

versed. The original source from which this is taken also contains this error. All functions used in this work are as defined in this book.

<sup>6</sup>H. S. Wilf, *Mathematics for the Physical Sciences*

(Wiley, New York, 1967), p. 131–136. Using the theorem stated in this book we carry out the  $y$  integration first. Then the  $\xi$  integration is elementary.

<sup>7</sup>R. J. Moore, *Phys. Rev. D* **2**, 313 (1970).

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## Energy Spectrum of the Hydrogen Atom in a Strong Magnetic Field\*

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We obtain the energy spectrum for the 14 lowest states of the hydrogen atom in a strong magnetic field  $B$ . Emphasis is placed on values of  $B \approx 10^6$ – $10^8$  G, characteristic of those found in magnetic white dwarfs. Our results for the ionization energy of the ground state are significantly better than those of Cohen, Lodenquai, and Ruderman for fields  $\leq 7 \times 10^{11}$  G.

The discovery of strong magnetic fields  $\approx 10^6$ – $10^7$  G in some white dwarfs,<sup>1</sup> and the possible existence of superstrong magnetic fields  $\sim 10^{12}$  G in pulsars, has stimulated interest in the behavior of atoms in intense magnetic fields. Mueller, Rau, and Spruch<sup>2</sup> considered the *total* ground-state binding energies of atoms and positive ions, whereas Cohen, Lodenquai, and Ruderman (CLR)<sup>3</sup> obtained the binding energy of the *last* electron for atoms and ions. These papers were largely motivated by the discovery of pulsars and thus concentrated on *superstrong* magnetic fields ( $B \gtrsim 3 \times 10^{10}$  G). By contrast, Rajagopal, Channugam, O'Connell, and Surmelian,<sup>4</sup> motivated by the discovery of magnetic white dwarfs,<sup>1</sup> initiated a program to study the behavior of atoms in fields of *any* strength, with emphasis on  $B$  values from about  $10^4$  to  $10^{12}$  G. That paper<sup>4</sup> concentrated on the ionization energies of hydrogen in magnetic white dwarfs, and the essence of the calculation was the use of a trial wave function which was hydrogenlike, in contrast to the oscillatorlike trial wave function used by CLR. By the use of merely a four-parameter trial function,<sup>4</sup> the values of the ionization energy obtained were significantly better than those of CLR for fields  $\leq 3 \times 10^{10}$  G. A general procedure for carrying out a multiparameter calculation was also outlined. It is our purpose here to present the results of such a calculation, not only for the ground state, but also for the next 13 lowest-energy eigenstates.

The Hamiltonian for the hydrogen atom (we neglect spin) in a magnetic field  $B$  oriented along the  $z$  axis is given by

$$H = \frac{p^2}{2m} - \frac{e^2}{r} + \omega_L L_z + \frac{1}{2} m \omega_L^2 r^2 \sin^2 \theta, \quad (1)$$

where  $\omega_L = eB/2mc$ . Since it is invariant under rotations about the  $z$  axis and under inversion, the eigenstates can be labeled by the eigenvalues of  $L_z$  and the parity. Thus, a general form of the trial solution may be written

$$\psi_m^\pm(\vec{r}) \equiv \psi_t = \sum_{ll} (a_l^{(t)} r^l + b_l^{(t)} r^{l+1}) \exp(-\beta_l^{(t)} r) Y_{lm}(\theta, \phi), \quad (2)$$

where  $a_l^{(t)}$ ,  $b_l^{(t)}$ , and  $\beta_l^{(t)}$  are parameters.

The sum on  $l$  in (2) over all even integers leads to the state with even parity (+), and the sum over odd  $l$ , to the odd-parity (–) state. Here  $m$  is the eigenvalue of  $L_z$ . This choice is consistent with the condition that the solution associated with  $Y_{lm}$  in the hydrogen atom must have the behavior  $r^l$  near  $r=0$ . It has the advantage that we obtain an explicit evaluation of the matrix elements entirely in terms of the well-known  $\Gamma$  functions. For superstrong  $B$  fields we used a partial-wave expansion with values of  $l$  up to  $20 + |m|$  included in summation (2). This was found to be necessary in order to obtain convergence of the expansion of  $\psi_m^\pm$ . Up to 9 Slater-type orbitals were employed in the description of the radial function for the ground state, and 12 for the excited states. Figure 1 gives the ionization energy  $E_I$  (in eV) of the ground state of hydrogen as a function of the magnetic field  $B$  (in G). We compare our results (curve *a*) with the results of CLR<sup>3</sup> (curve *b*) and with those using perturbation<sup>5</sup> theory (curve *c*). For  $B \leq 7 \times 10^{11}$  G, our value for the ground-state energy level is lower than that of CLR.<sup>3</sup> Previously, our result<sup>4</sup> was superior only for  $B \leq 3 \times 10^{10}$  G. Thus, we have significantly extended the range in which the Slater-

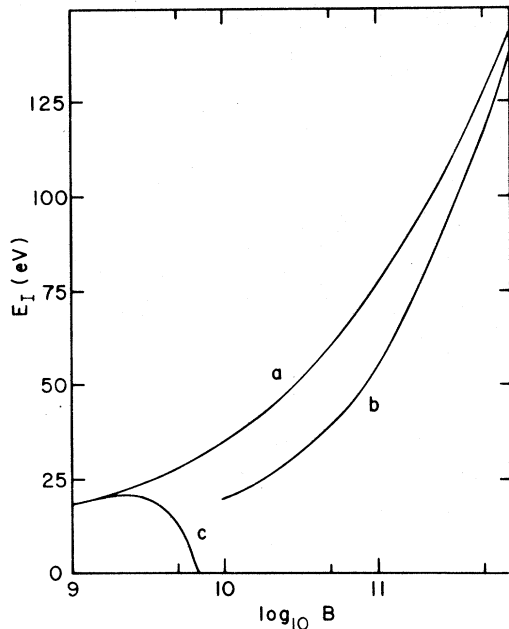


FIG. 1. The ionization energy of the ground state of hydrogen as a function of the magnetic field  $B$  calculated using our variational wave function (curve  $a$ ). Shown for comparison are the results of CLR (curve  $b$ ) and those obtained using perturbation theory (curve  $c$ ).

type variational results are better than those of CLR.<sup>3</sup>

In Fig. 2, we present the energy spectrum for the 13 lowest states above the ground state for  $B$  values from  $10^6$ – $10^8$  G. The labeling of the curves corresponds to the usual labels for the hydrogenic energy levels in the absence of a magnetic field.

We have compared these results with the perturbation results<sup>5</sup> and find that the latter are inadequate at  $B$  values of about  $10^8$  G and  $3 \times 10^7$  G

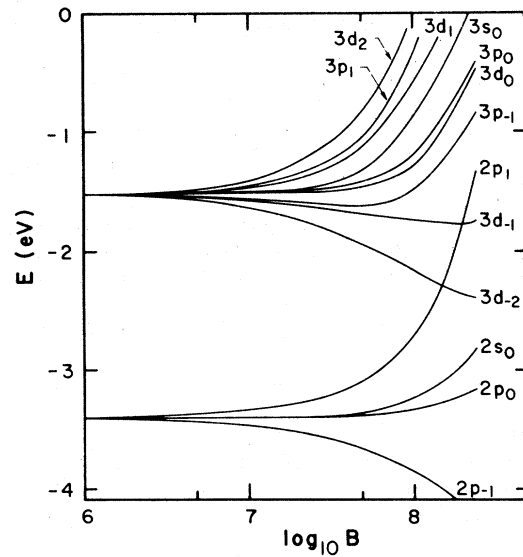


FIG. 2. The energy spectrum of hydrogen in a magnetic field for the 13 lowest states above the ground state.

for  $n=2$  and 3, respectively. This is in conformity with the well-known result<sup>5</sup> that, for a particular  $B$  value, the perturbation results are less reliable for the higher states. Presented elsewhere are results for oscillator strengths and transition probabilities.<sup>6</sup> Elsewhere, we will consider the implications relating to (a) the deduction of magnetic field values in white dwarfs from the quadratic Zeeman effect,<sup>7</sup> (b) the effect of a magnetic field on the opacities of the atmospheres of magnetic white dwarfs, and (c) solid state problems involving excitons.

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<sup>2</sup>R. O. Mueller, A. R. P. Rau, and L. Spruch, *Phys. Rev. Letters* **26**, 1136 (1971).

<sup>3</sup>R. Cohen, J. Lodenquai, and M. Ruderman, *Phys. Rev. Letters* **25**, 467 (1970).

<sup>4</sup>A. K. Rajagopal, G. Chanmugam, R. F. O'Connell, and G. L. Surmelian, *Astrophys. J.* (to be published).

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