THE ASTROPHYSICAL JOURNAL, 190:741–742, 1974 June 15

ENERGY SPECTRUM OF HYDROGEN-LIKE ATOMS IN A STRONG MAGNETIC FIELD

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Received 1973 December 11

ABSTRACT

An expression for the energy spectrum of hydrogen-like atoms in strong magnetic fields is derived in terms of the energy spectrum of hydrogen. The development of a parametric expression for the latter is thus motivated. We present such an expression for the ground-state energy of hydrogen in magnetic fields $B \simeq 10^7$–10^{12} gauss, obtained by least-squares fittings of numerical results obtained previously from a variational calculation.
Subject headings: magnetic fields — Zeeman effect

The probable existence of strong magnetic fields in some white dwarfs and in pulsars motivated a variational calculation of the energy spectrum of hydrogen in magnetic fields $B \simeq 10^7$–10^{12} gauss (Smith *et al.* 1972). We will now show that the energy spectrum of hydrogen-like atoms in a magnetic field may be simply obtained from the energy spectrum of hydrogen.

The Hamiltonian for a hydrogen-like atom in a magnetic field B oriented along the z-axis is (neglecting spin)

$$H(Z, B) = \frac{p^2}{2\mu} - \frac{Ze^2}{r} + \tfrac{1}{2}\mu\omega m + \hbar\omega^2 r^2 \sin^2\theta\,, \quad (1)$$

where $\omega = eB/2\mu c$ is the Larmor frequency, m is the z-component of the angular momentum and Z is the atomic number. When $B = 0$, a common choice for the units of length and energy are a_0 (the Bohr radius) and $E_0(2Ry)$, respectively. For $B \neq 0$, it is more convenient to choose units (a_0/Z) and Z^2E_0, so that

$$H(Z, B) = \frac{p^2}{2} - \frac{1}{r} + \frac{m}{2}\gamma + \tfrac{1}{8}\gamma^2 r^2 \sin^2\theta\,, \quad (2)$$

where

$$\gamma \equiv (B/B_0{}^*) \equiv \frac{1}{Z^2}(B/B_0)\,,$$

$$B_0 \equiv (\mu^2 ce^3/\hbar^3) = 2.350 \times 10^9 \text{ gauss}\,. \quad (3)$$

With this choice of units, we note that the right-hand side of equation (2) has no explicit Z dependence and thus we have a similar Schrödinger differential equation to solve for all hydrogen-like atoms. It follows immediately that $E(Z, B)$, the energy of a hydrogen-like atom of atomic number Z, in a magnetic field B, may be written in terms of the corresponding quantity for hydrogen, as follows:

$$E(Z, B) = Z^2E(1, B')\,, \quad (4)$$

where

$$B' \equiv (B/Z^2)\,. \quad (5)$$

This is our basic result. Since we have already obtained the energy spectrum of the 14 lowest states of hydrogen (Smith *et al.* 1972), thus in principle we have the corresponding results for all hydrogen-like atoms. In practice, it is clear that the extraction of such results would be greatly facilitated by development of parametric expressions for the numerically derived hydrogen spectrum in strong fields. Here we display such a development for the important case of the ground-state energy $E^{(0)}(1, B)$. A parametric expression for $E^{(0)}(1, B)$ is developed by a least-squares fitting of the numerical results obtained by Smith *et al.* (1972). For $10^7 \leqslant B \leqslant 10^{10}$ gauss, we obtain

TABLE 1

PARAMETERS APPEARING IN EQUATIONS (6) AND (7) FOR THE GROUND-STATE ENERGY OF HYDROGEN IN A MAGNETIC FIELD

A_1.......	−1.00000	D_1........	+0.236002
A_2.......	+0.415546	D_2........	−0.791129
A_3.......	−0.0783126	D_3........	+7.31174
A_4.......	+0.150095	D_4........	−4.13939
A_5.......	−0.0250024	D_5........	+1.75430

$$E^{(0)}(1, B) = A_1 + A_2\Omega^{3/2} + A_3\Omega^2 + A_4\Omega\log\Omega + A_5\Omega^{1/2}(\log\Omega)^2 \quad (6)$$

and for $10^{10} \leqslant B \leqslant 7 \times 10^{11}$ gauss (the upper limit is probably much higher than this) we get

$$E^{(0)}(1, B) = D_1\Omega\log\Omega + D_2(\log\Omega)^2 + D_3(\log\Omega)^3 + D_4(\log\Omega)^4 + D_5(\log\Omega)^5\,, \quad (7)$$

where

$$\Omega \equiv B/B_0\,, \quad (8)$$

and $E^{(0)}(1, B)$ is now in rydbergs.

The accuracy of the fit is better than 0.15 percent. The constants $A_1, A_2, \ldots, D_1, D_2\ldots$ are given in table 1. We note that perturbation theory results are valid for small B fields. For the particular case of He II, values given by equations (4)–(7) are identical to those obtained by a variational calculation (Surmelian and O'Connell 1973).

Finally, we note from equation (5) that as Z increases the upper limit of the range of the magnetic fields where equations (6) and (7) are valid is raised by a factor of Z^2. In figure 1 we present the ionization energy $I(Z, B) = \hbar\omega - E(Z, B)$ as a function of Z for B values equal to 10^8 and 10^{12} gauss.

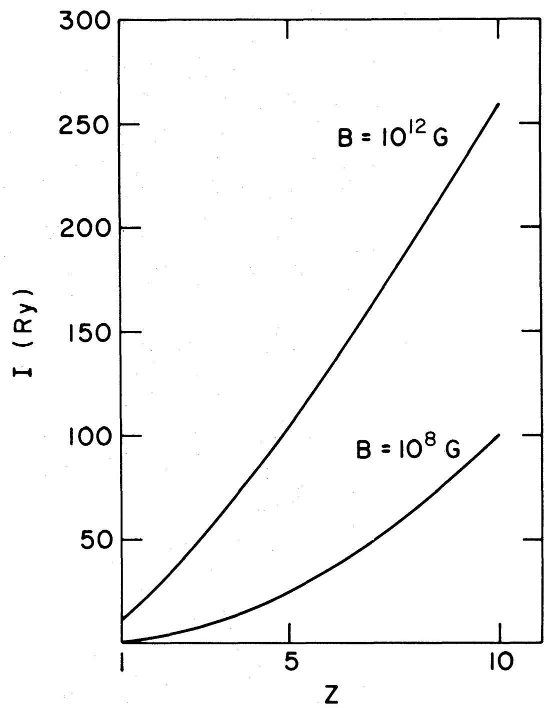

FIG. 1.—The ionization energy of the ground state of hydrogen-like atoms as a function of Z for magnetic-field values B of 10^8 and 10^{12} gauss.

REFERENCES

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