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HIGHLY EXCITED STATES OF ATOMS IN A MAGNETIC FIELD*

R. F. O'CONNELL†
Lawrence Livermore Laboratory, University of California, Livermore
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ABSTRACT

The Zeeman effect in the Ba I absorption spectrum has been observed by Garton and Tomkins. One striking feature of the spectrum they obtained is the existence of a sequence of σ lines, extending across the zero-field series limit into the continuum, with a regular spacing of about $1.5\hbar\omega$, where ω is the cyclotron frequency. A semiclassical explanation is given for this result.

Subject headings: atomic processes — Zeeman effect

The importance of the quadratic Zeeman effect for the study of electrons in solids in a magnetic field has been recognized for many years. Recently, its relevance to the study of atoms in magnetic white dwarfs and pulsars has become apparent. In most laboratory investigations of the Zeeman effect its influence has been negligible, though its importance for highly excited states did not escape notice (Van Vleck 1932).

Recently, Garton and Tomkins (1969) investigated this quadratic term by observing the Zeeman effect in the Ba I absorption spectrum. Their measurements extended to as high as $n = 75$ and they used a magnetic field of 2.4×10^4 gauss. One of the most striking features of the fascinating spectrum they obtained is the existence of a sequence of σ lines, extending across the zero-field series limit into the continuum, with a regular spacing of about $1.5\hbar\omega$, where ω is the cyclotron frequency. We recall that the Landau spacing of the energy levels of an electron in a pure magnetic field is $\hbar\omega$.

Our purpose in this communication is to present a semiclassical explanation for the origin of the $1.5\hbar\omega_c$ spacing. Now the corresponding n values are so large in this region of the spectrum that the motion of the excited electron is dominated by the magnetic field. For the case where the magnetic field cannot be treated as a perturbation on the Coulomb field, it is well known that either numerical or approximate solutions to the complete quantum-mechanical problem are necessary. A numerical approach, based on the use of variational techniques, has actually been used (Smith *et al.* 1972) to obtain the spectrum of the lower levels. However, the numerical work involved is prohibitive for the problem in hand, and hence we resort to a semiclassical approach.

Since we are focusing attention on the σ spectrum, we confine our attention to the motion of a single atomic electron in a plane perpendicular to the magnetic field B. In the absence of a Coulomb field, the energy of a nonrelativistic electron, E_m say, is given by (Sokolov and Ternov 1968)

$$E_m = n\hbar\omega\,, \tag{1}$$

where the magnetic quantum number $n = 0, 1, 2, \ldots$, and the Pauli spin contribution has been included. The corresponding radius of the "true circular orbit" is (Sokolov and Ternov 1968)

$$R_m = (2nB_0/B)^{1/2}a_0\,, \tag{2}$$

where

$$B_0 \equiv (m^2ce^3/\hbar^3) = 2.350 \times 10^9 \text{ gauss}\,, \tag{3}$$

a_0 is the Bohr radius, and m is the mass of the electron. In the presence of both magnetic and Coulomb fields, we write the energy E as

$$E = E_m + E_c\,, \tag{4}$$

where E_c is the Coulomb field contribution. For large n values, the influence of the magnetic field dominates and so we take the effect of the Coulomb field on the orbit radius to be negligible; i.e., we write

$$E_c = -Ze^2/R_m\,. \tag{5}$$

Noting that

$$e^2/a_0 = \hbar\omega(B/B_0)^{-1}\,, \tag{6}$$

we thus obtain

$$E = \hbar\omega[n - n^{-1/2}Z(2B/B_0)^{-1/2}]\,. \tag{7}$$

Hence

$$\frac{\delta E}{\delta n} = \frac{1}{n}\,[E_m - (E_c/2)]\,. \tag{8}$$

Finally, using equation (4) to eliminate E_c, we get

$$\delta E/\delta n = \tfrac{3}{2}\hbar\omega - (E/2n)\,. \tag{9}$$

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† Permanent address: Department of Physics and Astronomy, Louisiana State University, Baton Rouge, Louisiana 70803.

Thus, for $E \approx 0$, it follows that the spacing between levels is $1.5\hbar\omega$, in agreement with the results of Garton and Tomkins. That the agreement is so good is actually somewhat surprising since our approach should give best results for large E values. We have also treated the Ba valence electron as moving in the field of a hydrogenlike atom of nuclear charge Z. For $E \simeq 0$, our result for the level spacing is independent of Z.

As a final remark, we note that equation (8) predicts that the spacing decreases linearly with increasing E. It would be interesting to check this point experimentally. After this work was completed, Dr. B. Shore made me aware of papers by Edmonds (1970) and Starace (1973) who, utilizing a semiclassical calculation within the framework of the adiabatic approximation, *numerically* integrated the Bohr-Sommerfeld condition to get spacings of $1.58\hbar\omega$ and $1.5\hbar\omega$, respectively, at $E \simeq 0$.

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REFERENCES

Edmonds, A. R. 1970, *J. Phys.*, **31**, C4–71.
Garton, W. R. S., and Tomkins, F. S. 1969, *Ap. J.*, **158**, 839.
Smith, E. R., Henry, R. J. W., Surmelian, G. L., O'Connell, R. F., and Rajagopal, A. K. 1972, *Phys. Rev.*, **D6**, 3700.
Sokolov, A. A., and Ternov, I. M. 1968, *Synchrotron Radiation* (New York: Pergamon Press), pp. 72–73.
Starace, A. F. 1973, *J. Phys.*, **B6**, 585.
Van Vleck, J. H. 1932, *The Theory of Electric and Magnetic Susceptibilities* (Oxford: Oxford University Press).