Although in this theory the electromagnetic and weak corrections have to be considered together, it should be noted that up to the level of the quadratic divergence, electromagnetism and the rest are separately renormalizable.

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[1]S. Weinberg, Phys. Rev. Letters 19, 1264 (1967).
[2]G. 't Hooft, Nucl. Phys. B35, 167 (1971); B. W. Lee, Phys. Rev. D 5, 823 (1972).
[3]S. Weinberg, Phys. Rev. Letters 27, 1688 (1972).
[4]See Eq. (2) below.
[5]We have omitted the interaction with the scalar boson since it does not contribute to the quadratic divergences.
[6]Our metric and convention for Dirac matrices etc. are the same as in J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964).
[7]The apparent sixth-order divergence in $\Pi^{\mu\nu}$ does not really exist as can be seen by contracting the boson propagator momenta with the cubic boson coupling.
[8]The apparent quartic divergence in the vertex part also does not materialize for precisely the same reason as before.
[9]The lepton wave-function renormalization effect has to be considered, however, when examining the lepton-hadron universality.
[10]The Lagrangian in Eq. (1) corresponds to an anomalous-magnetic-moment interaction of the W boson with $\kappa_W = 1$. However, our conclusion for the electromagnetic correction to μ decay holds in fact for anomalous-magnetic-moment interaction with arbitrary κ_W.

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Radial Motion of a Spinning Test Body in the Field of a Black Hole

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The gravitational radiation emitted by a test particle falling radially into a Schwarzschild black hole has been analyzed by Zerilli. Here we investigate the effect of assigning spin to *both* the test particle and the source. We find that more (less) gravitational radiation is emitted if the spins are antiparallel (parallel).

The gravitational radiation emitted by a test particle falling radially into a black hole is of much current interest. In the case of a Schwarzschild black hole, the mathematical framework was outlined in detail by Zerilli.[1] Because of the complexity of Zerilli's equations, quantitative results must be obtained by numerical techniques. However, some very useful qualitative information may be obtained in a simple fashion, as Zerilli showed in his Appendix J. It is our purpose here to use the same type of approach to get some qualitative information on the effect of assigning spin to *both* the test particle and the source.

Our basic starting point is provided by the results of Barker and O'Connell,[2,3] who obtained an expression for the gravitational interaction between two spinning particles, correct to order G. This is what we refer[4] to as the *gravitational Breit-type interaction* (GBI), because it is the gravitational analog of Breit's expression for the electromagnetic interaction between two electrons. In particular, it contains spin-orbit and spin-spin terms.

Consider two particles, of masses m_1 and m_2 and spin angular momenta[5] J_1 and J_2. Consider $m_2 \gg m_1$ and let v be the velocity of m_1. We choose units $G=c=1$, so that $r_s^{(1)} \equiv (2Gm_1/c^2) = 2m_1$ and $r_s^{(2)} \equiv (2Gm_2/c^2) = 2m_2$, where $r_s^{(1)}$ and $r_s^{(2)}$ are the Schwarzschild radii of m_1 and m_2, respectively. Let

$$\vec{L} = \vec{r} \times \vec{v} \tag{1}$$

and

$$\vec{S}^{(1,2)} = (\vec{J}^{(1,2)}/m_{(1,2)}) . \tag{2}$$

Thus, the gravitational potential, in units of m_1, correct to first order in G, and assuming low velocities, may be written[2,4]

$$V(\vec{r}) = V_0 + V_1 + V_{so} + V_{ss} , \tag{3}$$

where

$$V_0 = -m_2/r , \tag{4}$$

$$V_1 = \tfrac{3}{2} v^2 V_0 , \tag{5}$$

$$V_{so} = -\frac{1}{r^2}(\tfrac{3}{2}\vec{S}^{(1)}\cdot\vec{L} + 2\vec{S}^{(2)}\cdot\vec{L})V_0, \tag{6}$$

$$V_{ss} = -\frac{1}{r^2}[3(\vec{S}^{(1)}\cdot\hat{r})(\vec{S}^{(2)}\cdot\hat{r}) - (\vec{S}^{(1)}\cdot\vec{S}^{(2)})]V_0 \tag{7}$$

are the Newtonian, relativistic, spin-orbit, and spin-spin contributions to the potential, respectively. It is illuminating to compare this result with the electromagnetic potential for positronium.[6] If we make the replacement $e^2 \to Gm^2$ in the latter we obtain terms of the same structure as above. The numerical coefficients are generally different, excepting the case of V_{ss}, where they are the same. In other words, as we had previously seen,[3] the gravitational spin-spin interaction may be obtained from the electromagnetic interaction between two magnetic dipoles by simply letting $e^2 \to -Gm^2$. For small velocities, we see that the dominant correction to V_0 is V_{ss}.

Consider m_1 falling radially towards m_2. It is now convenient to define the dimensionless quantities

$$\vec{a}^{(1,2)} = (\vec{S}^{(1,2)}/m_{(1,2)}). \tag{8}$$

Hence we see the spin-orbit term $V_{so}=0$, and

$$V_{ss} = -\left(\frac{m_1}{r}\right)\left(\frac{m_2}{r}\right)[3(\vec{a}^{(1)}\cdot\hat{r})(\vec{a}^{(2)}\cdot\hat{r}) - \vec{a}^{(1)}\cdot\vec{a}^{(2)}]V_0. \tag{9}$$

For simplicity, take $\vec{a}^{(1)}$ along $\hat{r}$, and then consider the two cases of $\vec{a}^{(2)}$ parallel and antiparallel to $\vec{a}^{(1)}$. Then

$$V = -\frac{m_2}{r}\left[1 + \tfrac{1}{2}v^2 \mp 2\left(\frac{m_1}{r}\right)\left(\frac{m_2}{r}\right) a^{(1)}a^{(2)}\right], \tag{10}$$

where $\mp$ refers to parallel and antiparallel, respectively, corresponding to repulsive and attractive gravitational spin forces. Now if m_1 falls from rest at ∞ to r, then, to order G, the maximum amount of gravitational radiation, E_g say, which can be emitted is given by $-V$. If we write

$$E_g \equiv E_g^{(0)} + E_g^{(S)}, \tag{11}$$

where $E_g^{(S)}$ is the contribution due to the spin, then

$$E_g^{(0)} = \frac{m_2}{r}(1 + \tfrac{3}{2}v^2) \tag{12}$$

and

$$E_g^{(S)} = \mp 2\left(\frac{m_1}{r}\right)\left(\frac{m_2}{r}\right)^2 a^{(1)}a^{(2)}. \tag{13}$$

In other words, *more* (less) gravitational radiation is emitted if the spins are *antiparallel* (parallel). If m_2 is an extreme Kerr black hole ($a^{(2)}=1$) then

$$E_g^{(S)} = \mp 2\left(\frac{m_1}{r}\right)\left(\frac{m_2}{r}\right)^2 a^{(1)}. \tag{14}$$

In general $a^{(1)}$ is arbitrary and may be greater than unity. In the particular case where m_1 is another extreme black hole, we get

$$E_g^{(S)} = \mp 2\left(\frac{m_1}{m_2}\right)\left(\frac{m_2}{r}\right)^3. \tag{15}$$

Thus, the effect of the spin-spin force on the amount of gravitational radiation emitted is comparable to the effect produced by the Newtonian forces,[1] when the black holes approach so closely that their events horizons (m_1 and m_2) essentially coincide. At such close distances, our linear-in-G analysis is no longer quantitatively reliable.

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[3] R. F. O'Connell, Phys. Letters 32A, 402 (1970).
[4] R. F. O'Connell, in Proceedings of the Varenna 1972 Summer School on Experimental Gravitation (unpublished).
[5] Our notation here is somewhat different from that of Ref. 2.
[6] V. B. Berestetskii, E. M. Lifshitz, and L. P. Pitaevskii, *Relativistic Quantum Theory* (Addison-Wesley, Reading, Mass., 1971), p. 286.