A COMPARISON BETWEEN DEBYE-HÜCKEL SCREENING AND THE STARK EFFECT ON THE DETERMINATION OF THE LAST OBSERVABLE SPECTRAL LINE FROM A HYDROGEN-LIKE PLASMA

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An analytic expression is derived for the critical temperature which marks the boundary between the two regimes in which the disappearance of spectral lines (emitted by transitions from levels near the continuum) from a hydrogen-like plasma is caused by either the Inglis Teller effect or the Debye-Hückel screening effect.

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Spectral lines emitted by transitions from levels near the continuum may disappear within a plasma because of two effects: (1) the broadening of the lines, causing closely lying states to merge together and (2) the displacement of the higher energy levels into the continuum due to the fact that in a plasma the bound electron moves in a screened potential rather than a pure Coulomb potential. The line merging effect has been treated by Inglis and Teller [1] and others [2]. The displacement of the upper energy levels into the contiuum has been considered by various authors [3, 4]. Since these two effects are both present in a plasma, they must be considered simultaneously. This was first accomplished by Margenau and Lewis [5] who reviewed the calculations of the maximum quantum number predicted by each of the two effects and then interpreted as the decisive effect that which predicted the smallest of the two maximum quantum numbers, appropriate to a specific density and temperature.

Here we present two improvements to the initial work of ref. [5]. First we obtain an improved value of the greatest possible "principal" quantum number of a screened potential in the light of our variational results [4]. Then we combine this result with the results of Inglis and Teller to obtain *analytically*, for a specified value of the plasma density, the critical temperature marking the boundary between the two different regimes where each of the effects dominate.

The quantum number n_D of the last bound state of a particle in the Debye-Hückel potential as a function of screening length D was determined as follows. Using the variational method described in ref. [4], and recalling that, for a given n, the state with the smallest l has the largest binding energy, we have calculated the value of the critical screening length below which each of the nine lowest s states disappears. These values were then used to determine the relation between n_D and D. We found the relation

$$n_D = 1.13\sqrt{D/a_0} = \sqrt{1.27D/a_0} = \sqrt{D/0.79a_0}, \qquad (1)$$

to be accurate to about 3%, where a_0 is the Bohr radius. In particular, we note that according to eq. (1) for $D < 0.79a_0$ there is no bound state, whereas our calculated variational result is $D < 0.84a_0$. Eq. (1) is in close agreement with a similar result obtiained in ref. [6]. Since [7]

$$D = 6.9\,(T/2N_e)^{1/2}\mathrm{cm}, \qquad (2)$$

for a two component plasma of electrons and singly charged ions ($N_{ion} = N_e$, where N denotes the number density) we have

$$n_D = 2.97\,(T/2N_e\,a_0^2)^{1/4}. \qquad (3)$$

Thus,

$$4\log n_D = \log\left(\frac{38.9\,T}{a_0^2}\right) - \log N_e. \qquad (4)$$

Thelast deiscernable level for the two component singly charged plasma due to the line merging caused by the Stark effect has a quantum number n_s deter-

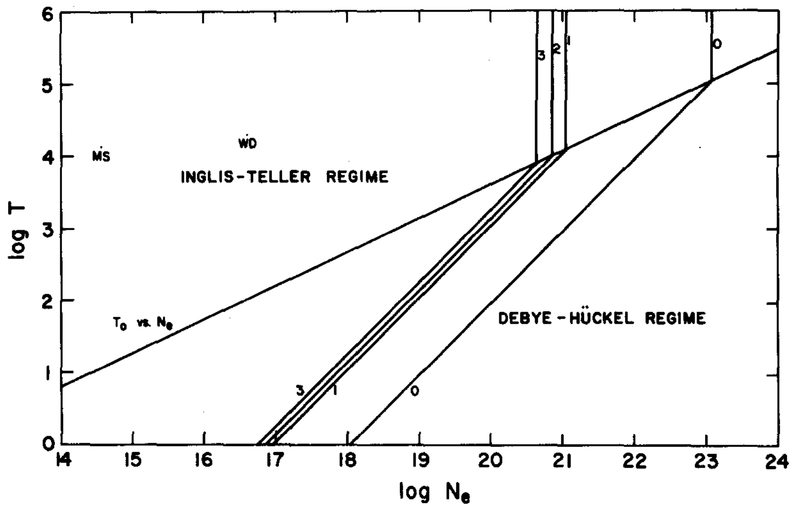

Fig. 1. The critical temperature T_0 in °K is shown as a function of N_e, the number of electrons per cc. To the right of the lines marked n (n=0, 1, 2, 3) only a maximum of n bound states exist. WD and MS denote the position of typical DA white dwarfs and early F-type main sequence stars.

mined by the electron density N_e as follows [1,2]

$$7.5 \log n_s = \log\left(\frac{2.7 \times 10^{-2}}{a_0^2}\right) - \log N_e$$

$$= 23.26 - \log N_e. \quad (5)$$

Despite the various corrections to the Inglis-Teller formula which have resulted from the modern theories of line broadening [8], the Inglis-Teller formula is still a good representation of the experimental results [2,8].

Subtracting eq. (4) from eq. (5)

$$7.5 \log n_s - 4 \log n_D = \log\left(\frac{1.32 \times 10^5}{T}\right). \quad (6)$$

Thus the critical temperature T_0 for which both effects predict the same maximum value of the "principal" quantum number n (i.e. $n_D = n_s$) occurs for

$$3.5 \log n_s = \log\left(\frac{1.32 \times 10^5}{T_0}\right). \quad (7)$$

Thus

$$T_0 = \frac{1.32 \times 10^5}{n_s^{7/2}} . \quad (8)$$

Using eq. (5), the critical temperature T_0 may be written

$$T_0 = 1.85 \times 10^{-6} N_e^{7/15} . \quad (9)$$

Therefore, for a particular density, if the temperature is less than T_0, then $n_s > n_D$. In this case the number of observed lines from a two component plasma composed of electrons and singly-charged ions is determined by the displacement of the higher levels into the continuum due to Debye-Hückel screening and not by the merging of the lines due to Stark broadening.

In fig. 1 we plot the critical temperature as a function of electron density for a two component singly charged plasma and thus show the region where each of the effects dominates. Also shown in the figure are numbered lines which denote the demarkation beyond which various bound states cease to exist. For example, at higher electron densities and lower temperatures than those of the line designated 0, there are no bound states. We see then that only for higher temperatures and lower electron densities than those given by line 2 are electric dipole transitions possible (i.e. below this region the 2p state does not exist).

In conlcusion, we have shown that for a specified density there exists a critical temperature below which the disappearance of spectral lines from a plasma is determined by the Debye-Hückel screening effect and not by the Inglis-Teller line merging effect. Furthermore, the use of the recent variational results [4] in determining n_D, eq. (1) yields a critical temperature which is 6.23 times smaller than that which would have been obtained using the n_D of ref. [5].

Finally, we would be remiss not to mention that uncertainties exist as to the use of a Debye-Hückel potential to describe the field of force in which the radiating electron of a hydrogen-like atom moves [9–11].

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