



ELSEVIER

23 July 2001

Physics Letters A 286 (2001) 87–90

PHYSICS LETTERS A

www.elsevier.com/locate/pla

Decoherence without dissipation

G.W. Ford^a, R.F. O’Connell^{b,*}

^a *Department of Physics, University of Michigan, Ann Arbor, MI 48108-1120, USA*

^b *Department of Physics and Astronomy, Louisiana State University, Baton Rouge, LA 70803-4001, USA*

Received 2 April 2001; received in revised form 5 June 2001; accepted 12 June 2001

Communicated by P.R. Holland

Abstract

The prototypical Schrödinger cat state, i.e., an initial state corresponding to two widely separated Gaussian wave packets, is considered. The decoherence time is calculated solely within the framework of elementary quantum mechanics and equilibrium statistical mechanics. This is at variance with common lore that irreversible coupling to a dissipative environment is the mechanism of decoherence. Here, we show that, on the contrary, decoherence can in fact occur at high temperature even for vanishingly small dissipation. © 2001 Elsevier Science B.V. All rights reserved.

PACS: 03.65.Bz; 05.30.-d; 05.40.+j

Quantum teleportation [1], quantum information and computation [2,3], entangled states [4], Schrödinger cats [5], and the classical–quantum interface [6]: topics at forefront of research embracing quantum physics, information science and telecommunications and all depending on an understanding of decoherence [7], i.e., how a quantum interference pattern is destroyed. In an introduction to the contents of a recent book devoted wholly to this subject, Joos surveys the current situation and, in discussing the mechanism of decoherence, states that “. . . irreversible coupling to (a dissipative) environment seems to have become widely accepted . . .”. Here, while we agree that coupling to the environment is necessary to establish thermal equilibrium, we show that at high temperature decoherence occurs even for vanishingly small dissipation. The situation is like that for an ideal gas: colli-

sions are necessary to bring the gas to equilibrium but do not appear in the equation of state, nor in the velocity distribution.

Much of the discussion of decoherence [8–11] has been in terms of the simple problem of a particle moving in one dimension that is placed in an initial superposition state (Schrödinger “cat” state) corresponding to two widely separated wave packets. The motivation for this choice is that it can be applied, say, to describe the interference pattern arising in Young’s two-slit experiment [8] or that arising from a quantum measurement involving a pair of “Gaussian slits” [12,13]. Of primary interest is the question of the classical–quantum interface, i.e., how the interference pattern is destroyed with the evolution of a classical state corresponding to two separately propagating packets. Decoherence refers to this destruction of the interference pattern and key questions are what is the origin of decoherence and what is the time scale for loss of coherence. The maintenance of coherence is an essential element in quantum teleportation, etc. Thus, an

* Corresponding author.

E-mail address: rfoc@stp.dias.ie (R.F. O’Connell).

understanding of all physical phenomena which can cause decoherence is essential. Our purpose here is to give an elementary calculation showing that at high temperature ($kT \gg \hbar\gamma$, where γ is the dissipative decay rate) decoherence occurs in a very short time that is, contrary to widely held belief, independent of the strength of coupling to the environment. Our starting point is the prototypical Schrödinger cat state, i.e., an initial state corresponding to two separated Gaussian wave packets. The corresponding wave function has the form

$$\psi(x, 0) = \frac{1}{[2(1 + e^{-d^2/8\sigma^2})]^{1/2}} \times \left(\frac{\exp\left\{-\frac{(x-d/2)^2}{4\sigma^2} + i\frac{mv}{\hbar}x\right\}}{(2\pi\sigma^2)^{1/4}} + \frac{\exp\left\{-\frac{(x+d/2)^2}{4\sigma^2} + i\frac{mv}{\hbar}x\right\}}{(2\pi\sigma^2)^{1/4}} \right), \quad (1)$$

where σ is the width of each packet, d is the separation between the centers of the two packets and v is the particle velocity. Next, we solve the free particle Schrödinger equation,

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2}, \quad (2)$$

with this initial state. The general solution is [14]

$$\psi(x, t) = \sqrt{\frac{m}{2\pi i \hbar t}} \int_{-\infty}^{\infty} dx' \exp\left\{-\frac{m(x-x')^2}{2i\hbar t}\right\} \times \psi(x, 0). \quad (3)$$

Hence, using (1) we obtain

$$\psi(x, t) = \frac{e^{i(mv/\hbar)x - i(mv^2 t/2\hbar)}}{[2(1 + e^{-d^2/8\sigma^2})]^{1/2}} \times \left(\frac{1}{[2\pi(\sigma + \frac{i\hbar t}{2m\sigma})^2]^{1/4}} \times \exp\left\{-\frac{(x-d/2-vt)^2}{4\sigma^2(1 + \frac{i\hbar t}{2m\sigma^2})}\right\} + \frac{1}{[2\pi(\sigma + \frac{i\hbar t}{2m\sigma})^2]^{1/4}} \times \exp\left\{-\frac{(x+d/2-vt)^2}{4\sigma^2(1 + \frac{i\hbar t}{2m\sigma^2})}\right\} \right). \quad (4)$$

Hence, the probability distribution, $P(x; t) = |\psi(x, t)|^2$, is

$$P(x; t) = \frac{1}{2(1 + e^{-d^2/8\sigma^2})\sqrt{2\pi(\sigma^2 + \frac{\hbar^2 t^2}{4m^2\sigma^2})}} \times \left(\exp\left\{-\frac{(x-d/2-vt)^2}{2(\sigma^2 + \frac{\hbar^2 t^2}{4m^2\sigma^2})}\right\} + \exp\left\{-\frac{(x+d/2-vt)^2}{2(\sigma^2 + \frac{\hbar^2 t^2}{4m^2\sigma^2})}\right\} + 2 \exp\left\{-\frac{(x-vt)^2 + d^2/4}{2(\sigma^2 + \frac{\hbar^2 t^2}{4m^2\sigma^2})}\right\} \times \cos \frac{\hbar t d(x-vt)}{4m\sigma^2(\sigma^2 + \frac{\hbar^2 t^2}{4m^2\sigma^2})} \right). \quad (5)$$

This is all within the realm of conventional quantum mechanics [14].

Next we consider the case of a particle in thermal equilibrium, but so weakly coupled to the environment that we can neglect dissipation. The principles of statistical mechanics then tell us that we obtain the corresponding probability distribution by averaging distribution (5) over a thermal distribution of velocities. The result is

$$P_T(x; t) \equiv \sqrt{\frac{m}{2\pi kT}} \int_{-\infty}^{\infty} dv \exp\left\{-\frac{mv^2}{2kT}\right\} P(x; t) = \frac{1}{2(1 + e^{-d^2/8\sigma^2})\sqrt{2\pi w^2}} \times \left(\exp\left\{-\frac{(x-d/2)^2}{2w^2}\right\} + \exp\left\{-\frac{(x+d/2)^2}{2w^2}\right\} + 2 \exp\left\{-\frac{x^2}{2w^2} - \frac{w^2 + \frac{kT}{m}t^2(\frac{\hbar t}{2m\sigma^2})^2 d^2}{(\sigma^2 + \frac{\hbar^2 t^2}{4m^2\sigma^2})w^2} \frac{d^2}{8}\right\} \times \cos \frac{\hbar t dx}{4m\sigma^2 w^2} \right), \quad (6)$$

where we have used a subscript T to emphasize that this is the probability distribution at finite temperature and where we have introduced

$$w^2(t) = \sigma^2 + \frac{kT}{m}t^2 + \frac{\hbar^2}{4m^2\sigma^2}t^2. \quad (7)$$

This probability distribution is the sum of three contributions, corresponding to the three terms within the parentheses. The first two clearly correspond to a pair of separately expanding wave packets, with $w^2(t)$ the width of each, while the third term, the one involving the cosine, is an interference term. The attenuation coefficient $a(t)$ is the ratio of the factor multiplying the cosine to twice the geometric mean of the first two terms. Thus

$$\begin{aligned} a(t) &= \exp\left\{-\frac{kTd^2}{8m\sigma^2w^2}t^2\right\} \\ &= \exp\left\{-\frac{\frac{kT}{m}t^2d^2}{8\sigma^4 + 8\sigma^2\frac{kT}{m}t^2 + \frac{2\hbar^2t^2}{m^2}}\right\}. \end{aligned} \quad (8)$$

At $t = 0$, we see that $a(0) = 1$ corresponding to maximum coherence and, as mentioned above, the goal of experimentalists is to maintain this coherence. However, for very short times, we see that $a(t) \cong \exp\{-t^2/\tau_d^2\}$, where the decoherence time is

$$\tau_d = \frac{\sqrt{8}\sigma^2}{\bar{v}d}, \quad (9)$$

in which $\bar{v} = \sqrt{kT/m}$ is the mean thermal velocity. This decoherence time is much different from that quoted extensively in the literature [7,11], namely $\gamma^{-1}\hbar^2/mkTd^2$, which is inversely proportional to γ (the dissipative decay rate). By contrast, τ_d given by (9) is independent of γ . The reason why existing calculations fail to obtain the form (8) for the attenuation coefficient at short times is that they are based on the assumption that the initial state of the particle is a pure state (of the form (1) with $v = 0$) and use a master equation to describe the time development [7–11]. Such a pure state is effectively at zero temperature and when the particle is suddenly coupled to a bath at temperature T , as described by the master equation, it takes a time of order γ^{-1} for the particle to warm up and acquire a thermal distribution of velocities. Such an approach therefore misses the initial thermal distribution of velocities responsible for the rapid loss of

coherence we have obtained. The result we have obtained does follow from a new approach [13], which is not based on a master equation and which incorporates both arbitrary temperature and arbitrary dissipation. In fact, the exact general formula for the attenuation coefficient, expressed in terms of the mean square displacement and the nonequal-time commutator, is given by [13]

$$a_{\text{exact}}(t) = \exp\left\{-\frac{s(t)d^2}{8\sigma^2w_{\text{exact}}^2(t)}\right\}, \quad (10)$$

where now the width of a single wave packet is given by

$$w_{\text{exact}}^2(t) = \sigma^2 - \frac{[x(t_1), x(t_1 + t)]^2}{4\sigma^2} + s(t), \quad (11)$$

in which

$$s(t) = \langle\{x(t_1) - x(t_1 + t)\}^2\rangle \quad (12)$$

is the mean square displacement. For the special case of a free particle without dissipation, where $s(t) = (kT/m)t^2$ and $[x(t_1), x(t_1 + t)] = i\hbar t/m$, this reduces to (8) above. However, even in the presence of dissipation, for times short compared with γ^{-1} , where γ is a typical dissipative decay rate, the motion is again that of a free particle. For such free motion, there is a rapid decay of coherence with characteristic time τ_d given by (9). It should be stressed that since decoherence decay times are always much smaller than dissipative decay times, we always have $\gamma\tau_d \ll 1$. For example, if we consider an electron at room temperature (300 K), then $\bar{v} = 6.8 \times 10^6$ cm/s so that if we take $d = 1$ cm and $\sigma = 0.4$ Å, then using (9) we obtain $\tau_d = 6.9 \times 10^{-24}$ s, which is orders of magnitude smaller than typical γ^{-1} values. Even for $T = 1$ K (which fulfills our definition of high temperature, i.e., $kT \gg \hbar\gamma$, for $\gamma \ll 10^{11}$ s $^{-1}$) we obtain $\tau_d = 1.2 \times 10^{-22}$ s.

In summary, we have presented a simple derivation of the result for decoherence without dissipation, working solely within the framework of elementary quantum mechanics and equilibrium statistical mechanics.

References

- [1] A. Zeilinger, Sci. Am. 282 (4) (2000) 50.

- [2] C.H. Bennett, *Phys. Today* 48 (10) (1995) 24.
- [3] *Quantum Information, Special Issue of Phys. World* 11 (3) (1998).
- [4] S. Haroche, *Phys. Today* 51 (7) (1998) 36.
- [5] A. Zeilinger, *Nature* 408 (2000) 639.
- [6] A. Tegmark, J.A. Wheeler, *Sci. Am.* 284 (2) (2001) 68.
- [7] D. Giulini, E. Joos, C. Kiefer, J. Kupisch, I.-O. Stamatescu, H.D. Zeh, *Decoherence and the Appearance of a Classical World in Quantum Theory*, Springer, New York, 1996.
- [8] A.O. Caldeira, A.J. Leggett, *Phys. Rev. A* 31 (1985) 1059.
- [9] D.F. Walls, G.J. Milburn, *Phys. Rev. A* 31 (1985) 2403.
- [10] C.M. Savage, D.F. Walls, *Phys. Rev. A* 32 (1985) 2316.
- [11] W.H. Zurek, *Phys. Today* 44 (10) (1991) 36.
- [12] R.P. Feynman, A.R. Hibbs, *Quantum Mechanics and Path Integrals*, McGraw-Hill, New York, 1965.
- [13] G.W. Ford, J.T. Lewis, R.F. O'Connell, *Phys. Rev. A*, in press.
- [14] E. Merzbacher, *Quantum Mechanics*, 3rd ed., Wiley, New York, 1998.