GRG Vol.6, No.1 (1975), pp. 99 - 102.

CAN QUANTUM GRAVITATIONAL FORCES STOP GRAVITATIONAL COLLAPSE?†

R.F. O'CONNELL

Department of Physics and Astronomy,
Louisiana State University,
Baton Rouge, Louisiana 70803

ABSTRACT

We consider, in lowest order of the gravitational coupling constant G, the gravitational potential between two neutrons. As we have previously pointed out [1], the quantum (including spin) contributions to the gravitational field dominate for distances smaller than the Compton wavelength of the neutron. At such distances the gravitational force between two neutrons may be repulsive. In particular, the gravitational forces which are analogous to the familiar Darwin and Fermi forces of quantum electrodynamics are capable of stopping gravitational collapse. Our discussion is within the framework of Einstein's theory, but on a microscopic level. We conclude that gravitational collapse may be halted without the necessity of extending Einstein's theory à la Cartan or otherwise.

Some four years ago we pointed out [1] that the gravitational force between two electrons may be repulsive at distances smaller than the Compton wavelength of the electron, and mentioned the possible relevance of this possibility to the problem of gravitational collapse (GC). In view of the recent interest [2-4] in the possibility of halting GC by extending Einstein's theory à la Cartan, we wish to return to these considerations and argue that GC may be halted without the necessity of extending Einstein's theory. This is achieved by treating the latter theory on a microscopic level.

We consider, in the lowest order of the gravitational coupling constant G, the gravitational potential between two neutrons (regarded, for our present discussion, as point particles). We have

† Presented at the Seventh International Conference on Gravitation and Relativity, June 1974, Tel-Aviv, Israel.

previously noted [5,6] the advantage of formulating our discussion in terms of what we called the gravitational Breit-type interaction (GBI).

The electromagnetic interaction between two electrons, which is accurate to first order in α^2 (where $\alpha^2 = e^2/\hbar c$), with inclusion of relativistic and spin effects, was derived by Breit [7] from quantum electrodynamic field theory. His result for the potential may be written, in the notation of reference [6], as

$$V_{EM} = \frac{e^2}{r}\{1 + U_R^{(1)} + U_R^{(2)} + U_{SO} + U_{SS} + U_{Darwin} + U_{Fermi}\}, \qquad (1)$$

where (in the centre of mass system with $\vec{P}_1 = -\vec{P}_2 = \vec{P}$ and $\vec{L} = (\vec{r}\times\vec{P})/m$)

$$U_R^{(1)} = \frac{P^2}{2m^2}, \qquad U_R^{(2)} = \frac{(\vec{P}\cdot\vec{r})^2}{2r^2m^2}, \qquad (2a)$$

$$U_{SO} = -\frac{3}{4}\frac{1}{r}\frac{\lambda\!\!\!^{-}_c}{r}[\vec{\sigma}^{(1)}\cdot\vec{L} + \vec{\sigma}^{(2)}\cdot\vec{L}], \qquad (2b)$$

$$U_{SS} = -\frac{1}{4}\left(\frac{\lambda\!\!\!^{-}_c}{r}\right)^2[3(\vec{\sigma}^{(1)}\cdot\hat{r})(\vec{\sigma}^{(2)}\cdot\hat{r}) - \vec{\sigma}^{(1)}\cdot\vec{\sigma}^{(2)}], \qquad (2c)$$

$$U_{Darwin} = -\pi r\lambda\!\!\!^{-}_c{}^2\delta(\vec{r}), \qquad (2d)$$

and

$$U_{Fermi} = -\frac{2}{3}\pi r\lambda\!\!\!^{-}_c{}^2(\vec{\sigma}^{(1)}\cdot\vec{\sigma}^{(2)})\delta(\vec{r}), \qquad (2e)$$

are the relativistic, spin-orbit, spin-spin, Darwin, and Fermi contributions to the potential, respectively. The Darwin term is generally considered to be due to the zitterbewegung of the electron and arises from the use of a relativistic theory. The Fermi term gives rise to the well-known hyperfine splitting.

The corresponding gravitational potential V_g may be written as [5]

$$V_g = -\frac{Gm^2}{r}\{1 + 14U^{(1)} + \frac{7}{3}U_{SO} + U_{SS} + 7U_{Darwin} + U_{Fermi}\}. \qquad (3)$$

In particular, we see that the gravitational quadratic spin terms may be obtained from the corresponding electromagnetic terms by simply letting $e^2 \rightarrow -GM^2$. We note that the Darwin term and both quadratic spin terms (and the spin-orbit term when the velocities are relativistic) are essentially of the same order of magnitude, and comparable to the Newtonian term for $r < \lambda_c$.

The magnitude and sign of the overall force will, of course, depend on the spin orientations. In the special case of a Friedmann universe with random spin orientation we expect that all the quantum corrections will average to zero except the Darwin term. The latter is spin-independent and repulsive, and capable of overcoming the Newtonian attraction for $r \leqslant \lambda_c$. In other words, gravitational collapse may be halted by quantum forces — and without the necessity of having spin alignment.

We remark that the Fermi spin-spin contact term is similar in form to the term which arises from using Cartan's extension of Einstein's theory to introduce torsion. We emphasize that our discussion is within the framework of Einstein's theory but on a microscopic level — V_g is obtained from a consideration of one-graviton exchange in the gravitational scattering of two neutrons, similar to the way V_{EM} is obtained from a consideration of one-photon exchange in Møller scattering. However, the form of the spin squared term in the modified Friedmann equation in Cartan theory, ΔE_c say, is unusual not only in the sense that it is of order G^2 but also that it is of order N^2 (N = total number of baryons in the universe). The form of the corresponding modification due to the Darwin or Fermi terms, ΔE say, is of first order in both G and N. In fact

$$\frac{\Delta E_c}{\Delta E} \sim N\left(\frac{r_s}{r}\right), \tag{4}$$

where r_s is the Schwarzchild radius.

A measure of confidence in the correctness of V_g is achieved from the following consideration. Barker and O'Connell [5,6] used equations (2,3) as a starting-point to obtain the gravitational interaction between macroscopic spinning bodies. From the latter they obtained results for (a) the Lense-Thirring contribution to the precession of perihelion, and (b) the precession of a gyroscope in the field of a rotating body. These results are in complete agreement with those obtained by using Einstein's equations supplemented by the Corinaldesi-Papapetrou [8] equations of motion of a spinning body in a gravitational field. Thus, at least, the correctness of the U_{SO} and U_{SS} gravitational terms are confirmed.

To conclude, we feel that our analysis makes feasible the idea that gravitational collapse may be halted simply by including the quantum forces which arise from a microscopic treatment of Einstein's equation. Ultimately, of course, one would like to carry out a more exact calculation (the considerations above were confined to order G) and also take account of the finite size of neutrons.

ACKNOWLEDGMENTS

The author would like to acknowledge the stimulation and benefit he received from an extended correspondence with Dr. F.W. Hehl. He would also like to thank Professors D. Sciama and A. Trautman for helpful communications.

REFERENCES

1. O'Connell, R.F. (1970). *Phys. Lett.*, **32A**, 402.
2. Trautman, A. (1973). *Nature Phys. Sci.*, **242**, 7; Kopczyński, W. (1972). *Phys. Lett.*, **39A**, 219.
3. Isham, C.J., Salam, A. and Strathdee, J. (1973). *Nature Phys. Sci.*, **244**, 82.
4. Heyl, F.W. (1973). *Gen. Rel. Grav.*, **4**, 333; Heyl, F.W., Van der Heyde, P. and Kerlick, G.D. (preprint).
5. Barker, B.M. and O'Connell, R.F. (1970). *Phys. Rev.*, **D2**, 1428.
6. O'Connell, R.F. (1974). In *Experimental Gravitation: Proceedings of The International School of Physics "Enrico Fermi", Course 56*, (ed. B. Bertotti), (Academic Press, New York), pp. 496-514.
7. Breit, G. (1929). *Phys. Rev.*, **34**, 553; (1932). *Phys. Rev.*, **39**, 616; Berestetskii, V.B., Lifshitz, E.M. and Pitaevskii, L.P. (1971). *Relativistic Quantum Theory*, (Cambridge University Press), p. 284.
8. Papapetrou, A. (1951). *Proc. Roy. Soc.*, **A209**, 248; Corinaldesi, E. and Papapetrou, A. (1951). *Proc. Roy. Soc.*, **A209**, 259.