THE GRAVITATIONAL FIELD OF THE ELECTRON

R. F. O'CONNELL *
Institute of Theoretical Astronomy, University of Cambridge

Received 17 July 1970

We show that spin contributions to the gravitational field of an electron dominate for distances between the classical radius and the Compton wavelength of the electron. At such distances the gravitational force between two electrons may be repulsive, depending on the spin orientations.

As pointed out by Martin and Pritchett [1], "it is not correct to refer, as is sometimes done, to the Reissner-Nordstrom metric as the gravitational field of the electron". These authors presented a more realistic metric by taking into account the magnetic dipole moment associated with the spin of the electron. However, they neglected to take into account what proves to be a more important contribution and that is the gravitational field due to the spin itself. Expressing each contribution to the metric in units of the mass contribution $(2Gm/rc^2)$, a simple calculation based on the concept of equivalent mass **, shows that the charge and magnetic dipole contributions go roughly as (r_0/r) and $\{(r_0/r)(\lambda_c/r)^2\}$, respectively (where r_0 and λ_c refer to the classical radius and Compton wavelength of the electron, respectively), in agreement with the conclusions of ref. [1]. It is our purpose here to point out that the spin contribution goes as (λ_c/r) and thus dominates the metric for $\lambda_c > r > r_0$.

Our approach is based on the detailed calculations of Barker and O'Connell [2] of the equations of motion of a gyroscope (and this analysis in turn is based on the elegant work of Gupta [3], who reinterpreted Einstein's theory as a theory of gravitation in flat space, thus making it readily amenable to the application [4] of well-known techniques from quantum electrodynamics). A simple extension of the results of ref. [2] enables us to write down ‡ an expression, correct to first order in the gravitational coupling constant and to first order in β ($\beta = v/c$ where v is the non-relativistic velocity of one body with respect to the other), the spin-independent and spin-dependent parts of the total force $\boldsymbol{F}$ between two point bodies each with mass m and spin $\frac{1}{2}\hbar$ (the directions of the spin being denoted by $\boldsymbol{\sigma}^{(1)}$ and $\boldsymbol{\sigma}^{(2)}$). Denoting the unit vector along r by $\boldsymbol{n}$, we find that

$$\boldsymbol{F} = \boldsymbol{F}^{(n)} + \boldsymbol{F}^{(E)} + \boldsymbol{F}^{(1)} + \boldsymbol{F}^{(2)} + \boldsymbol{F}^{(1,2)}, \qquad (1)$$

where

$$\boldsymbol{F}^{(n)} = -(Gm^2/r^2)\boldsymbol{n}, \qquad (2)$$

and

$$\boldsymbol{F}^{(1,2)} = 3|\boldsymbol{F}^{(n)}|\{5(\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{n})(\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{n})\boldsymbol{n} - (\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}) - (\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{n})\boldsymbol{\sigma}^{(2)} - (\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{n})\boldsymbol{\sigma}^{(1)}\}\left(\frac{\lambda_c}{r}\right)^2 \qquad (3)$$

are the dominant spin-independent and spin-dependent terms, respectively. It turns out that $F^{(E)} \sim \beta F^{(n)}$, and both $F^{(1)}$ and $F^{(2)} \sim \beta(\lambda_c/r)F^{(n)}$ and, in addition, $F^{(1)}$ and $F^{(2)}$ depend linearly on $\sigma^{(1)}$ and $\sigma^{(2)}$ respectively, and their directions depend on the spin orientations. There is a contribution (λ_c/r) from each electron to the right-hand-side of eq. (3) from which readily follows our already stated conclusion that the spin contribution to the metric goes as (λ_c/r) relative to the mass contribution. It also follows that for $r < \lambda_c$ the overall gravitational

* Permanent address: Department of Physics and Astronomy, Louisiana State University, Baton Rouge, USA.

** The equivalent mass is proportional to $c^{-2}(E^2 + H^2)r^3$, where E and H are the electric and magnetic fields due to the charge and magnetic dipole moment, respectively. Also $E \sim (e/r^2)$ and $H \sim \{(e\hbar/2mc)/r^3\} \sim E(\lambda_c/r)$.

‡ The details follow closely those of ref. [2] except that here we treat quantum mechanical spinning particles of equal mass whereas in ref. [2] we concentrated on classical rotating masses, one being much heavier than the other.

force between two electrons can be repulsive. If the spins are anti-parallel, then the spin-spin force is repulsive (attractive) when the spins are oriented perpendicular (parallel) to the line joining the spin positions. For parallel spins the reverse is true.

For $r < \lambdabar_c$, the electrons will undoubtedly be relativistic and it is thus likely that the $F^{(1)}$ and $F^{(2)}$ terms (which are very complex in structure [2]) will also contribute, making it more difficult to say which orientation of spins will give repulsion. With application to the problem of gravitational collapse in mind it is tempting to apply the above analysis to the gravitational interaction of neutrons. The difficulty (apart from neglecting strong interactions, as we are interested here in seeing how far we can go with gravitational theory alone) is that neutrons have structure and $r < \lambdabar_c^{(n)}$ (the Compton wavelength of the neutron) corresponds to the interior of the neutron. It would thus be of interest to obtain a generalization of the interior Schwarzschild solution to include spin effects.

The author is pleased to acknowledge the collaboration with Dr. Bruce M. Barker in the derivation of the detailed calculations [2] upon which the above conclusions are based. The author would like to thank Professor F. Hoyle for his hospitality at the Institute of Theoretical Astronomy.

References

[1] A. W. Martin and P. L. Pritchett, J. Math. Phys. 9 (1968) 593.
[2] B. M. Barker and R. F. O'Connell, The equations of motion of a gyroscope, to be published.
[3] S. N. Gupta, in: Recent developments in general relativity (Pergamon Press, 1962) gives a summary and bibliography of this work.
[4] E. Corinaldesi, Proc. Phys. Soc. (London) A69 (1956) 189;
B. M. Barker, S. N. Gupta and R. D. Haracz, Phys. Rev. 149 (1966) 1027.

* * * * *

NUCLEAR MAGNETIC RESONANCE STUDIES OF ^{35}Cl, ^{37}Cl, ^{79}Br AND ^{81}Br

J. BLASER, O. LUTZ and W. STEINKILBERG
Physikalisches Institut der Universität Tübingen, 74 Tübingen, Gmelinstr. 6, Germany

Received 17 July 1970

The Larmor frequencies of ^{35}Cl, ^{37}Cl, ^{79}Br and ^{81}Br have been measured relative to the Larmor frequency of ^{2}H in aqueous solution and nuclear magnetic moments have been calculated for these nuclei. The solvent isotope effect on the chemical shift of the halide nuclei in aqueous solution has been investigated.

Recently, Deverell [1] has used various methods for calculating the nuclear magnetic shielding constants (absolute chemical shift) of Cl^- and Br^- ions in aqueous solutions. These shielding constands could be evaluated experimentally [2-3], by comparing the nuclear magnetic moment of the free atom derived from atomic beam or optical pumping techniques, with nuclear magnetic resonance (NMR) measurements of the nuclear magnetic moment of the ion in solution.

The nuclear magnetic moments of the chlorine and bromine isotopes have not been measured with sufficient accuracy by either of these methods. Therefore we have determined the nuclear magnetic moments of ^{35}Cl, ^{37}Cl, ^{79}Br and ^{81}Br in the ions in solution by the NMR method.

Our spectrometer [3] is able to detect the nuclei ^{2}H, ^{37}Cl, ^{79}Br and ^{81}Br at a field of 18.07 kOe * merely by variation of the radio frequency. The Larmor frequencies of the halide nuclei and the Larmor frequency of ^{2}H have been measured in definite solutions of halide salts in D_2O alternately in the same probe at constant field only by varying the radio frequency. Table 1 shows the ratios of the Larmor frequencies in

* The magnetic field was held constant with the aid of a ^{7}Li NMR probe [4].