Astron. & Astrophys. 18, 198–200 (1972)

Magnetic Properties of a Degenerate Electron Gas and Implications for Metals, White Dwarfs, and Neutron Stars

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Received October 11, 1971, revised January 18, 1972

In previous work we have considered the absolute stability of the Landau Orbital Ferromagnetic states for both a non-relativistic and a relativistic electron gas at $T = 0$. We now extend this work to include the effects of a) non-zero temperatures, b) Coulomb interactions, c) effective mass values other than unity. As before we conclude that the non-magnetic ($B = 0$) state rather than the magnetic Landau Orbital Ferromagnetic states forms the thermodynamically most stable state in thermal equilibrium. Thus these Landau Orbital Ferromagnetic states are observable only as possible metastable states in metals, white dwarfs, and neutron stars, which could account for the occurrence of large magnetic fields in some white dwarfs but not others.

Key words: LOFER states — Gibbs free energy — thermodynamic stability — white dwarfs — neutron stars — pulsars

I. Introduction

With the proposal by Gold (1968, 1969) that pulsars are rapidly rotating neutron stars possessing intense magnetic fields and the discovery of large magnetic fields in white dwarfs (Kemp *et al.*, 1970; Angel and Landstreet, 1971), the origin of these large magnetic fields in white dwarfs and neutron stars has become an astrophysical problem of great interest. A mechanism leading to a ferromagnetic state in a degenerate electron gas, called the Landau Orbital Ferromagnetic (LOFER) states, has been proposed by Lee *et al.* (1969). However in two preceding papers (O'Connell and Roussel, 1971a and 1971b) we have shown that in the case of a noninteracting completely degenerate electron gas in thermal equilibrium, which is the model considered by Lee *et al.*, these LOFER states correspond to a larger value of the thermodynamic potential than the nonmagnetic state. In other words, the LOFER states do not correspond to the most absolutely stable state of the system and may exist only as metastable states. We now extend this work to include interactions and non-zero temperatures.

In Section II we discuss the thermodynamics of an electron gas in a magnetic field and derive an expression for the thermodynamic potential per unit volume of a non-zero temperature electron gas and discuss the stability of the LOFER states for such a system. In Section III we deal with the LOFER state stability of a completely degenerate electron gas interacting directly through a Coulomb potential. In Section IV we discuss the effects of effective electron mass on LOFER state stability and present our overall conclusions. In particular, we provide a possible answer to the question as to why large magnetic fields are found in some white dwarfs but not others.

* NSF pre-doctoral fellow.

II. Non-Zero Temperature and LOFER State Stability

Previous to Shoenburg (1962), the magnetization of a system, M, had been assumed to be a function of the magnetic field H, so that the magnetic induction field was given by

$$B = H + 4\pi M(H)\,. \tag{1}$$

Such a system is called a magnetically noninteracting system. Shoenburg (1962) proposed that the magnetization is actually a function of the magnetic induction field B. Thus Eq. (1) becomes

$$B = H + 4\pi M(B)\,. \tag{2}$$

A system which is described by Eq. (2) is called a magnetically interacting system because, besides the interaction between the electrons and the magnetic field H resulting in the magnetization

$M(H)$, the electrons interact magnetically among themselves. The total effect of all the interactions is to produce a magnetization $M(B)$. This magnetically interacting system should not be confused with a directly interacting system, a system in which the electrons would interact directly with each other. A directly interacting system may be considered the framework of either a magnetically noninteracting or a magnetically interacting system.

Now (Pippard, 1963; Lee, 1970)

$$\Omega(\mu, T, B) = \Omega_0^*(\mu, T, B) + 2\pi M^2(T, \mu, B) \quad (3)$$

where $\Omega(\mu, T, B)$ is the thermodynamic potential in the magnetically interacting system, which must be a minimum for a stable state (Landau and Lifshitz, 1969), and $\Omega_0^*(\mu, T, B)$ is a function obtained from the noninteracting thermodynamic potential $\Omega_0^*(\mu, T, H)$ by replacing H with B. In our previous papers (O'Connell and Roussel, 1971a and 1971b) we used G instead of Ω but we now adhere to what seems to be the more commonly used symbol Ω.

Let us first consider a non-zero temperature system. For simplicity we confine ourselves to the non-relativistic case but similar results hold in the extreme relativistic case. Neglecting direct interactions, $\Omega_0^*(\mu, T, B)$ and $M(\mu, T, B)$ are given by (Wilson, 1953)

$$\Omega_0^*(\mu, T, B) = -\frac{(2\mu)^{5/2}}{15\pi^2} \cdot \left(1 + \frac{5}{4}\pi^2 b^{-2} - \frac{15\pi^{1/2}}{4} b^{-5/2} \Sigma_1\right), \quad (4)$$

$$M(\mu, T, B) = -\frac{(2\mu)^{1/2}\alpha B}{4\pi^{7/2}} b^{1/2} \Sigma_2 \quad (5)$$

and the number of particles is given by

$$n(\mu, T, B) = -\frac{(2\mu)^{3/2}}{3\pi^2}\left[1 + \frac{3\pi^{1/2}}{2} b^{-3/2} \Sigma_2\right], \quad (6)$$

where

$$\Sigma_1 = \sum_{r=1}^{\infty} \frac{\cos(br - \pi/4)}{r^{5/2}} \frac{2r\pi^2 kT/(B/B_c)}{\sinh[2r\pi^2 kT/(B/B_c)]}. \quad (7)$$

and

$$\Sigma_2 = \sum_{r=1}^{\infty} \frac{\sin(br - \pi/4)}{r^{3/2}} \frac{2r\pi^2 kT/(B/B_c)}{\sinh[2r\pi^2 kT/(B/B_c)]}. \quad (8)$$

Thus

$$2\pi M^2(\mu, T, B) = -\frac{(2\mu)^{5/2}}{15\pi^2}\left\{-\frac{15\alpha}{2^{5/2}\pi^2}\mu^{1/2} b^{-1} |\Sigma_2|^2\right\}. \quad (9)$$

Using the same procedure as in our previous papers,

$$\Omega(\mu, T, B) = -\frac{(2\mu)^{5/2}}{15\pi^2} \cdot \left\{1 - \frac{15\pi^{1/2}}{8} b^{-3/2} |\Sigma_2| + \frac{5}{4}\pi^2 b^{-2} + \ldots\right\}. \quad (10)$$

Realizing we are considering $b \gg 1$, it is obvious from Eq. (10) that the LOFER state possesses a larger thermodynamic potential than the non-magnetic ($B = O$) state.

III. Coulomb Interaction Effects

Let us now consider a completely degenerate non-relativistic electron gas interacting through a Coulomb potential. The non-interacting thermodynamic potential per unit volume is given by (Sondheimer and Wilson, 1953)

$$\Omega_0^*(\mu, B) = -\frac{(2\mu)^{5/2}}{15\pi^2}\left\{1 + \frac{5\pi^2}{4} b^{-2} - \frac{15\pi^{1/2}}{4} b^{-5/2} \Sigma_3\right\} \quad (11)$$

where

$$\Sigma_3 = \sum_{r=1}^{\infty} \frac{\cos(br - \pi/4)}{r^{5/2}}. \quad (12)$$

To this we must add the correction due to Coulomb interactions $\Omega_0^{*c}(\mu, B)$ which is given by (Ichimura and Tanaka, 1961; and Isihara *et al.* 1971)

$$\Omega_0^{*c}(\mu, B) = -\frac{(2\mu)^{5/2}}{15\pi^2}\left\{\frac{15\alpha}{\pi^{1/2} 2^{3/2} \mu^{1/2}} b^{-3/2} \Sigma_4\right\}, \quad (13)$$

where

$$\Sigma_4 = \sum_{r=1}^{\infty} \frac{\sin(br - \pi/4)}{r^{3/2}}. \quad (14)$$

The magnetization $M(\mu, B)$, to correct order, is given by

$$M(\mu, B) = -\frac{\alpha^{5/4} 2^{1/2}}{\pi^3} \mu^{3/2} B^{-1/2} \Sigma_5. \quad (15)$$

where

$$\Sigma_5 = \sum_{r=1}^{\infty} \frac{\cos(br - \pi/4)}{r^{1/2}}. \quad (16)$$

In the usual manner we find

$$\Omega(\mu, B)_{B=0} = [\Omega_0^*(\mu, B) + 2\pi M^2(\mu, B)]_{B=0} = \Omega_0^*(\mu, 0) = -\frac{(2\mu)^{5/2}}{15\pi^2}. \quad (17)$$

For the LOFER state solution, Eq. (3) becomes

$$\Omega(\mu, B)_{\substack{\text{LOFER}\\ \text{state}}} = -\frac{(2\mu)^{5/2}}{15\pi^2}\left\{1 - \frac{15\alpha^3}{2^{3/2}\pi^4\mu^{1/2}} b\, |\Sigma_5|^2 + \frac{15\alpha}{2^{3/2}\pi^{1/2}\mu^{1/2}} b^{-3/2} \Sigma_4\right\}. \quad (18)$$

Again realizing we are considering $b \gg 1$, it is apparent that the nonmagnetic solution has the lowest thermodynamic potential.

IV. Effective Mass Corrections

To include effective mass considerations we need only to introduce the "effective mass" m^* consistently in the theory in place of m. (See for example Kittel, 1953). Thus we have a new parameter b^* where

$$b^* = \frac{\pi^5}{\alpha\,|\Sigma_2^*|^2\,\alpha^2\,(\mu/m^*)} \tag{19}$$

and where

$$\Sigma_2^* = \sum_{r=1}^{\infty} \frac{\sin\,(b^* r - \pi/4)}{r^{3/2}}\,. \tag{20}$$

Thus

$$b^* = \frac{m^*}{m}\left(\frac{\Sigma_2}{\Sigma_2^*}\right)^2 b\,. \tag{21}$$

It follows that

$$b^* \approx \frac{m^*}{m}\,b\,. \tag{22}$$

and since $b \gg 1$ it can be shown that $b^* \gg 1$.

The thermodynamic potential for the magnetically interacting non-relativistic case obtained by the usual method is

$$\Omega\,(\mu, 0, B) = -\frac{m^{*3/2}\,(2\,\mu)^{5/2}}{15\,\pi^2}\left\{1 - \frac{15\,\pi^{1/2}}{8}\,|\Sigma_2^*|\,b^{*-3/2} + \frac{5}{8}\,\pi^2\left(3\,\frac{m^{*2}}{m} - 1\right) b^{*-2}\right\}. \tag{23}$$

Since $b^* \gg 1$ it is apparent from Eq. (23) that the nonmagnetic solution has a lower thermodynamic potential than the LOFER state solutions for any reasonably physical value of m^*/m. A similar result follows for the extreme relativistic limit.

To conclude, the inclusion of several interaction effects does not alter our previous conclusion that the LOFER states do not correspond to the most absolutely stable state of the system. This conclusion leaves only the possibility that LOFER states may correspond to metastable states of the system so that once the system is somehow excited into one of these states, its relatively long life-life may permit it to be observed. It also provides us with a possible answer as to why large magnetic fields are found in some white dwarfs but not others-it could be due to the occurrence of some, as yet unknown, circumstance resulting in the system going into a metastable Lofer state instead of the lower energy $B = 0$ state.

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