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INTERNAL MAGNETIC FIELDS OF PULSARS, WHITE DWARFS, AND OTHER STARS

R. F. O'CONNELL
Department of Physics and Astronomy, Louisiana State University
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ABSTRACT

The possibility that the very early magnetic fields in pulsars and other stars have a large number of nodes m in the radial function, has been considered by some authors. However, these analyses have not taken into account the fact that, as we shall show here, such large m values imply large internal magnetic fields. Such large fields could affect the interior dynamics and may also give rise to stability problems.

Subject headings: magnetic fields — pulsars — white dwarfs

In the analyses of stellar magnetic fields, the possibility has been considered that magnetic fields with a large number of nodes m in the radial function could exist. In particular, Chanmugam (1973) has considered the possibility that the very early pulsar fields might have m values of 2000 or more. However, all these analyses have not taken into account the fact that, as we shall prove here, such large m values imply large internal magnetic fields. Such large fields could affect the theoretical mass-radius relation in white dwarfs (Ostriker and Hartwick 1968; Mestel 1965) and the interior dynamics of pulsars (Vandakurov 1970), and they could destroy proton superconductivity in pulsars (Baym, Pethick, and Pines 1969). However, they could also give rise to stability problems.

The salient features of our argument are best displayed by assuming a constant electrical conductivity σ. It also serves to make clear that our basic conclusion is applicable to *all* stars. In the general case of a poloidal magnetic field, the only nonzero component of the electric field may be written (Wrubel 1952)

$$E_\phi = g_n(r) P_n{}^1(\mu) e^{-t/\tau}\,, \tag{1}$$

where $P_n{}^1(\mu)$ is the associated Legendre function of the first kind, $\mu = \cos\theta$ and τ is the decay time. This leads to an expression for B which depends on both $g(r)$ and its radial derivative. For constant σ, and for matching on to an external dipole field, we have (Lamb 1883)

$$g_n(x) = A(2\lambda/\pi)^{1/2} j_n(\lambda x)\,, \tag{2}$$

where j_n is the spherical Bessel function, $x = (r/R)$, R is the radius of the star, and $\lambda^2 \equiv (4\pi\sigma R^2/\tau c^2)$. For a dipolar field (Lamb 1883), $\lambda = (m+1)\pi$ (where $m = 0, 1, 2, 3, \ldots$ denotes the number of nodes of the radial function g) and hence $\tau_m = 4\sigma R^2/\pi c^2(m+1)^2$. The constant A is determined from $g_n(1)$.

Using these results, one can show that the central field $B_c = 0$ for $n > 1$. For $n = 1$ (dipolar) we obtain

$$g_1(1) = A(-1)^m (2/\pi\lambda)^{1/2}\,, \tag{3}$$

$$g_1(0) = \lim_{x\to 0} Ax(\lambda^3/2\pi)^{1/2}\,. \tag{4}$$

Further algebra leads to our basic result for the ratio of central and surface fields, viz.,

$$(B_c/B_s) \equiv N_m = [\pi^2(m+1)^2/(3\mu^2+1)^{1/2}]\,. \tag{5}$$

In particular, $B_c \sim 10(m+1)^2 B_e$, where B_e is the equatorial field. This leads to an N_0 value ~ 10 to be compared with the values of 80 and 92 derived using radially dependent σ values appropriate to the Sun (Cowling 1945; Wrubel 1952). However, the important $(m+1)^2$ dependence is expected to be a very good approximation even if we include the radial dependence of σ. This expectation is borne out by the fact that our results for (N_1/N_0) and (N_2/N_0) are within a factor of ~ 2 of the numerical results (Wrubel 1952; Chanmugam and Gabriel 1973).

We turn now to some implications of our results. For white dwarfs, we see that large interior fields (Ostriker and Hartwick 1968) are possible if m is sufficiently large, in contrast to the conclusion (Chanmugam and Gabriel 1973) based on the use of m values no greater than 2. For example, taking $B_e \sim 10^5$ gauss for a white dwarf, we see that B_e values $> 10^{11}$ gauss may be achieved with values of $(m+1) \gtrsim 300$. However, recalling that $\tau_m \alpha (m+1)^{-2}$, we see that the lifetimes of these fields are proportionally smaller. Thus the question of whether a white dwarf star has a large internal field is certainly not settled and depends on many speculative questions which we shall not go into here.

The likelihood that the magnetic field is highly turbulent after a supernova collapse leading to the formation of a white dwarf or neutron star, led Chanmugam (1973) to consider the possibility that the very early pulsar fields have m values of 2000 or more. Such large m values imply large internal fields. For example, taking $B_e \sim 10^{12}$ gauss for a pulsar, we see that an $m+1$ value $\sim 10^3$ would imply a B_c value for this component of 10^{19} gauss. Such a B_c value is so large that questions of stability arise, at least over the central portions of the star (Chandrasekhar and Fermi 1953; Tayler 1973). Even an $m+1$ value ~ 60 would imply a B_c value of 3.6×10^{16} gauss, which is equal to the critical magnetic field beyond which the protons in the pulsar interior cease to be superconducting (Baym *et al.* 1969).

To summarize, we have shown that magnetic field components with large *m* values (number of nodes in the radial function) imply large internal fields which could be relevant in discussions of the implications of models which invoke such large *m* values.

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R. F. O'Connell: Louisiana State University, Department of Physics and Astronomy, Baton Rouge, LA 70803