THE GRAVITATIONAL FIELD OF THE ELECTRON

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We show that spin contributions to the gravitational field of an electron dominate for distances between the classical radius and the Compton wavelength of the electron. At such distances the gravitational force between two electrons may be repulsive, depending on the spin orientations.

As pointed out by Martin and Pritchett [1], "it is not correct to refer, as is sometimes done, to the Reissner-Nordström metric as the gravitational field of the electron". These authors presented a more realistic metric by taking into account the magnetic dipole moment associated with the spin of the electron. However, they neglected to take into account what proves to be a more important contribution and that is the gravitational field due to the spin itself. Expressing each contribution to the metric in units of the mass contribution \((2Gm/r^2c^2)\), a simple calculation based on the concept of equivalent mass **, shows that the charge and magnetic dipole contributions go roughly as \((r_\text{O}/r)\) and \(\{(r_\text{O}/r)(\lambda_\text{C}/r)^2\}\), respectively (where \(r_\text{O}\) and \(\lambda_\text{C}\) refer to the classical radius and Compton wavelength of the electron, respectively), in agreement with the conclusions of ref. [1]. It is our purpose here to point out that the spin contribution goes as \((\lambda_\text{C}/r)\) and thus dominates the metric for \(\lambda_\text{C} > r > r_\text{O}\).

Our approach is based on the detailed calculations of Barker and O'Connell [2] of the equations of motion of a gyroscope (and this analysis in turn is based on the elegant work of Gupta [3], who reinterpreted Einstein's theory as a theory of gravitation in flat space, thus making it readily amenable to the application [4] of well-known techniques from quantum electrodynamics). A simple extension of the results of ref. [2] enables us to write down an expression, correct to first order in the gravitational coupling constant and to first order in \(\beta (\beta = v/c\) where \(v\) is the non-relativistic velocity of one body with respect to the other), the spin-independent and spin-dependent parts of the total force \(F\) between two point bodies each with mass \(m\) and spin \(\frac{1}{2}h\) (the directions of the spin being denoted by \(\sigma(1)\) and \(\sigma(2)\)). Denoting the unit vector along \(r\) by \(n\), we find that

\[
F = F(n) + F(E) + F(1) + F(2) + F(1, 2),
\]

where

\[
F(n) = -(Gm^2/r^2) n,
\]

and

\[
F(1, 2) = 3|F(n)| \left[5\sigma(1) \cdot n)(\sigma(2) \cdot n) n - (\sigma(1) \cdot \sigma(2))
\]

\[
- (\sigma(1) \cdot n)\sigma(2) - (\sigma(2) \cdot n)\sigma(1)\right] \frac{\lambda_\text{C}^2}{r^2}
\]

are the dominant spin-independent and spin-dependent terms, respectively. It turns out that \(F(E) \sim \beta F(n)\), and both \(F(1)\) and \(F(2) \sim \beta(\lambda_\text{C}/r) F(n)\) and, in addition, \(F(1)\) and \(F(2)\) depend linearly on \(\sigma(1)\) and \(\sigma(2)\) respectively, and their directions depend on the spin orientations. There is a contribution \((\lambda_\text{C}/r)\) from each electron to the right-hand-side of eq. (3) from which readily follows our already stated conclusion that the spin contribution to the metric goes as \((\lambda_\text{C}/r)\) relative to the mass contribution. It also follows that for \(r < \lambda_\text{C}\) the overall gravitational

\[**\]The equivalent mass is proportional to \(c^{-2}(E^2 + H^2)r^3\), where \(E\) and \(H\) are the electric and magnetic fields due to the charge and magnetic dipole moment, respectively. Also \(E \sim (e/r^2)\) and \(H \sim \left(te\hbar/2mc/r^3\right) \sim E(\lambda_\text{C}/r)\).

\[\dagger\]The details follow closely those of ref. [2] except that here we treat quantum mechanical spinning particles of equal mass whereas in ref. [2] we concentrated on classical rotating masses, one being much heavier than the other.
force between two electrons can be repulsive. If
the spins are anti-parallel, then the spin-spin
force is repulsive (attractive) when the spins are
oriented perpendicular (parallel) to the line
joining the spin positions. For parallel spins the
reverse is true.

For \( r < \lambda_c \), the electrons will undoubtedly be
relativistic and it is thus likely that the \( f^{(1)} \) and
\( f^{(2)} \) terms (which are very complex in struc-
ture [2]) will also contribute, making it more
difficult to say which orientation of spins will
give repulsion. With application to the problem
of gravitational collapse in mind it is tempting
to apply the above analysis to the gravitational
interaction of neutrons. The difficulty (apart
from neglecting strong interactions, as we are
interested here in seeing how far we can go
with gravitational theory alone) is that neutrons
have structure and \( r < \lambda^{(n)} \) (the Compton wave-
length of the neutron) corresponds to the interior
of the neutron. It would thus be of interest to ob-
tain a generalization of the interior Schwarz-
schield solution to include spin effects.

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NUCLEAR MAGNETIC RESONANCE STUDIES OF \( ^{35}\mathrm{Cl}, ~^{37}\mathrm{Cl}, ~^{79}\mathrm{Br} \) AND \( ^{81}\mathrm{Br} \)

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The Larmor frequencies of \( ^{35}\mathrm{Cl}, ~^{37}\mathrm{Cl}, ~^{79}\mathrm{Br} \) and \( ^{81}\mathrm{Br} \) have been measured relative to the Larmor
frequency of \( ^{2}\mathrm{H} \) in aqueous solution and nuclear magnetic moments have been calculated for these nuclei.

The solvent isotope effect on the chemical shift of the halide nuclei in aqueous solution has been investi-
gated.

Recently, Deverell [1] has used various meth-
ods for calculating the nuclear magnetic shielding
constants (absolute chemical shift) of \( \mathrm{Cl}^- \) and \( \mathrm{Br}^- \)
ions in aqueous solutions. These shielding con-
stants could be evaluated experimentally [2-3],
by comparing the nuclear magnetic moment of
the free atom derived from atomic beam or opti-
cal pumping techniques, with nuclear magnetic
resonance (NMR) measurements of the nuclear
magnetic moment of the ion in solution.

The nuclear magnetic moments of the chlorine
and bromine isotopes have not been measured
with sufficient accuracy by either of these meth-
ods. Therefore we have determined the nuclear
magnetic moments of \( ^{35}\mathrm{Cl}, ~^{37}\mathrm{Cl}, ~^{79}\mathrm{Br} \) and \( ^{81}\mathrm{Br} \)
in the ions in solution by the NMR method.

Our spectrometer [3] is able to detect the
nuclei \( ^{2}\mathrm{H}, ~^{37}\mathrm{Cl}, ~^{79}\mathrm{Br} \) and \( ^{81}\mathrm{Br} \) at a field of
18.07 kOe * merely by variation of the radio fre-
quency. The Larmor frequencies of the halide
nuclei and the Larmor frequency of \( ^{2}\mathrm{H} \) have been measured in definite solutions of halide salts in
\( \text{D}_{2} \text{O} \) alternately in the same probe at constant
field only by varying the radio frequency. Table 1 shows the ratios of the Larmor frequencies in

* The magnetic field was held constant with the aid of a
\( ^{7}\mathrm{Li} \) NMR probe [4].