Comment on "Completely Positive Quantum Dissipation"

In a recent Letter [1] Vacchini used a specific model for the interaction of a quantum particle with its environment to determine a master equation in Lindblad form [2]. However, since we have obtained the most general form of such an equation [3], we were motivated to compare it to Vacchini's form.

Ignoring the energy shift (which is of no consequence for the present discussion), and following Vacchini by taking the high temperature limit, the general form of our master equation for an oscillator potential is [see Eq. (3) of Ref. [3] except now we simplify the notation by considering all momenta to be in units of $m\omega_0$ where ω_0 is the oscillator potential]

$$\begin{aligned}\frac{\partial\overline{\rho}}{\partial t} &= \frac{1}{i\hbar}[H,\overline{\rho}] - \frac{\gamma(\omega_0)}{4\pi} m\omega_0 \left\{ i([x, p\overline{\rho} + \overline{\rho}p] - [p, x\overline{\rho} + \overline{\rho}x]) + \frac{2kT}{\hbar\omega_0}([p,[p,\overline{\rho}]] + [x,[x,\overline{\rho}]]) \right\} \\ &\equiv \frac{1}{i\hbar}[H,\overline{\rho}] - \frac{\gamma(\omega_0)}{4\hbar} m\omega_0 \left\{ i(A - B) + \frac{2kT}{\hbar\omega_0}(C + D) \right\}, \qquad (1)\end{aligned}$$

in an obvious notation. Also, the bar was introduced to indicate that $\overline{\rho}$ is the slowly varying mean. In addition, we recall that (1) has the Lindblad form of the master equation familiar in quantum optics [4].

Vacchini's master equation [Eq. (4) of Ref. [1]] is also in Lindblad form and, for the case of an oscillator potential, may be written in the form [following the notation used in (1) except that we use ρ to indicate the density matrix in this case and γ_ν to indicate Vacchini's decay constant]

$$\left(\frac{\partial\rho}{\partial t}\right)_{\text{Vacchini}} = \frac{1}{i\hbar}[H,\rho] - \frac{\gamma_\nu}{\hbar} m\omega_0 \times \left\{ iA + \frac{2kT}{\hbar\omega_0}\left[C + \left(\frac{\hbar\omega_0}{4kT}\right)^2 D \right] \right\}, \qquad (2)$$

where A, B, C, and D are defined as in (1).

The form of (2) is clearly different from that of (1). Also, the form of (1) is invariant under the canonical transformation $x \to x\cos\theta + p\sin\theta$, $p \to -x\sin\theta + p\cos\theta$ which keeps the Hamiltonian invariant. Thus, it is not possible to transform (2) into (1) by a rotation in phase space. Furthermore, (1) was shown to be "...applicable to an oscillator in a general dissipative environment..." [5]. Thus, one is led to question the viability of (2). In fact, as may be verified, Vacchini's equation, as distinct from (1), does not lead to the correct equilibrium state: $\rho_{\text{eq}} = \exp\{-H/kT\}$; i.e., detailed balance does not hold. We conclude that Vacchini's equation is not an acceptable master equation. In his Reply [6] to this Comment, Vacchini does not dispute our results but disagrees with our conclusion regarding the necessity for equipartition (detailed balance); his remarks are essentially based on Lindblad's claim that the use of a master equation exhibiting complete positivity leads to the conclusion that translation invariance and equipartition cannot be simultaneously satisfied. Our viewpoint is that there are equations other than Lindblad's [2] which exhibit positivity and need to be examined in this light. Furthermore, for weak coupling situations, we believe that the canonical distribution must be always maintained, as it is by workers in quantum optics [3,4] and other areas [7], because it is a fundamental hypothesis of equilibrium statistical mechanics.

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