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Comment on ''The Lax-Onsager Regression ''Theorem'' Revisited''

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We have just read the paper by Lax (to appear in this volume [1]) and we are pleased to note that Lax and ourselves are in basic agreement; the Lax formula is correct and extremely useful for systems where the coupling is weak and only frequencies near resonance are of interest. Indeed, it is the only approximation we have for the strongly driven systems of quantum optics. We should stress that we never had objections to necessary and appropriate approximations such as the Lax formula; our objection is when these approximations are elevated to general principles (not by Lax himself who, in his initial [2] (referred to as Q II by Lax) and subsequent publications [3] (referred to as Q III by Lax) [4] (referred to as Q XI by Lax) [5] (referred to as Q IV by Lax) was careful to note the domain of applicability of his formula).

However, we are puzzled when Lax says that the key statement (viz. the title) of Ref. [6] can be misleading. All we did in Ref. [6] was present an exactly solvable example demonstrating that the Onsager regression hypothesis fails in the quantum case. We then went on to point out that there is a correct and broadly applicable quantum generalization of the Onsager hypothesis: the fluctuation-dissipation theorem of Callen and Welton [7]. We do not agree that this is simply a mathematical statement.

It is true that much of our work has involved the message that quantum noise is universal: its spectrum is that of Planck plus, where appropriate, zero-point noise. It might be helpful to note that for *classical* stochastic processes, as described by a Langevin equation, there are two equivalent characterizations of a Markov process: (a) no memory in the dissipative term and (b) white noise power spectrum. In fact, in 1965, using a model of coupled oscillators to describe a heat bath, Ford, Kac and Mazur [8] showed that, in the context of the quantum Langevin equation, one can have (a) but never (b).

The Lax model for a maser or laser is based on the idea that accurate results are possible even if the noise is not white provided that it does not vary too much over the width of the line. Rather than trying to characterize the nature of this approximation, we feel it is best to refer to the detailed exposition given by Lax [4,5] and summarized by him in his present paper [1]. For a detailed and exact treatment of a quantum particle moving in an *arbitrary potential* and coupled to a heat bath, we refer to Ref. [9], which also discusses various models not found in the quantum optics literature. In addition, as well as presenting a general result for the symmetric autocorrelation of the random force, we also wrote down its nonequal-time commutator so that the corresponding result for the normally ordered operators (used to

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describe photodetectors, as emphasized by Lax [1]) immediately follows.

Lax has verbally pointed out to us that the conclusions section of our present paper [10] could be subject to misinterpretation in that it might be assumed (incorrectly) that ''the prefactor'' is a significant element in our calculation of the spectrum of resonance fluorescence. On the contrary, the conditions underlying the Lax approximation are met and it clearly does an admirable job. This is consistent with our original statement [6] ''there seem to be no flagrant errors in its application–in the field of quantum optics'', based on the fact that many results in quantum optics depend only on the quantum noise spectrum evaluated at a resonance frequency. For the same reason, our remark applies to laser theory, as Lax has now demonstrated in detail [1]. Also, Lax [1] does admit that ''there can be large discrepancies away from the area of the peak frequencies'' but goes on to show that their contribution to laser theory is negligible. However, we feel it is important to stress that the situation is very different in a variety of other areas. Examples of where quantum noise, memory and off-resonance effects play an important role include work on radiation reaction in the equation of motion for an electron [11,12], Stark shifts due to blackbody radiation [11,13], memory effects in transport theory [14], tunneling in a non-Markoffian dissipative environment [15], anomalous diffusion [16] and the spin-boson problem [17]. A particularly striking example is the measured temperature-dependent shift of spectral lines due to coupling with an ambient blackbody radiation field [18]. There, the *off-resonance* high frequencies are *solely* responsible for the shift [11,13]. In fact, this problem provides an answer to Lax's wish [1] that ''it would be nice to see an error analysis of some real problem to prove that an exact procedure was necessary'': the usual frequency-independent damping model of quantum optics (corresponding to no memory in the dissipative term in the quantum Langevin equation) simply gave a zero result for the energy shift [11,19] making it necessary to consider the exact QED problem. In particular, it was necessary to incorporate elements not normally encountered in the quantum optics literature viz. mass renormalization and also a calculation of both the free energy and the energy due to interaction with the reservoir. Ref. [20] reviews some of this work [11–15] and also complements Lax's historical review [1].

Finally, we are pleased to note that Lax was careful to state that the results given in his (A.11) are really assumptions and not derivable from (A.9) and (A.10), as most of the textbooks (incorrectly) claim to do. This point was recently addressed by us in detail and in fact, we have shown explicitly that ''the power spectrum of the correlation resulting from the RWA Hamiltonian is not that of white noise'' [21].

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