\overset{\circ}{.}974\ \mathrm{yr}^{-1})\,g_{\mathrm{DS}}(m_1, m_2)\,\mathbf{n}\,, \qquad (6)$$

where

$$g(m_1, m_2) = \{f(m_1, m_2)/f(m_\odot, m_\odot)\} \qquad (7)$$

and $m_\odot$ is the mass of the Sun.

When $m_1 = m_2 = m$ (which Hulse and Taylor 1975 consider is not unlikely),

$$g_{\mathrm{DS}} = (m/m_\odot)^{2/3}\,. \qquad (8)$$

Finally, we note that the precession of the periastron is (Robertson 1938; Barker and O'Connell 1975)

$$\mathbf{\Omega}^{*(E)} = \frac{3G\bar{\omega}(m_1 + m_2)}{c^2a(1 - e^2)}\,\mathbf{n}\,. \qquad (9)$$

Hence, using equation (3), we have

$$\mathbf{\Omega}^{*(E)} = \frac{3G^{2/3}\bar{\omega}^{5/3}}{c^2(1 - e^2)}\,(m_1 + m_2)^{2/3}\mathbf{n}\,. \qquad (10)$$

Thus, for PSR 1913+16,

$$\mathbf{\Omega}^{*(E)} = (3\overset{\circ}{.}34\ \mathrm{yr}^{-1})\,g_E(m_1, m_2)\,\mathbf{n}\,, \qquad (11)$$

where

$$g_E = \{(m_1 + m_2)/2m_\odot\}^{2/3}\,. \qquad (12)$$

In the equal-mass case we note that $\mathbf{\Omega}^{*(E)} = (24/7)\,\mathbf{\Omega}_{\mathrm{DS\ av}}^{(1)}$, whereas in the large-mass approximation $\mathbf{\Omega}^{*(E)} = 2\mathbf{\Omega}_{\mathrm{DS\ av}}^{(1)}$.

It is clear that a measurement of $\mathbf{\Omega}^{*(E)}$ will enable one to obtain a value for $M \equiv m_1 + m_2$. In addition, a measurement of $\mathbf{\Omega}_{\mathrm{DS\ av}}$ gives a value for $f_{\mathrm{DS}}(m_1, m_2)$. Thus we can deduce values for the individual masses, viz.,

$$m_1 = M\{[4 - (3f_{\mathrm{DS}}/M^{2/3})]^{1/2} - 1\}\,, \qquad (13)$$

and $m_2 = M - m_1$. On the other hand, if a measurement of M is combined with other measurements to obtain the individual masses (Hulse and Taylor 1975), then a measurement of f_{DS} is in essence a test of general relativity.

REFERENCES

Barker, B. M., and O'Connell, R. F. 1975, *Phys. Rev. D*, in press.
Hulse, R. A., and Taylor, J. H. 1975, *Ap. J.* (*Letters*), **195**, L51.
Robertson, H. P. 1938, *Ann. Math.*, **39**, 101.

B. M. Barker and R. F. O'Connell: Department of Physics and Astronomy, Louisiana State University, Baton Rouge, LA 70803