Reply to "Comment on 'Quantum measurement and decoherence' "

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While agreeing with our exact expression for the time dependence of the motion of a free particle in an initial superposition state, corresponding to two identical Gaussians separated by a distance d, at temperature T, Gobert *et al.*, in the preceding Comment [Phys. Rev. A **70**, 026101 (2004)], dispute our conclusions on decoherence time scales. However, the parameters they used to generate their figures are outside the regime of validity of our interpretation of the results and, moreover, are not of physical interest in that they correspond to $T\approx 0$. The point is that in their figures they have chosen the thermal de Broglie wavelength $\lambda_{th}=\hbar\sqrt{mkT}$ to be equal to slit spacing d, whereas we have clearly stated [in the paragraph preceding Eq. (21) of our paper] that decoherence occurs and that our expression for the decoherence time applies only in the limit where d is large compared not only with the slit width σ but also with the thermal de Broglie wavelength, $d\gg\lambda_{th},\sigma$.

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For a free particle in an initial superposition state, corresponding to a pair of Gaussians each of width σ and separated by a distance d, at temperature T, we presented an exact expression for the coordinate probability, $P(x,t)$ [1,2]. Gobert *et al.* (hereafter referred to as GDA), in the preceding Comment [3], agree with this result, which is valid for arbitrary temperature T and arbitrary strength of the coupling to the heat bath as measured by the dissipative decay rate γ. GDA also agree that their results for the corresponding reduced density matrix are consistent with our results for $W(q,p,t)$, the Wigner function for the probability in Wigner phase space [4] (we note that the Wigner distribution function is the Fourier transform of the density matrix and contains the same information [5]). Based on our result for $P(x,t)$ we concluded that decoherence without dissipation can occur for all cases of interest. GDA dispute this conclusion. Our purpose here is to show their analysis is flawed due to a choice of parameters that are not of physical interest in that they correspond to $T\approx 0$. The point is that their figures correspond to a slit spacing d equal to the thermal de Broglie wavelength

$$\lambda_{\mathrm{th}}=\frac{\hbar}{\sqrt{mkT}}. \tag{1}$$

But we have clearly stated [in the paragraph preceding Eq. (21) of our paper] that decoherence occurs and that our expression for the decoherence time applies only in the limit where d is large compared not only with the slit width σ but also the thermal de Broglie wavelength, $d\gg\lambda_{th},\sigma$. Indeed, decoherence is fully developed even for the very modest ratio d/λ_{th}. To illustrate this we show in Fig. 1 a plot of $P(x,t)$ [multiplied by σ to make it dimensionless] versus x (divided by σ) at a time $t=2m\sigma d/5\hbar\equiv t_{\mathrm{mix}}/5$ for three different values of the ratio d/λ_{th}. The solid curve corresponds to $d/\lambda_{\mathrm{th}}=5$ and there we see that there is no hint of an interference pattern, that is, decoherence has occurred. The two dashed curves, which are nearly indistinguishable, correspond to the GDA choice $d/\lambda_{\mathrm{th}}=1$ and the zero-temperature case $d/\lambda_{\mathrm{th}}=0$, the zero temperature case having the slightly larger interference amplitude. In all three cases we have neglected the coupling to the bath (that is, set $\gamma=0$) and made the GDA choice $d=20\sigma$. We assert that this figure refutes those of GDA, showing that under the conditions we have stated decoherence without dissipation does occur. GDA ask how can it be that the interference pattern has not disappeared in their Fig. 1. The answer of course is that they have chosen values of the parameters corresponding to an effectively zero temperature, for which in the absence of dissipation there is of course no decoherence. As our Fig. 1 shows, if they were to choose only a modestly higher temperature, corresponding to $d/\lambda_{\mathrm{th}}=5$, they would see no interference. We forbear to present a repeat of GDA's Fig. 1, since for zero temperature the figure would be indistinguishable from theirs, while for $d/\lambda_{\mathrm{th}}=5$ the figure would be utterly featureless, consisting of two Gaussians propagating independently without interference. Perhaps a numerical illustration might be helpful. Since the magnitude of the de Broglie wavelength λ_{th} plays a key role in the analysis, we use Eq. (1) to write

$$\lambda_{\mathrm{th}}^2=(5.1\times 10^{-21}\ \mathrm{cm})^2\left(\frac{1\ \mathrm{g}}{m}\right)\left(\frac{300\ \mathrm{K}}{T}\right). \tag{2}$$

As an illustrative example, Zurek, in his oft-cited paper [6], chooses $m=1$ g, $d=1$ cm, and $T=300$ K, corresponding to the very large ratio $d/\lambda_{\mathrm{th}}=2\times 10^{20}$. Put another way, the GDA choice of $d/\lambda_{\mathrm{th}}=d$ would require a temperature of 8.1×10^{-39} K. This is perhaps an extreme choice of parameters, but it illustrates the fact that the condition $d\gg\lambda_{\mathrm{th}}$ is not unique to our discussion but has generally been understood to apply in earlier discussions of decoherence. In general, the

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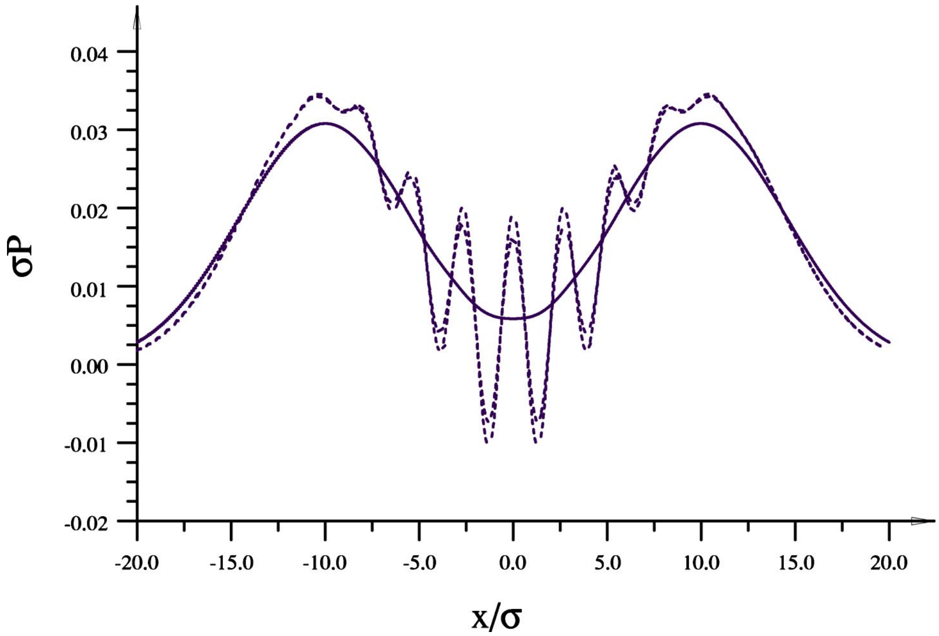

FIG. 1. $P(x,t)$ the coordinate probability at time t (multiplied by σ to make it dimensionless) as a function of x (divided by σ) is shown at a time $t=t_{\mathrm{mix}}/5$. The solid curve, which shows no hint of interference, corresponds to $\lambda_{\mathrm{T}}=d/5$. The two dashed curves, which are nearly indistinguishable, correspond to the GDA choice of parameters: $\lambda_{\mathrm{T}}=d$ and to the zero temperature case $\lambda_{\mathrm{T}}=\infty$, with the zero temperature interference pattern having a slightly larger amplitude.

decoherence times τ_{d} are typically much smaller than the decay time γ^{-1}. But this is so only under our condition $\lambda_{\mathrm{th}} \ll d$, no matter how one defines decoherence. Next, we turn to the question of how decoherence should be defined. As we have stressed more than once, "…a quantitative measure of decoherence depends not only on the specific system being studied but also on whether one is considering coordinate, momentum, or phase space" [4]. On the other hand, based on their density matrix analysis (which is nothing more than that given previously by us for Wigner phase space [4]), GDA claim that this implies that there is no decoherence in physical space, which as we have seen is not true. Moreover, while again recognizing that there are various ways of defining decoherence, our definition of an attenuation coefficient $a(t)$ was such as to ensure that $a(t)=1$ for all times in the case where $T=0$ and $\gamma=0$, corresponding to a pure Schrödinger cat state at all times (as discussed in detail in Sec. VI B. of Ref. [7]). This choice leads to a time scale which is convenient for this system but other definitions of decoherence time (which will also involve the temperature) might also be considered, depending on what particular features of the time development are of interest. For example, it is instructive to write our general result for $P(x,t)$ in the form [1,8]

$$P(x,t) = \frac{1}{2(1+e^{-d^2/8\sigma^2})}\left\{P_0\left(x-\frac{d}{2},t\right)+P_0\left(x+\frac{d}{2},t\right) + 2e^{-d^2/8w^2(t)}a(t)P_0(x,t)\cos\frac{[x(0),x(t)]xd}{4i\sigma^2 w^2(t)}\right\}, \quad (3)$$

where P_0 is the probability distribution for a single wave packet, given by

$$P_0(x,t) = \frac{1}{\sqrt{2\pi w^2(t)}}\exp\left\{-\frac{x^2}{2w^2(t)}\right\}. \quad (4)$$

Here and in Eq. (3) $w^2(t)$ is the variance of a single wave packet, which in general is given by

$$w^2(t) = \sigma^2 - \frac{[x(0),x(t)]^2}{4\sigma^2} + s(t), \quad (5)$$

where σ^2 is the initial variance, $[x(0),x(t)]$ is the commutator, and

$$s(t) = \langle\{x(t)-x(0)\}^2\rangle \quad (6)$$

is the mean square displacement. Also, $a(t)$, which can be defined as the ratio of the factor multiplying the cosine in the interference term to twice the geometric mean of the first two terms [2] is given by the following exact general formula:

$$a(t) = \exp\left\{-\frac{s(t)d^2}{8\sigma^2 w^2(t)}\right\}. \quad (7)$$

As we emphasized in Ref. [2], it is only for the case of ohmic dissipation, high temperature ($kT \gg \hbar\gamma$) and $d \gg \lambda_{\mathrm{th}}$, σ, that Eq. (7) reduces to

$$a(t) \to \exp\left\{-\left(\frac{t}{\tau_d}\right)^2\right\}, \quad (8)$$

where

$$\tau_d = \frac{\sqrt{8}\,\sigma^2}{\sqrt{kT/m}\,d}, \quad (9)$$

the result quoted by GDA in their Eq. (4). But, this result clearly does not apply for $d=\lambda_{\mathrm{th}}$, the GDA choice of parameters.

GDA continually refer to Ford, Lewis, and O'Connell (FLO's) "–imperfect preparation of the initial state–". What perhaps they mean is that the initial state of FLO is not a pure state, which is certainly the case. But a pure state necessarily corresponds to a particle at zero temperature. FLO argue that the picture of a particle at zero temperature suddenly coupled to a heat bath at a high temperature T is not a realistic picture of the physical situation. The essence of FLO's paper is to show that one can, by measurement, prepare an initial state that is entangled with the bath, with a temperature equal to that of the bath, but with a probability distribution identical with that of the pure state. They argue that this is a more realistic initial state. We conclude that the comment of GDA is simply misleading due to their unrealistic choice of parameters and we reiterate our previous conclusion that decoherence can occur without dissipation.

Note added. In response to our Reply, Gobert *et al.* added the case $T=10E$ (i.e., $d=\sqrt{3}\ \lambda_{\rm th}$) to their Fig. 3 (along with associated comments). While not denying our conclusion that the interference pattern disappears when the thermal wavelength is only modestly greater than the slit spacing, they then argue that $a(t)$ is not a good measure of decoherence. However, we point out that their revised Fig. 3 shows that under the condition $d \gg \lambda_{\rm th}$ the amplitude of the interference term as measured by the function $a(t)$ does indeed fall to near zero in a time of the order of our choice of the decoherence time. In summary, on a positive note, we were pleased to see that Gobert *et al.* agreed with our analytic results while giving us the opportunity to clarify in more detail the interpretation of such results, especially our emphasis on observable coordinate probabilities, rather than density matrix elements, as constituting the best measure of decoherence.

[1] G. W. Ford, J. T. Lewis, and R. F. O'Connell, Phys. Rev. A **64**, 032101 (2001), referred to in the text as FLO.

[2] G. W. Ford and R. F. O'Connell, Phys. Lett. A **286**, 87 (2001).

[3] D. Gobert, J. von Delft and V. Ambegaokar, Phys. Rev. A **70**, 026101 (2004).

[4] M. Murakami, G. W. Ford, and R. F. O'Connell, Laser Phys. **13**, 180 (2003).

[5] M. Hillery, R. F. O'Connell, M. O. Scully, and E. P. Wigner, Phys. Rep. **106**, 121 (1984).

[6] W. H. Zurek, Phys. Today **44**(10), 36 (1991).

[7] G. W. Ford and R. F. O'Connell, Am. J. Phys. **70**, 319 (2002).

[8] G. W. Ford and R. F. O'Connell, J. Opt. B: Quantum Semiclassical Opt. **5**, S609 (2003).