ENERGY SPECTRUM OF He II IN A STRONG MAGNETIC FIELD AND BOUND-BOUND TRANSITION PROBABILITIES

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Abstract. The ground state energy of the He II (singly ionized helium) atom is determined in magnetic fields up to 10^{12} G. The 13 lowest excited states and bound-bound transition probabilities are calculated in magnetic fields from 10^7 to 10^9 G.

The discovery of strong magnetic fields in pulsars and white dwarfs (Kemp, 1970b; Kemp *et al.*, 1970) has led to an investigation of the properties of atoms and ions in magnetic fields so strong that conventional perturbation theory is inadequate. The ground state energy of hydrogen has now been determined for fields up to 10^{12} G (Rajagopal *et al.*, 1972; Smith *et al.*, 1972). The lowest excited states and the corresponding bound-bound transition probabilities for hydrogen in the region 10^7–10^8 G have also been calculated (Smith *et al.*, 1972, 1973).

In this paper we present a similar calculation of the ground state energy of singly ionized helium in fields of up to 10^{12} G; and, in addition, the first 13 excited states and bound-bound transition probabilities in fields of 10^7–10^9 G.

The Hamiltonian for a hydrogen-like ion of charge Z in a magnetic field B oriented along the z-axis is (neglecting spin)

$$H_0 = \frac{p^2}{2\mu} - \frac{Ze^2}{r} + \omega_L L_z + \tfrac{1}{2}\mu\omega_L^2 r^2 \sin^2\theta\,, \tag{1}$$

where

$$\omega_L = \frac{eB}{2\mu c}\,.$$

As previously pointed out (Rajagopal *et al.*, 1972) the eigenstates can be labelled by the eigenvalues of L_z and the parity. The trial solution is of the form

$$\psi_m^{\pm}(\mathbf{r}) \equiv \psi_t = \sum_{il} \{a_i^{(t)} r^l + b_i^{(t)} r^{l+1}\}\, e^{-\beta_i^{(t)} r}\, Y_{lm}(\theta, \phi)\,, \tag{2}$$

where $l=0, 2, 4 \ldots$ for even parity, $l=1, 3, 5 \ldots$ for odd parity, the $\beta^{(t)}$'s are the variational parameters and $a^{(t)}$, $b^{(t)}$ the coefficients determined in the calculation.

Figure 1 gives the ionization energy of the ground state of He II as a function of the magnetic field B (curve (a)), as compared to that using perturbation theory (curve (b)).

In Figure 2 we present the energy spectrum for the 13 lowest excited states for

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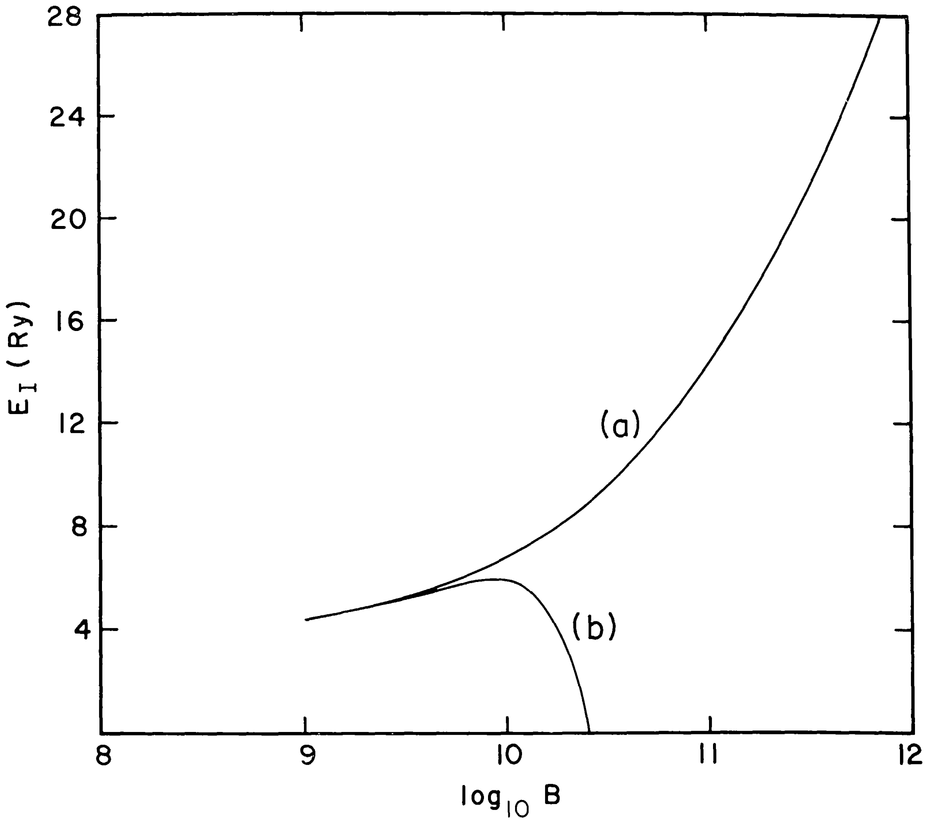

Fig. 1. The ionization energy of the ground state of HeII as a function of the magnetic field B (curve (*a*)), as compared to that using perturbation theory (curve (*b*)).

fields from 10^6 to 10^9 G. The labelling corresponds to the hydrogenic energy levels in the absence of a magnetic field. The calculation of the transition probabilities is similar to that of hydrogen (Smith *et al.*, 1973).

In the electric dipole approximation, the probability per unit time for an atom to undergo a transition from state m to m' and emit radiation of frequency $\omega_{m'm} = (E_m - E_{m'})/\hbar$ in the polarization direction $\hat{e}_q$ into a solid angle $\mathrm{d}\Omega$ is (cf. Smith *et al.*, 1973)

$$A_{m'm}\,\mathrm{d}\Omega = \frac{e^2\hbar}{2\pi\mu^2 c^3}\,\omega_{m'm}\left|\langle m'|\left(\nabla + \frac{i\mu}{\hbar}\,\omega_L\hat{B}\times\mathbf{r}\right)\cdot\hat{e}_q|m\rangle\right|^2\mathrm{d}\Omega\,, \tag{3}$$

where

$$\hat{e}_{\pm 1} = \mp\frac{1}{\sqrt{2}}\,(\hat{e}_x \pm i\hat{e}_y)\,;\ \hat{e}_0 = \hat{e}_z\,.$$

We may rewrite $A_{m'm}$ in terms of the dipole length matrix element $\mathbf{R}_{m'm} = \langle m'|\mathbf{r}|m\rangle$ as

$$A_{m'm}\,\mathrm{d}\Omega = \frac{e^2}{2\pi\hbar c^3}\,\omega^3_{m'm}\,|\mathbf{R}_{m'm}\cdot\hat{e}_q|^2\,\mathrm{d}\Omega\,, \tag{4}$$

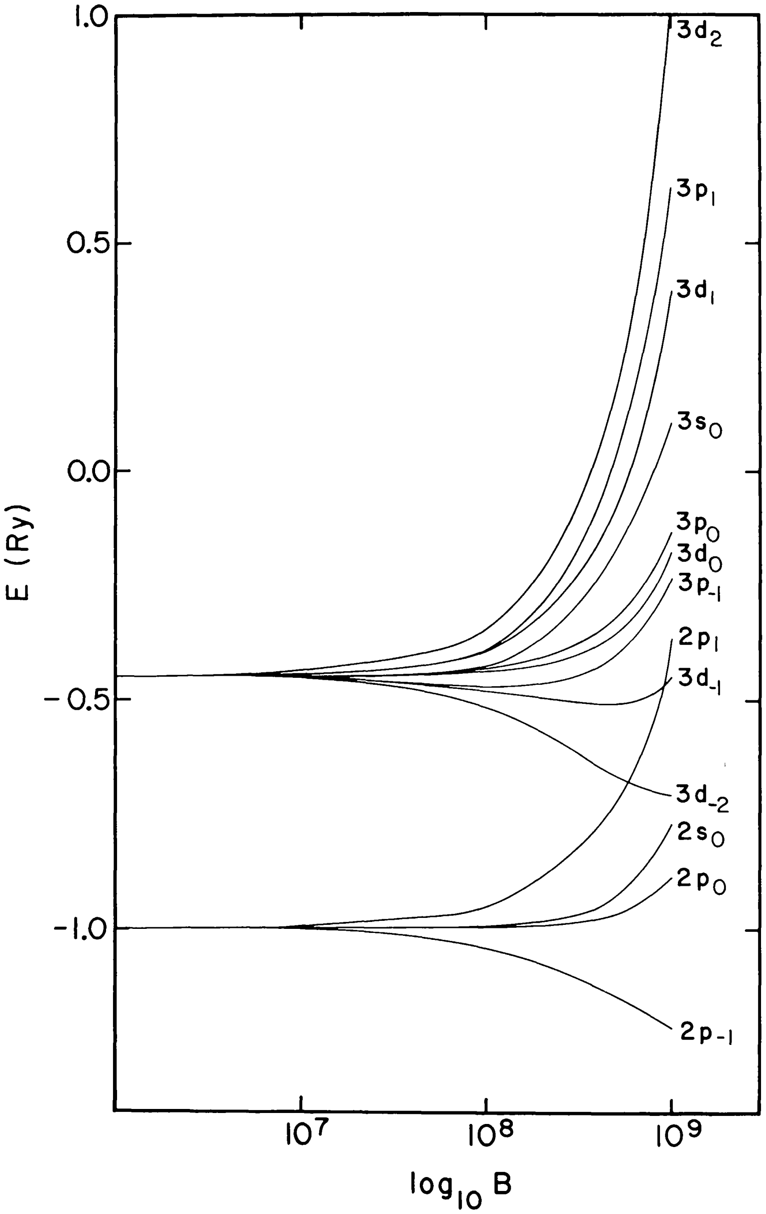

Fig. 2. The energy spectrum for the 13 lowest excited states for fields from 10^6 to 10^9 G.

For spontaneous emission in the dipole approximation we have two of the usual selection rules: parity change and $\Delta m = 0, \pm 1$. Using (1) and (2) we have calculated transition probabilities in the dipole length and momentum approximations. As in the case of hydrogen, we find that with the exception of a few cases, where the energy level seperations are small, the results are the same. Tables I, II, III give the wavelengths, in Å or μ, on the first row and, $A_{m'm}$ in $10^8\ \mathrm{s}^{-1}$, in the dipole length approximation on the second row; and Table IV gives $A_{m'm}$ in the dipole length and momentum approximations (whenever there is a difference), for fields of 10^7, 10^8, 10^9 G.

TABLE I

Transition probability $A_{m'm}$ from Equation (4) for $B = 10^7$ G

		$2p_{-1}$	$2p_0$	$2p_1$	$3p_{-1}$	$3p_0$	$3p_1$
$1s_0$	Wavelength	303.16 Å	302.73 Å	302.30 Å	255.72 Å	255.42 Å	255.11 Å
	$A_{m'm}$ [$10^8 s^{-1}$]	11.9	12.0	12.0	3.19	3.20	3.21
$2s_0$	Wavelength	21.347 μ	5028.0 μ	21.392 μ	1647.0 Å	1634.6 Å	1622.0 Å
	$A_{m'm}$ [$10^8 s^{-1}$]	0.554(−6)[a]	0.424(−13)	0.551(−6)	0.420	0.430	0.440
$3d_{-2}$	Wavelength	1647.1 Å			21.167 μ		
	$A_{m'm}$ [$10^8 s^{-1}$]	1.21			0.256(−5)		
$3d_{-1}$	Wavelength	1634.6 Å	1647.2 Å		1118.0 μ	21.369 μ	
	$A_{m'm}$ [$10^8 s^{-1}$]	0.619	0.605		0.868(−11)	0.124(−5)	
$3d_0$	Wavelength	1622.3 Å	1634.7 Å	1647.3 Å	21.968 μ	2628.7 μ	20.803 μ
	$A_{m'm}$ [$10^8 s^{-1}$]	0.115	0.832	0.110	0.187(−7)	0.196(−11)	0.221(−7)
$3s_0$	Wavelength	1621.9 Å	1634.3 Å	1646.9 Å	21.298 μ	951.96 μ	21.441 μ
	$A_{m'm}$ [$10^8 s^{-1}$]	0.137	0.329(−1)	0.131	0.374(−5)	0.150(−10)	0.367(−5)
$3d_1$	Wavelength		1622.1 Å	1634.6 Å		21.369 μ	1118.0 μ
	$A_{m'm}$ [$10^8 s^{-1}$]		0.633	0.619		0.124(−5)	0.868(−11)
$3d_2$	Wavelength			1622.1 Å			21.575 μ
	$A_{m'm}$ [$10^8 s^{-1}$]			1.27			0.241(−5)

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[a] $1.0(-1) = 1.0 \times 10^{-1}$

TABLE II

Transition probability $A_{m'm}$ from Equations (4) for $B = 10^8$ G

		$2p_{-1}$	$2p_0$	$2p_1$	$3p_{-1}$	$3p_0$	$3p_1$
$1s_0$	Wavelength	306.83 Å	302.62 Å	298.26 Å	257.39 Å	254.89 Å	251.34 Å
	$A_{m'm}$ $[10^8 \mathrm{s}^{-1}]$	11.6	12.0	12.6	3.30	3.31	3.54
$2s_0$	Wavelength	2.1147 μ	50.610 μ	2.1597 μ	1728.2 Å	1621.2 Å	1487.6 Å
	$A_{m'm}$ $[10^8 \mathrm{s}^{-1}]$	0.565(−3)[a]	0.416(−7)	0.530(−3)	0.372	0.448	0.584
$3d_{-2}$	Wavelength	1739.4 Å			1.9601 μ		
	$A_{m'm}$ $[10^8 \mathrm{s}^{-1}]$	1.04			0.294(−2)		
$3d_{-1}$	Wavelength	1619.6 Å	1747.9 Å		11.785 μ	2.1419 μ	
	$A_{m'm}$ $[10^8 \mathrm{s}^{-1}]$	0.644	0.512		0.738(−5)	0.113(−2)	
$3d_0$	Wavelength	1513.8 Å	1625.3 Å	1763.7 Å	2.8861 μ	28.137 μ	1.6966 μ
	$A_{m'm}$ $[10^8 \mathrm{s}^{-1}]$	0.142	0.847	0.898(−1)	0.964(−5)	0.162(−5)	0.474(−4)
$3s_0$	Wavelength	1483.2 Å	1590.1 Å	1722.3 Å	2.0709 μ	9.9171 μ	2.2073 μ
	$A_{m'm}$ $[10^8 \mathrm{s}^{-1}]$	0.186	0.394(−1)	0.119	0.373(−2)	0.130(−4)	0.308(−2)
$3d_1$	Wavelength		1502.1 Å	1619.6 Å		2.1320 μ	11.785 μ
	$A_{m'm}$ $[10^8 \mathrm{s}^{-1}]$		0.807	0.644		0.114(−2)	0.738(−5)
$3d_2$	Wavelength			1495.9 Å			2.3488 μ
	$A_{m'm}$ $[10^8 \mathrm{s}^{-1}]$			1.64			0.171(−2)

[a] $1.0(-1) = 1.0 \times 10^{-1}$.

TABLE III

Transition probability $A_{m'm}$ from Equation (4) for $B = 10^9$ G

		$2p_{-1}$	$2p_0$	$2p_{+1}$	$3p_{-1}$	$3p_0$	$3p_{+1}$
$1s_0$	Wavelength	328.22 Å	293.93 Å	251.09 Å	242.21 Å	236.40 Å	197.45 Å
	$A_{m'm}$ [10^8 s^{-1}]	11.1	14.5	24.8	3.23	4.01	5.97
$2s_0$	Wavelength	2025.07 Å	7226.1 Å	2261.1 Å	1700.0 Å	1449.6 Å	656.10 Å
	$A_{m'm}$ [10^8 s^{-1}]	0.371	0.153 (− 1)[a]	0.267	0.260	0.886	4.52
$3d_{-2}$	Wavelength	1791.6			1909.4 Å		
	$A_{m'm}$ [10^8 s^{-1}]	0.796			0.319		
$3d_{-1}$	Wavelength	1197.9 Å	2085.5 Å		4046.5 Å	2867.6 Å	
	$A_{m'm}$ [10^8 s^{-1}]	1.51	0.262		0.226	0.525 (−1)	
$3d_0$	Wavelength	867.35 Å	1272.5 Å	4873.9 Å	1.6889 μ	2.3592 μ	1140.6 Å
	$A_{m'm}$ [10^8 s^{-1}]	0.903	1.18	0.525 (−2)	0.393 (−3)	0.301 (−2)	1.28
$3s_0$	Wavelength	687.22 Å	909.20 Å	1926.0 Å	2679.1 Å	3681.1 Å	1777.3 Å
	$A_{m'm}$ [10^8 s^{-1}]	1.21	8.24	0.551 (−1)	0.323 (−1)	0.450	0.111
$3d_{+1}$	Wavelength		706.51 Å	1197.9 Å		1703.0 Å	4046.5 Å
	$A_{m'm}$ [10^8 s^{-1}]		6.73	1.51		0.251	0.226
$3d_{+2}$	Wavelength			669.31 Å			2426.1 Å
	$A_{m'm}$ [10^8 s^{-1}]			15.3			0.155

[a] 1.0 (−1) = 1.0 × 10^{-1}.

TABLE IV

Transition probability $A_{m'm}$ [10^8 s^{-1}] from Equations (3) and (4) for $B = 10^9$G, in the dipole length (L) and the dipole momentum (p) approximations.

Transition	L	P
$3d_{-1}$–$3p_0$	0.525 (−1)	0.509 (−1)
$3d_0$–$3p_{-1}$	0.396 (−3)[a]	0.376 (−3)
$3d_0$–$3p_0$	0.301 (−2)	0.352 (−2)
$3d_0$–$3p_1$	1.28	1.22
$3s_0$–$3p_{-1}$	0.323 (−1)	0.620 (−2)
$3s_0$–$3p_0$	0.450	0.400
$3s_0$–$3p_1$	0.111	0.212 (−1)
$3d_1$–$3p_0$	0.251	0.243

[a] 1.0 (−1) = 1.0 × 10^{-1}.

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