PLANETS, STARS AND NEBULAE

studied with photopolarimetry

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COMPUTATION OF STRONG MAGNETIC FIELDS IN WHITE DWARFS

R. F. O'CONNELL
Louisiana State University

Our work on the properties of atoms, ions, and electrons in the presence of a strong magnetic field is briefly surveyed.

The discovery of polarized radiation from certain white dwarfs has brought the study of white dwarfs into prominence once again. So far, a total of four white dwarfs has been found to exhibit a fractional polarization[1]

$$q \equiv [P_+(\omega) - P_-(\omega)]/[P_+(\omega) + P_-(\omega)], \qquad (1)$$

where $P_\pm(\omega)$ are the intensities of right and left circularly polarized light of angular frequency ω. As far as I am aware, no attempt has been made to explain the observations other than by invoking the presence of a magnetic field in the white dwarf.[2]

In his initial pioneering work, Kemp (1970) predicted that

$$q \simeq -(\Omega/\omega), \text{ if } \Omega \ll \omega \qquad (2)$$

where $\Omega = (eB/2\mu c)$ is the Larmor frequency, and where B and μ denote the magnetic field and electron mass, respectively. From the observed q values, it was deduced that B in all cases is in the range 10^6–10^8 gauss. However, it soon became apparent that the spectral dependence of q is rather complicated and not in conformity with the predictions of Kemp's model.[3] Attempts made to improve the theoretical predictions by taking into account radiative transfer and by a more exact treatment of Kemp's model have met with only limited success. A detailed report of this work has recently been given (O'Connell 1972) and thus will not be repeated here. As a result, it became clear that the way to proceed was by a detailed examination of the properties of atoms, ions, and electrons

[1]See p. 981. [2]Also see p. 54. [3]See p. 982.

in the presence of a strong magnetic field. The progress already made in this direction is also summarized by O'Connell (1972). In particular, the energy spectrum of the hydrogen atom in a strong magnetic field has been obtained (Smith et al. 1972). In addition, the theory of transition probabilities in a strong magnetic field has been formulated and applied to bound-bound transitions in the hydrogen atom (Smith et al. 1973). Similar calculations have been carried out for He II (Surmelian and O'Connell 1973).

The thrust of our present efforts is mainly two-pronged: (a) calculation of photoionization in a strong magnetic field, and (b) calculation of the quadratic Zeeman effect (QZE) to an accuracy greater than that given by perturbation theory, when the magnetic field is large.

The photoionization calculation is being carried out by Henry, Roussel, and O'Connell. We believe that this process will be the basic ingredient of a realistic physical model that will explain the many intriguing facets of the polarized radiation from magnetic white dwarfs. Introducing several simplifying assumptions, we have obtained analytic results that give a q value with the same spectral dependence as predicted by Kemp's model. We expect to obtain a more complicated—and hence, in the present context, more interesting—q dependence on wavelength when we include B^2 terms and Coulomb field effects in our calculation.

An accurate calculation of the QZE is being carried out by Surmelian and O'Connell. Displacements of *line* spectra due to this effect may be used, as another method, to detect B fields (Preston 1970). The existing expression used for the QZE is obtained by treating the B^2 term in the Hamiltonian as a first-order perturbation. It follows that, for a hydrogen atom in a magnetic field, the energy of the electron may be written (in units $\hbar = c = \mu = 1$)

$$E \equiv E_0 + E_1 + E_2 = \frac{\alpha^2}{2n^2}\left[-1 + mn^2\left(\frac{B}{B_0}\right) + F_{nlm}n^6\left(\frac{B}{B_0}\right)^2\right], \tag{3}$$

where

$$B_0 \equiv \frac{\mu^2 ce^3}{\hbar^3} \equiv \alpha^2 B_c = 2.350 \times 10^9 \text{ gauss}, \tag{4}$$

α is the fine-structure constant, nlm are the usual hydrogen-atom-quantum numbers, and

$$F_{nlm} = \frac{5\left\{1 + \frac{1}{5n^2}[1 - 3l(l + 1)]\right\}\{l(l + 1) + m^2 - 1\}}{4(2l - 1)(2l + 3)}. \tag{5}$$

For large n and $l = 1$, we have $F = (1 + m^2)/4$, so that (in Rydbergs)

$$E_2 = \frac{1}{4} n^4 (1 + m^2) \left(\frac{B}{B_0}\right)^2, \tag{6}$$

which is the formula used by Preston (1970).

Now, it is generally stated that, since

$$(E_2/E_1) \sim [(B/B_0)n^4], \tag{7}$$

perturbation theory breaks down at a critical field, B_H say (H for hydrogen), given by (B_0/n^4). However, this is not quite correct, as the value of the coefficients of the B terms also play an important role in determining the exact value of B_H. It is important to determine B_H exactly so that we know precisely when Equation (7) is no longer trustworthy. A knowledge of B_H is also very useful in considering other problems such as photoionization, for example, because much work may be saved if perturbation-theory results are known to be reliable. On the other hand, this knowledge will also prevent us from using perturbation theory blindly in situations where it is not justified. Thus, we proceed to an accurate evaluation of B_H.

We define B_H as the magnetic field for which

$$|E_2| = |E_0 + E_1|. \tag{8}$$

This is certainly an upper limit, and we emphasize that perturbation-theory results will probably not be very accurate at values of B say five times smaller than B_H. It is advantageous to consider separately the cases $m = 0$ and $m \neq 0$, in the derivation of B_H from Equations (3) and (8). It is also convenient to write

$$B_H \equiv (B_0/y). \tag{9}$$

1. The case where $m = 0$. Thus,

$$y = n^3 |F_{nl0}^{1/2}|. \tag{10}$$

For large n and $l = 1$, it follows that $y \approx n^3$—not n^4.

2. The case where $m \neq 0$. For many cases of interest $|E_0| \gg |E_1|$. Hence,

$$y = n^3 |F_{nlm}^{1/2}|. \tag{11}$$

For large n and $l = 1$ (hence, $|m| = 1$), it follows that $y \approx n^3/\sqrt{2}$.

For use in interpreting the observational data, a knowledge of the QZE is useful for n values up to 10. Thus we have calculated the corresponding values of B_H over this range. In Tables I and II, we present, for selected values of n, the values of B_H corresponding to both $l = n - 1$, $m = 1$ and $l = |m| = 1$.

TABLE I
Values of B_H *From Equations (9) and (10)*

n	l	m	B_H (gauss)
2	1	0	6.8×10^8
2	0	0	4.4×10^8
5	4	0	4.7×10^7
5	0	0	2.9×10^7
6	5	0	2.7×10^7
6	0	0	1.7×10^7
9	8	0	8.4×10^6
9	0	0	5.0×10^6
10	9	0	6.2×10^6
10	0	0	3.6×10^6

TABLE II
Values of B_H *From Equations (3) and (8)*

n	l	m	B_H (gauss)
2	1	1	3.2×10^8
5	4	4	1.7×10^7
5	1	1	2.3×10^7
6	5	5	9.9×10^6
6	1	1	1.4×10^7
9	8	8	2.8×10^6
9	1	1	4.2×10^6
10	9	9	2.1×10^6
10	1	1	3.1×10^6

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