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Magnetic Moment of a Magnetized Electron Gas and Magnetic Fields in White Dwarfs and Neutron Stars.

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It has been suggested by LEE, CANUTO, CHIU and CHIUDERI (¹) that the magnetization due to the magnetic moment associated with Landau levels will create in white dwarfs and neutron stars values of the magnetic induction B ranging from about 10^7 G up to fields of the order of $B_c = m^2c^3/e\hbar = 4.4 \cdot 10^{13}$ G. In some recent unpublished work we disagreed with this conclusion, but we now find that our criticism is invalid. Our previous analysis was based on an expansion of the various thermodynamical quantities but this is wrong because it turns out that these quantities are not analytic functions of B for small B.

We now present analytic arguments which in fact support the numerical results of LEE *et al.* (¹) (and apologies to these authors for our previous invalid criticism). We will concentrate initially on the nonrelativistic regime for which well-known and correct results exist. Using the results of WILSON (²) (which include the effects of spin), taking $T = 0$ (the most favourable condition for the existence of « LOFER » (¹)) and using units $\hbar = c = m_e$ (mass of the electron) $= 1$ (so that $B_c = \alpha^{-\frac{1}{2}}$, where α is the fine-structure constant) we find that the magnetic moment M and the total number of particles per unit volume N may be written

$$M = \frac{(2\mu)^{\frac{1}{2}}}{6\pi^2}\,\alpha B\left\{1 - \frac{3}{2}\pi^{-\frac{1}{2}} b^{\frac{1}{2}} \sum_{r=1}^{\infty} \frac{\sin(br - \pi/4)}{r^{\frac{1}{2}}}\right\}, \tag{1}$$

$$N = \frac{(2\mu)^{\frac{3}{2}}}{3\pi}\left\{1 + \frac{3\pi^{\frac{1}{2}}}{2} b^{-\frac{3}{2}} \sum_{r=1}^{\infty} \frac{\sin(br - \pi/4)}{r^{\frac{3}{2}}}\right\}. \tag{2}$$

(¹) H. J. LEE, V. CANUTO, H. Y. CHIU and C. CHIUDERI: *Phys. Rev. Lett.*, **23**, 390 (1969); V. CANUTO, H. Y. CHIU, C. CHIUDERI and H. J. LEE: *New source of intense magnetic fields in neutron stars* (to be published).

(²) A. H. WILSON: *The Theory of Metals*, 2nd edition (Cambridge, 1965), p. 171.

where

$$b \equiv [2\pi\mu/(B/B_c)] \tag{3}$$

and μ denotes the chemical potential.

Now, since N is a constant, it is clear that μ in general depends on B and the same remark applies to the μ appearing in the various results of CHIU *et al.* [3]. However, in the case where $b \gg 1$, it is clear from eq. (2) that μ may be regarded as a constant and, following LEE *et al.* [1], this is the case we will consider henceforth. An estimate of the maximum absolute value of the oscillatory part of M, $M^{(\mathrm{osc})}$ say [1], can be made by noting that

$$\sum_{r=1}^{\infty} \frac{\sin(br - \pi/4)}{r^{\frac{3}{2}}} \lesssim \frac{1}{\sqrt{2}} \sum_{r=1}^{\infty} r^{-\frac{3}{2}} \simeq 1.77\,. \tag{4}$$

If we now accept the equation [1] $B = H + 4\pi M(B)$ and take $H = 0$, then it follows from eqs. (1) and (4) that the maximum value of B may be found from the equation

$$B + dB^{\frac{1}{2}} = 0\,, \tag{5}$$

where

$$d \simeq \{4\pi |M^{(\mathrm{osc})}_{\max}|/B^{\frac{1}{2}}\} = 3.54\,\pi^{-2}\alpha^{\frac{1}{2}}\mu\,. \tag{6}$$

Now for $b \gg 1$, it is clear that d is essentially independent of B. Thus, the solutions of eq. (5) are $B = 0$ and $B^{\frac{1}{2}} = -d$. The latter solution implies $B = d^2$. It follows that

$$\frac{B}{B_c} = 0.13\,\alpha^2\mu^2 = 6.93\cdot 10^{-4}\mu^2\,. \tag{7}$$

If we assume that eq. (7) is also correct in the relativistic regime, then for very dense bodies we get

$$B_8 = 3(\varrho_6/\mu_e)^{\frac{2}{3}}\,, \tag{8}$$

where B_8 is the maximum magnetic field in units of 10^8 G, ϱ_6 is the density in units of 10^6 g/cm^3 and μ_e = (number of nucleons/number of electrons). The $\varrho^{\frac{2}{3}}$ dependence of $B_{\max}$ is in complete agreement with the numerical behaviour obtained by LEE *et al.* [1] and our somewhat larger absolute values for $B_{\max}$ probably result from an over-estimate of $|M^{(\mathrm{osc})}_{\max}|$ by a factor of about 2.5.

That eq. (7) is in fact the correct relativistic equation finds support from the work of LIFSHITZ and KOSEVICH [4], who derive the magnetic moment for an arbitrary disper-

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(3) H. Y. CHIU and V. CANUTO: *Phys. Rev. Lett.*, **21**, 110 (1968); H. Y. CHIU and V. CANUTO: *Astrophys. Journ.*, **153**, L157 (1968); V. CANUTO, H. Y. CHIU and L. FASSIO-CANUTO: *Astrophys. Space Sci.*, **3**, 258 (1969); H. Y. CHIU, V. CANUTO and L. FASSIO-CANUTO: *Phys. Rev.*, **176**, 1438 (1968).

(4) I. M. LIFSHITZ and A. M. KOSEVICH: *Žurn. Ėksp. Teor. Fiz.*, **29**, 730 (1955) (translation: *Sov. Phys. JETP*, **2**, 636 (1955)).

sion law and this work may be applied with appropriate modifications to treat the case of the energy eigenvalues of a relativistic electron. As in the nonrelativistic case, for $b \gg 1$, the amplitude of the oscillatory part of the thermodynamical potential is small relative to the nonoscillatory part but the reverse is true in the expression for the magnetic moment. This is due to the crucial fact that the argument of the cosine term in the expression for the thermodynamical potential (see eq. (2.20) of ref. [4]) is proportional to S_m/B, where S_m is the extremal value of the cross-sectional area of the Fermi surface and, for $\mu \gg 1$ (dense bodies), S_m is proportional to μ^2. When this argument is differentiated with respect to B it results in a relatively large amplitude for $M^{(\mathrm{osc})}$. In addition, we have numerically verified that μ may also be considered as constant in the relativistic region, provided that $b \gg 1$.