SUPERFLUIDITY IN NEUTRON STARS

G. CHANMUGAM, R.F. O'CONNELL and A.K. RAJAGOPAL

Department of Physics and Astronomy, Louisiana State University, Baton Rouge, Louisiana 70803, USA

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Arguments are presented to show that the BCS theory of superfluidity in its original form may not be applicable to neutron star matter over a wide range of density.

The recent discovery of pulsars, which are believed to be neutron stars, has stimulated considerable interest in neutron star matter and in particular its superfluidity [1—3]. Several authors [3—6] have calculated numerous properties of superfluid neutron matter such as the energy gap $\Delta$, critical temperature $T_C$. All of these calculations (with the exception of ref. [4]) have assumed that the use of the BCS [7] Hamiltonian is justified in describing superfluid neutron star matter. The purpose of this letter is to point out that the BCS theory in its original form may not be applicable to neutron star matter at densities $\lesssim 10^{14}$ g/cm$^3$.

An important parameter in the BCS theory is the coherence length $\xi = \hbar v_F / \Delta \pi$ where $v_F$ is the Fermi velocity. In metals $\xi$ is so large ($\sim 10^{-4}$ cm) that there are a large number of overlapping Cooper pairs over a range of the order of $\xi$. Thus, if the number density of electrons in a metal is $(\frac{4}{3} \pi r_s^3)^{-1}$, we have $\xi/r_s \approx 10^4$ in a typical superconductor. On the other hand in neutron star matter (for definiteness we shall use the results of Østgaard [6]) we have* for the neutron Fermi momentum $k_F = 0.5$ fm$^{-1}$, $\Delta \approx 3.13$ MeV so that $\xi/r_s \approx 0.6$. Here $r_s$ corresponds to neutrons. One notes also that $\Delta/E_F \approx 0.6$ whereas $\Delta/E_F \approx 10^{-3}$ to $10^{-4}$ in metals.

In order for the BCS theory to be applicable, the corrections to the BCS ground state energy due to the difference $H'$ between the Hamiltonian $H$ and the reduced Hamiltonian $H_{\text{red}}$ must be small. This can be estimated in perturbation theory and one requires that $W'$, the correction to the energy due to $H'$, be small compared with $W_0$ (where $W_0$ is the difference between the normal and superfluid free energy). This has been estimated by BCS [7, eq. (A 14)] for metals and it was found that $W'/W_0 \approx 10^{-3}$, even after over-estimating $W'$. For our system a more realistic estimate would be made by carrying out the sums [7, eq. (A 14)] in the evaluation of $W'$ over a region $2\Delta$ (instead of $6\Delta$) wide since here $\Delta/E_F \approx 1$ unlike for metals. We then find that

$$W' \approx \frac{3 [N(0) V]^2}{1 - \exp (-2/N(0) V)} \frac{\Delta}{E_F}.$$

Here $N(0)$ is the density of states per unit energy at the Fermi surface, and $V$ is the average matrix element for pairs in a shell of thickness $\hbar \omega$ near $E_F$. For neutron star matter, at $k_F = 0.5$ fm$^{-1}$, one has $W'/W_0 \approx 1.2$ which means that the BCS approximation breaks down. The physical reason for this is that $\xi/r_s$ is of order unity which implies very little overlap of Cooper pairs. In fact for neutron star matter (where $\hbar \omega \approx E_F$) one notes that $W'/W_0 \to \infty$ as $\xi/r_s \to 0$ while $W'/W_0 \to 0$ as $\xi/r_s \to \infty$.

We also note that $W'/W_0$ decreases with $k_F$. At $k_F = 1.0$ fm$^{-1}$, $W'/W_0 \approx 0.1$. Hence, the BCS theory in its original form may not be applicable in neutron star matter when $k_F \approx 1.0$ fm$^{-1}$. This region falls entirely within the crust of the neutron star [8]. Yang and Clark [4] have approached the problem in an essentially different way and as such it is difficult to compare their results with the BCS approach discussed in this letter. Finally, we emphasize that our remarks are restricted to the model of superfluidity involving s-wave paring.

References


