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PRESENT STATUS OF THE THEORY OF THE RELATIVITY-GYROSCOPE EXPERIMENT†

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§(1): *INTRODUCTION*

In 1960, Schiff [1] proposed the relativity-gyroscope experiment as a new test of gravitational theories. Preparations for this experiment, to be carried out by Everitt and Fairbank [2,3], are now in an advanced stage. Now it turns out that the main contribution to the precession, that due to the gravitational field of a non-rotating earth and referred to as the de Sitter contribution [4], comes from the same terms in the metric [4] that contribute to the well-known gravitational deflection of light, and thus it is not of compelling interest. On the other hand, the experiment will be sensitive to the terms arising from the earth's rotation, the so-called Lense-Thirring off-diagonal metric terms, and in fact it is the only experiment that envisages testing these terms. For this reason the relativity-gyroscope experiment is unique and important.

It is proposed to launch a satellite containing two pairs of gyroscopes into a polar orbit around the Earth; the spin of one pair (gyro 1) will be parallel to the Earth's axis and the spin of the other pair (gyro 2) will be perpendicular to the plane of the orbit. Based on the original calculations [1], it seemed that gyro 2 would be responsive only to the de Sitter terms but, as we shall show, this is no longer true when perturbations are taken into account. However, we also show that this clean separation of the two main contributions is not important but what is important is to choose an orbit whose angular momentum vector is least affected by the perturbations. This dictates, once more, a choice of polar orbit.

In section 2 we write down a general expression for the precession of a gyroscope in Einstein theory, giving a detailed discussion

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of the various perturbations which must be taken into account before one can meaningfully compare the observations with the theoretical predictions. They are due to

(i) the quadrupole moment of the earth, which manifests itself in two ways, giving rise to

(a) the direct effect [5], which is an instantaneous contribution to the precession, due to the additional gravitational field, and
(b) the indirect effect [6,7], which comes only into consideration when the de Sitter term is averaged over a period of the motion,

(ii) the gravitational field of the sun [8], and

(iii) the apparent precession of the gyro due to bending of light from the reference star by the sun's gravitational field [9].

In section 3 we examine the numerical predictions, again confining attention to Einstein theory. In section 4 we consider a general formulation which will incorporate the predictions of other gravitational theories. By way of example, we treat the Brans-Dicke scalar-tensor theory, both analytically and numerically. In section 5 we discuss the relative merits of various orbits. We conclude that the polar orbit is the best choice and then we generalize our previous results to include eccentricity terms in this important case.

§(2): *THE PRECESSION OF A GYROSCOPE IN EINSTEIN THEORY - ANALYTICAL RESULTS*

The rate of change of the spin axis $\vec{S}$ of a gyroscope may be written [1]

$$\frac{d\vec{S}}{dt} = \vec{\Omega} \times \vec{S}, \tag{1}$$

where $\vec{\Omega}$ is the angular velocity of precession.

In Einstein theory ($\vec{\Omega} \equiv \vec{\Omega}_E$), we have

$$\vec{\Omega}_E = \vec{\Omega}_T + \vec{\Omega}_{DS} + \vec{\Omega}_{LT} + \vec{\Omega}_Q + \vec{\Omega}_{DS}{}^S \tag{2}$$

where $\vec{\Omega}_T, \vec{\Omega}_{DS}, \vec{\Omega}_{LT}, \vec{\Omega}_Q$, and $\vec{\Omega}_{DS}{}^S$ are the so-called Thomas, de Sitter, Lense-Thirring, quadrupole-moment [5 - 7] and sun [8] contributions, respectively. It is possible to have Ω_T essentially zero [1] by putting the gyroscope in a satellite. Explicitly,

$$\vec{\Omega}_{DS} = \frac{3Gm_2}{2c^2r^3} (\vec{r} \times \vec{v}), \tag{3}$$

$$\vec{\Omega}_{LT} = \frac{GS^{(2)}}{c^2 r^3}\,(3\cos\phi\,\hat{r} - \vec{n}^{(2)}), \tag{4}$$

$$\vec{\Omega}_{Q} = \frac{3Gm_2}{2c^2 r^3}\,(\vec{R}\times\vec{v}), \tag{5}$$

$$\vec{\Omega}_{DS}{}^{S} = \frac{3GM_\odot}{2c^2 d^3}\,(\vec{d}\times\vec{v}_E), \tag{6}$$

and

$$\vec{R} = \frac{3J_2}{2r}\,[(1 - 5\cos^2\phi)\hat{r} + 2\cos\phi\,\vec{n}^{(2)}], \tag{7}$$

where m_2, $\vec{S}^{(2)} = I^{(2)}\vec{\omega}^{(2)}$, $I^{(2)}$, and $\vec{\omega}^{(2)}$ refer respectively to the mass, spin, moment-of-inertia and angular velocity of the earth; $\vec{v}$ is the velocity of the gyro around the earth and $\vec{v}_E$ is the velocity of the earth around the sun, r is the earth-gyro distance, d is the earth-sun distance, and $M_\odot$ is the mass of the sun. Unit vectors in the $\vec{r}$ and $\vec{S}^{(2)}$ directions are denoted by $\hat{r}$ and $\vec{n}^{(2)}$, respectively, and $\hat{r}\cdot\vec{n}^{(2)} \equiv \cos\phi$. The quadrupole moment of the earth, J_2, is defined from writing the Newtonian potential of the earth in the form

$$\Phi(r) = \frac{Gm_2}{r} + \frac{GJ_2 m_2}{2r^3}\,(1 - 3\cos^2\phi). \tag{8}$$

In order to obtain the secular precession of the spin we average these equations over a complete period of a distorted circular orbit† (the distortion being due to the quadrupole moment). It turns out that, to the accuracy required, all contributions except $\vec{\Omega}_{DS}$ may be averaged over the undistorted orbit, which is purely circular. From henceforth we will regard all the $\vec{\Omega}$ terms as being averaged over a period of the motion. It is convenient to write

$$\vec{\Omega}_{DS} = \vec{\Omega}_{DS}^{(0)} + \Delta\vec{\Omega}_{DS} \tag{9}$$

where $\vec{\Omega}_{DS}$ and $\vec{\Omega}_{DS}^{(0)}$ refer to the average over the distorted and undistorted orbits, respectively. It follows that

$$\vec{\Omega}_{DS}^{(0)} = (3Gm_2\omega/2c^2 a)\vec{n}, \tag{10}$$

† For the case of "distorted" elliptic orbits, the algebra is more cumbersome without adding anything new physically and so we refer the reader to reference [7] for details. However, in section 5, we will present the results appropriate to this more general orbit for the specific but important case of a polar orbit.

$$\vec{\Omega}_{LT} = \frac{GS^{(2)}}{2c^2a^3}\,[\vec{n}^{(2)} - 3\cos\theta\,\vec{n}], \tag{11}$$

$$\vec{\Omega}_Q = \left(\frac{3Gm_2\omega}{2c^2a}\right)\left(\frac{3J_2}{2a^2}\right)[\tfrac{1}{2}(5\cos^2\theta - 1)\vec{n} - \cos\theta\,\vec{n}^{(2)}] \tag{12}$$

and

$$\vec{\Omega}_{DS}{}^{S} = \frac{3GM_{\odot}\omega_E}{2c^2a_E}\,\vec{n}^{(S)}, \tag{13}$$

where $\vec{n}$ and $\vec{n}^{(2)}$ are unit vectors along the orbital angular momentum of the gyro and the orbital angular momentum of the earth, respectively. In addition, $\cos\theta = \vec{n}\cdot\vec{n}^{(2)}$, ω and ω_E denote the average orbital angular velocities of the satellite and the earth respectively and "a" is defined so as to ensure that Kepler's law holds in its normal form viz.

$$\omega = \frac{2\pi}{T} = (Gm_2/a^3)^{\frac{1}{2}}, \tag{14}$$

where T is the period. The corresponding quantity for the earth's orbit is denoted by a_E. It is curious to note that $\vec{\Omega}_{DS}{}^{S}$ is half of the *perihelion* precession of the earth around the sun†.

Now θ is actually the angle of inclination of the orbit with respect to the earth's equatorial system. In the case of an equatorial or polar orbit ($\theta = 0$ and $\pi/2$, respectively), we find [6,7] that $\Delta\Omega_{DS}$ is a simple multiple of $\vec{\Omega}_{DS}^{(0)}$. Explicitly [6,7],

$$\Delta\vec{\Omega}_{DS} = -\frac{1}{2}(J_2/a^2)\vec{\Omega}_{DS}^{(0)} \qquad \textit{for} \quad \theta = 0, \tag{15}$$

and

$$\Delta\vec{\Omega}_{DS} = \frac{1}{4}(J_2/a^2)\vec{\Omega}_{DS}^{(0)} \qquad \textit{for} \quad \theta = \pi/2. \tag{16}$$

The situation in the case of arbitrary θ is complicated by the fact that the node of the orbit regresses and, as a consequence, a gyro which was originally placed along $\vec{n}$ will not stay along $\vec{n}$ simply because $\vec{n}$ itself is not a fixed vector in space. The analysis of such an orbit is presently being carried out by Barker and O'Connell [10].

We turn now to the apparent precession of the gyro due to bending of light from the reference star by the sun's gravitational field. A frame of reference is obtained by rigidly attaching the

† See reference [7], equation (78a).

gyro-housing to a telescope which is trained on some reference star and thus one is interested in difference measurements between the gyro and telescope readings [2]. However, it has recently been shown by O'Connell and Surmelian [9] that the deflection of the light from the reference star due to the sun's gravitational field can give rise to an apparent precession of the gyroscope, $\vec{\Omega}_{DEF}$ say. In Einstein theory ($\vec{\Omega}_{DEF} \equiv \vec{\Omega}^{E}_{DEF}$), we have

$$\vec{\Omega}^{E}_{DEF} = (4.1 \times 10^{-3} \cot \frac{\alpha}{2}) \text{ sec.}, \tag{17}$$

where α is the angle between the earth-sun direction and the earth-star direction ($\pi \geqslant \alpha > 0$). Thus, over a 6-month period, this could amount to as much as 1.75 sec., depending on the angle which the sun-star line makes with the ecliptic. Clearly, such a change must be included in the calculation of the *true* angle of precession of the gyroscope.

§(3): *THE PRECESSION OF A GYROSCOPE IN EINSTEIN THEORY - NUMERICAL RESULTS*

The magnitude of the quadrupole moment is given by [11]

$$J_2/R^2 = (1082.64 \pm 0.08) \times 10^{-6}, \tag{18}$$

where R is the earth's equatorial radius. For the calculation of $I^{(2)}$ we use a realistic mass distribution for the earth [12] and we find that it reduces the value of $\vec{\Omega}_{LT}$ by about 17% below the value one would obtain by assuming a model of homogeneous density for the earth. To be specific, let us consider the satellite in a circular polar orbit ($\theta = \pi/2$) 300 miles above the earth. Then (in units of sec/yr),

$$\vec{\Omega}^{(0)}_{DS} = 7.0\, \vec{n}, \tag{19}$$

$$\Delta\vec{\Omega}_{DS} = 1.65 \times 10^{-3}\, \vec{n}, \tag{20}$$

$$\vec{\Omega}_{LT} = 43.8 \times 10^{-3}\, \vec{n}^{(2)}, \tag{21}$$

$$\vec{\Omega}_{Q} = -4.95 \times 10^{-3}\, \vec{n}, \tag{22}$$

and

$$\vec{\Omega}^{(S)}_{DS} = 19.2 \times 10^{-3}\, \vec{n}^{(S)}, \tag{23}$$

As we have previously pointed out the most interesting term is the Lense-Thirring term $\vec{\Omega}_{LT}$. Now we shall see in section 4 that Brans-Dicke (BD) theory [13] predicts a reduction in this value by a factor [4] of 1/16, i.e. by 2.7×10^{-3} sec/yr. Thus to distinguish between the Einstein and BD theories the experiment should be capable of measuring such small precession angles. In fact, measurement accurate to 10^{-3} sec/yr will be possible [2,3] by use of the London moment readout technique. Originally it was thought [2,3] that a gyro in polar orbit oriented along $\vec{n}$ would be responsive only to $\vec{\Omega}_{LT}$ but now we demonstrate that this is no longer true when the sun's perturbation is taken into account. Since the magnitude of $\vec{\Omega}_{DS}{}^{S}$ is 19.2×10^{-3} sec/yr and since the Earth's equator is inclined at an angle λ of 23.44° to the ecliptic, the $\vec{n}^{(2)}$ component (i.e. the component in the same direction as $\vec{\Omega}_{LT}$) is 0.917 $\Omega_{DS}{}^{S}$, i.e., 17.6×10^{-3} sec/yr. It is more than 6 times as large as the difference between the Lense-Thirring contributions arising from the Einstein and BD theories. In addition, $\vec{\Omega}_{DS}{}^{S}$ also has a component in the $\vec{n} \times \vec{n}^{(2)}$ direction, to which gyro No.2 (which measures $\vec{\Omega}_{LT}$) is responsive. The magnitude of this component is $\sin\lambda \cos u \times \Omega_{DS}{}^{S}$, i.e., $7.64 \times 10^{-3} \times \cos u$ sec/yr, where u is the angle between the autumnal equinox and the $\vec{n}$ direction.

§(4): THE PRECESSION OF A GYROSCOPE IN AN ARBITRARY THEORY OF GRAVITATION

Eddington [14] and others [1,15] have treated arbitrary theories of gravitation in a convenient manner by writing down a metric of the most general, static, spherically symmetric, isotropic form. This is achieved by introducing three dimensionless numerical parameters α, β, and γ. A simple generalization, to include the earth's rotation, is effected by the introduction of another parameter δ. Thus, in the isotropic metric with $\vec{\omega}^{(2)}$ directed along the positive z axis,

$$\begin{aligned} ds^2 &= [1 - 2\alpha(m/r) + 2\beta(m/r)^2 + \cdots]c^2dt^2 \\ &\quad - [1 + 2\gamma(m/r) + \cdots](dx^2 + dy^2 + dz^2) \\ &\quad + \delta\,\frac{4GI^{(2)}\omega^{(2)}}{c^3r^3}\,[x dy - y dx]c dt, \end{aligned} \tag{24}$$

where $m = (Gm_2/c^2)$. In Einstein theory $\alpha = \beta = \gamma = \delta = 1$ and in BD theory [13,4] $\alpha = 1$, $\beta = 1$, $\gamma = [(\omega+1)/(\omega+2)]$, and $\sigma = [(2\omega+3)/(2\omega+4)]$, where ω is the dimensionless coupling constant which appears in BD theory.

Using equation (24), it follows that the angular velocity of precession of a gyroscope in an arbitrary theory of gravitation is given by [1,4]

$$\vec{\Omega} = \vec{\omega}_T + \frac{1}{3}(\alpha + 2\gamma)\{\vec{\Omega}_{DS} + \vec{\Omega}_Q + \vec{\Omega}_{DS}{}^{S}\} + \delta\vec{\Omega}_{LT}. \tag{25}$$

In addition

$$\vec{\Omega}_{DEF} = \frac{1}{2}(\alpha + \gamma)\vec{\Omega}^{E}_{DEF}. \tag{26}$$

The corresponding quantities in BD theory are [4]

$$\vec{\Omega}_{BD} = \vec{\Omega}_T + \frac{4 + 3\omega}{6 + 3\omega} (\vec{\Omega}_{DS} + \vec{\Omega}_Q + \vec{\Omega}_{DS}{}^S) + \frac{3 + 2\omega}{4 + 2\omega} \vec{\Omega}_{LT},$$

and

$$\vec{\Omega}^{BD}_{DEF} = \left[\frac{3 + 2\omega}{4 + 2\omega}\right] \vec{\Omega}^{E}_{DEF}. \tag{27}$$

A common choice for ω is [16] $\omega = 6$, for which we obtain

$$\vec{\Omega}_{BD} = \vec{\Omega}_T + \frac{11}{12} (\vec{\Omega}_{DS} + \vec{\Omega}_Q + \vec{\Omega}^S_{DS}) + \frac{15}{16} \vec{\Omega}_{LT}, \tag{28}$$

and

$$\vec{\Omega}^{BD}_{DEF} = \frac{15}{16} \vec{\Omega}^{E}_{DEF}. \tag{29}$$

§(5): *DISCUSSION*

Our results contain three parameters in the general case. However, for an acceptable theory of gravitation we must have $\alpha = 1$. Thus, we require the observations to provide us with values for γ and δ. As equation (26) makes clear, the well-known light deflection experiments provide us with a measure of γ. The unique feature of the gyro experiment is that it is capable of providing us with a value for δ, the earth rotation term. Referring back to section 1 we see the original hope was that gyro 2 would provide a unique measure of δ, regardless of the value of γ (which would be measured by gyro 1, regardless of the value of δ) but, as the discussion at the end of section 3 made clear, this hope is destroyed by the presence of the $\vec{\Omega}_{DS}{}^S$ term.

The following question now immediately suggests itself: What is the best orbit to select for the purpose of achieving the "cleanest" test of $\vec{\Omega}_{LT}$? The answer is an orbit inclined at an angle $\lambda = 23.44°$ to the equator so that $\vec{n}$ and $\vec{n}^{(S)}$ point in the same direction (i.e. $\theta = \lambda$). If a gyro is again placed in the $\vec{n}$ direction *it will now be sensitive to both* $\vec{\Omega}_{DS}$ *and* $\vec{\Omega}_{DS}{}^S$. However, now the Lense-Thirring contribution to the precession of the gyro is reduced by a factor $\sin\lambda = 0.398$ to a value 17.4×10^{-3} sec/yr and the difference between the Einstein and BD contributions in this case is 1.1×10^{-3}, very close to the limits of the expected experimental accuracy. Furthermore, there is now a contribution from $\vec{\Omega}_Q$ amounting to 3.6×10^{-2} sec/yr. But undoubtedly the greatest objection of all to the use of this orbit - and to all other orbits other than the polar and equatorial orbits - arises from the fact that the angular momentum vector $\vec{L}$ does not remain fixed in space. The secular variation of $\vec{L}$ with time may be written in the form [7]

$$\frac{d\vec{L}}{dt} = \frac{d\Omega'}{dt} (\vec{n}^{(2)} \times \vec{L}), \tag{30}$$

where Ω' denotes the longitude of the ascending node and where we have neglected the small variations in the angle of inclination θ. Now the principal contribution [7] to $(d\Omega'/dt)$ is due to the quadrupole moment of the earth. Explicitly [7]

$$\begin{aligned}\frac{d\Omega'}{dt} &= -\frac{3J_2}{2a^2}\cos\theta \\ &= -\left(9.96\times\left(\frac{R}{a}\right)^{7/2}\cos\theta\right)\text{ deg/day.}\end{aligned} \tag{31}$$

Since $\vec{n}$ is the unit vector along $\vec{L}$, we can thus write

$$\frac{d\vec{n}}{dt} = -\left[4.98\times\left(\frac{R}{a}\right)^{7/2}\sin 2\theta\right]\vec{j}\text{ deg/day,} \tag{32}$$

where $\vec{j}$ is a unit vector in the $(\vec{n}^{(2)}\times\vec{n})$ direction.

It is clear that $(d\vec{n}/dt)$ vanishes for both the polar and equatorial orbits. It is a maximum for $\theta = 45°$, and for a satellite in a circular orbit, 300 miles above the earth, in an orbit of this inclination we find

$$\left(\left|\frac{d\vec{n}}{dt}\right|\right)_{\max} = 3.86\text{ deg/day} \tag{33}$$

A similar orbit at an inclination 23.44° to the equator results in

$$\left(\left|\frac{d\vec{n}}{dt}\right|\right)_{\theta = 23.44°} = 2.82\text{ deg/day.} \tag{34}$$

Since the satellite period at 300 miles is 0.0655 days, it follows that

$$\left(\left|\frac{d\vec{n}}{dt}\right|\right)_{\theta = 23.44°} = 11.1\text{ min/(satellite period).} \tag{35}$$

This is a measure of the rate at which gyro 2 (originally placed along $\vec{n}$) goes out of alignment with $\vec{n}$, making the analysis of the observations much more complicated [10].

Thus we are left with a choice between the polar and equatorial orbits. Since our main objective is the measurement of δ, this demands that the magnitude of $\vec{\Omega}_{LT}$ be as large as possible. From equation (11) we see that this is achieved by choosing $\theta = 0$, i.e. an equatorial orbit. However, in this case $\vec{\Omega}_{LT}$ is along the same direction as $\vec{\Omega}_{DS}$, which would clearly curtail the information desired by the use of two gyros. In the polar case, one has the obvious advantage that $\vec{\Omega}_{LT}$ is at right angles to $\vec{\Omega}_{DS}$ while the magnitude of $\vec{\Omega}_{LT}$ is still half of what it would be in the equatorial case. We conclude that the best choice of orbit is the polar orbit. In addition, it should be emphasized that, since for a 300 mile polar orbit, for example,

$$\frac{|\vec{\Omega}_{LT}|}{|\vec{\Omega}_{DS}|} = 6 \times 10^{-3}, \tag{36}$$

and since we desire an accurate result for the δ term, it is necessary to know the actual inclination of the orbit to within about 30 seconds of arc.

Because of the importance of the polar orbit we now write down the results appropriate to the more general case of a satellite in a "distorted" *elliptical* polar orbit. Letting e be the eccentricity of what would be a pure ellipse in the absence of the earth's quadrupole moment and applying the results of reference [7] to the case of a polar orbit ($\theta = \pi/2$), we find†

$$\vec{\Omega}_{DS}^{(0)} = \frac{3Gm_2\omega}{2c^2a(1-e^2)}\vec{n}, \tag{37}$$

$$\vec{\Omega}_{LT} = \frac{GS^{(2)}}{2c^2a^3(1-e^2)^{3/2}}\vec{n}^{(2)}, \tag{38}$$

and

$$\vec{\Omega}_Q = -\left[\frac{3Gm_2\omega}{2c^2a(1-e^2)}\right]\left(\frac{3J_2}{4a^2}\right)\left[\frac{1+\frac{e^2}{4}+\frac{10}{4}e^2\cos^2 t}{(1-e^2)^2}\right]\vec{n} \tag{39}$$

where t is the angle between $\vec{n}^{(2)}$ and a vector from the focus of the elliptic orbit towards the perihelion. The e dependence of $\Delta\vec{\Omega}_{DS}$ is presently being studied by Barker and O'Connell. In addition, we note that the small value of the eccentricity of the earth's orbit around the sun, equal to 0.017, can be neglected.

As a final remark, we point out that other more conventional effects which also contribute to the precession of the gyroscope include

(a) the aberration of the starlight. This effect can be calculated very precisely. The semi-major axis of the small elliptic orbit through which the star seems to shift is 20.5" due to the earth's orbital motion, and of the order of 5" due to the satellite's motion. The effect of the gravitational field on the magnitude of this effect is found to be negligible [17].

† The quantity e should read e^2 in equation (28) of reference [7].

(b) parallax due to the earth's orbital motion. This is seldom greater than 0.6", even for the nearest stars.

(c) the proper motion of the star itself. Very few stars move more than 1" per year.

It is clear that observations taken exactly one year apart serve to eliminate the effects of parallax, light deflection, and aberration due to the earth's motion. Now, in this connection it should be emphasized that if the satellite deviates by a very small angle ε from an exact polar orbit then, as we see from equation (30), both the node and the angular momentum unit vector regress at the same rate. This has the result, as is clear form equations (19) and (32), that the direction of the de Sitter contribution $\vec{\Omega}^{(0)}_{DS}$ changes by an amount given by (for the usual 300 mile orbit)

$$\delta\vec{\Omega}^{(0)}_{DS} = -(1.67 \times 10^{-3}\varepsilon)'' \,\vec{J}/\mathrm{yr}, \tag{40}$$

where ε is measured in seconds. Consequently, for observations taken one year apart, this change in $\vec{\Omega}_{DS}$ cannot be ignored except for values of ε less than $\approx 0.5''$. If the latter criterion is not fulfilled then the analysis of reference [*10*] must be used.

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