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Effect of a Constant Magnetic Field on the Neutron Beta Decay Rate and its Astrophysical Implications

RECENT advances in the production of large magnetic fields in the laboratory[1,2] have generated interest in the effect of intense magnetic fields on various phenomena[3,4]. The largest field that can be produced in the laboratory[1,2] at present is about 10^6 G, which is considerably lower than the quantum critical field value[5] of $H_c = m^2c^3/e\hbar = 4{\cdot}4 \times 10^{13}$ G; but the "cosmic laboratory" may be a source of much stronger fields and it has been suggested[6] that magnetic fields as large as 10^{14}–10^{16} G may exist in neutron stars. Hoyle[7] has cited the possibility of a large primordial magnetic field, and Brownell and Callaway[8] speculate that neutron stars and the dense early universe may be ferromagnetic. One of us (R. F. O.) has examined[9] various effects of a large magnetic field and has indicated an effect of magnetic fields which has been often ignored in astrophysical investigations, namely, that the rates of all elementary particle processes will be affected. Pursuing this idea, we examine here the effect of a magnetic field on the β decay rate of a neutron. This is a fundamental process in many astrophysical phenomena and in particular it is very important[10] in a problem of current interest, that is, the production of He in the "big-bang" expansion of the universe[11]. In addition, our calculations should be applicable to other elementary particle processes. We present here only the main ideas; the calculation details will be published elsewhere[12].

We start with the V–A Hamiltonian density[13] but now use the exact wave functions for an electron in a constant magnetic field[14] ($H = H_z$, say). These wave functions are characterized[14] by discrete quantum numbers n and s and by the continuous variable k which denotes the z component of the linear momentum of the electrons. We treat the nucleons non-relativistically, as is usual. Integrating over the neutrino variables and summing over the quantum number s of the electrons, as well as the electron and proton spin states, we obtain the transition probability/s to an electron Landau level[14] n and momentum interval k

to $k+\mathrm{d}k$, for an unpolarized source of neutrons. Integration over k [consistent with $k\binom{\max}{\min}=\pm\sqrt{W_0^2-1-4\gamma n}$] and summing over all allowed Landau levels to find the total neutron β decay rate ($\hbar=c=m_e=1$) gives

$$w(H)=\frac{\gamma g_v^2}{\pi^3}(1+3\lambda^2)\sum_{n=0}^{n(\max)}(1-\tfrac{1}{2}\delta_{n0})\Big\{W_0^2(P_0^2-4\gamma n)^{1/2}-\frac{2}{3}(P_0^2-4\gamma n)^{3/2}-W_0(1+4\gamma n)\sinh^{-1}[(P_0^2-4\gamma n)/(1+4\gamma n)]^{1/2}\Big\} \tag{1}$$

where[13] $\lambda\equiv g_A/g_V\simeq 1{\cdot}18$, $W_0=2{\cdot}53$ is the total decay energy, $\gamma\equiv\frac{1}{2}\frac{H}{H_c}$, $P_0^2=W_0^2-1$, and where $n(\max)$ is the largest integer occurring in $P_0^2/4\gamma$. Using the Euler summation formula

$$w(H)=\Big\{1+0{\cdot}47\Big(\frac{H}{H_c}\Big)^2+\cdots\Big\}\,w(O)\ \text{for}\ H\ll H_c \tag{2}$$

where $w(O)$ is the zero field decay rate[13]. It is clear that the correction to $w(O)$ is negligible for the largest fields attainable at present in the laboratory ($H<10^{-7}\,H_c$). The situation, however, is quite different for fields of the order of those speculated to exist in the early universe and in neutron stars. When $H>2{\cdot}7\,H_c$, the only term in the sum that contributes is $n=0$ (this corresponds to the "one-dimensional gas" behaviour discussed by Manley[14]) and

$$\begin{aligned} w(H>2{\cdot}7\,H_c)&=\frac{g_v^2}{4\pi^3}\frac{H}{H_c}(1+3\lambda^2)\,\{P_0+\tfrac{1}{3}P_0^3-W_0\sinh^{-1}P_0\}\\ &=0{\cdot}77\,\frac{H}{H_c}\,w(O) \end{aligned} \tag{3}$$

For a magnetic field of arbitrary strength, numerical evaluation of equation (1) leads to the results presented in Fig. 1. An important feature is that $w(H)$ is always greater than $w(O)$, and increases linearly with H for $H>2{\cdot}7\,H_c$.

What are the possible effects of our results on the production of He in an expanding universe? He is produced at a temperature of 10^8–10^9 °K about 10^2 s after the start of the "big-bang"[10,11]. The value of the primordial magnetic field[7] and its dependence on time is a matter of speculation, but if a strong magnetic field does exist at this time then, as is clear from Fig. 1, $w(H)$ may be substantially larger than $w(O)$ and a larger decay rate for the neutron would reduce the amount of He produced[10], which agrees with experiment[10,11]. But other factors must be considered. First, the magnetic field affects the thermodynamic energy distribution of the electron gas of the expanding universe. In addition, the magnetic field also affects the He production rate because the usual Friedman homogeneous model

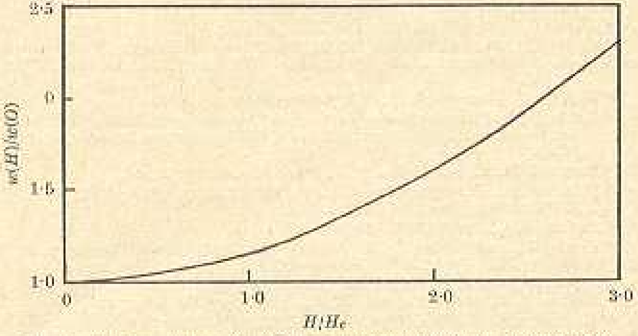

Fig. 1. The dependence of $w(H)$, the neutron β decay rate in a magnetic field, on the magnetic field H.

for an expanding universe must be replaced by an anisotropic model[14] (which in general reduces the He production rate[16]).

What is the possibility of maintaining a magnetic field greater than H_c? Because of the fact that $H > H_c$ implies that $2\mu_B H > mc^2$ (where $\mu_B = e\hbar/2mc$ is the Bohr magneton), it is sometimes stated that spontaneous pair production will occur in magnetic fields greater than H_c with the result that the magnetic field is destroyed, but this is incorrect because it considers only the Pauli spin energy of the electron and neglects the orbital energy (because of the quantization of orbits the latter can never be zero). One of us (R. F. O.) re-examined this question[3], including not only these contributions to the energy but also the energy arising from the interaction of the anomalous magnetic moment of the electron with the magnetic field. It was concluded that spontaneous pair production will not occur for values of $H \simeq H_c$ but will only occur for values of H greater than $4\pi\alpha^{-1}H_c (\simeq 5 \times 10^{16}$ G). For fields greater than this value electron–positron pairs can be created at the expense of the thermal energy of the system and these pairs may then act to destroy the magnetic field which helped to give them birth.

Finally, the effect of a magnetic field on elementary particle rates will also have application to the cooling rate of neutron stars if large magnetic fields exist there[4].

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