

Slanted coupling of one-dimensional arrays of small tunnel junctions

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We have studied the electrostatic problem of the slanted coupling of two one-dimensional (1D) arrays with equal junction capacitances C , equal stray capacitances C_0 , equal coupling capacitances C_c , and with both arrays biased. In the weak coupling limit ($C_c/C \ll 1$), we obtain an analytic solution for the potential profile and the corresponding Gibbs free energy, and we derive threshold voltages for various charge transport modes. Our results show that C_0 , C_c , and the bias voltage V_1 all play important roles in determining the threshold voltage of the system. In the small stray capacitance limit ($C_0/C \ll 1$), the threshold voltage is proportional to $1/C$, while in the large stray capacitance limit ($C_0/C \gg 1$), the threshold voltage becomes independent of C . Also, in the small C_c/C limit, single electron tunneling always has a lower threshold voltage than that of the electron-hole and mixed tunneling. In addition, we find that V_1 has a more dramatic effect on the electron-hole tunneling threshold voltage than on that of the single electron tunneling, i.e., at some favored value of C_c/C a small change in V_1 can switch the transport of the system from single electron to electron-hole transport. © 1998 American Institute of Physics.

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I. INTRODUCTION

In the study of Coulomb blockade of single electron tunneling, capacitively coupled one-dimensional (1D) arrays of small tunnel junctions have attracted a lot of attention.¹⁻⁷ These systems can be used to study both time correlated and space correlated single electron tunneling, and holds promise for being the basis of a new metrological standard.¹ Nevertheless, as shown in recent experiments,⁷ it is exceedingly difficult to keep a single charge state propagating in a 1D array, because of the extreme sensitivity to stray charges. Thus much effort has been made to develop a more sophisticated system of 1D arrays.²⁻⁵ One promising experimental system consists of two parallel 1D arrays (see Fig. 1), where each electrode of one array is coupled to the electrodes of the other array with one [straight, Fig. 1(a)] or two [slanted, Fig. 1(b)] capacitors C_c . When a single electron is placed on an electrode in the first array, the total energy will be lower if a hole is added (an electron is removed from) a neighboring electrode in the other array. As a result, the electron and the hole (e-h) tend to move together in the arrays resulting in two identical currents of opposite signs in the two arrays.^{1,2} Recently, Delsing *et al.*³ have reported that they have observed indications of correlated electron-hole transport. However, in the straight coupling case, the low energy state can be conserved only if the e-h tunnels simultaneously, which is a second-order cotunneling process and a less likely event. It was noted by Delsing *et al.*³ that the slanted cou-

pling is a better choice where the low energy e-h state can be preserved even for sequential tunneling (so that we are still dealing with a lowest order process), and it is very important to understand the system theoretically. Nevertheless, to date theoretical investigations have dealt mainly with the case of straight coupling and zero bias on one of the arrays. Here, we study the electrostatic problem of the slanted coupled two 1D arrays, and investigate the charge profiles and threshold voltages of various charge solitons. In particular, we show that the electrostatic problem can be solved in the weak coupling limit, and we give a quantitative analysis for the case when both arrays are biased. In Sec. II, we present the general formalism underlying our method and investigate the charge profiles and threshold voltages of various charge solitons. Our results are summarized in Sec. III.

II. FORMULATION

The system with which we are concerned is illustrated in Fig. 1(b), where two 1D arrays each with m small tunnel junctions, with equal junction capacitances C and equal stray capacitances C_0 , are doubly coupled to each other by the coupling capacitance C_c . Specifically, the i th island of the first array is coupled through C_c to both the $(i-1)$ th and i th island of the second array. The bias potentials of the two edges of the upper (first) array are, respectively, Φ_0 and Φ_m , while that of the lower (second) array are φ_0 and φ_m . Also, the tunnel resistance R of each junction is assumed to be the same and $R \gg h/e^2$, which ensures that the wave function of an excess electron on an island is localized there. We denote

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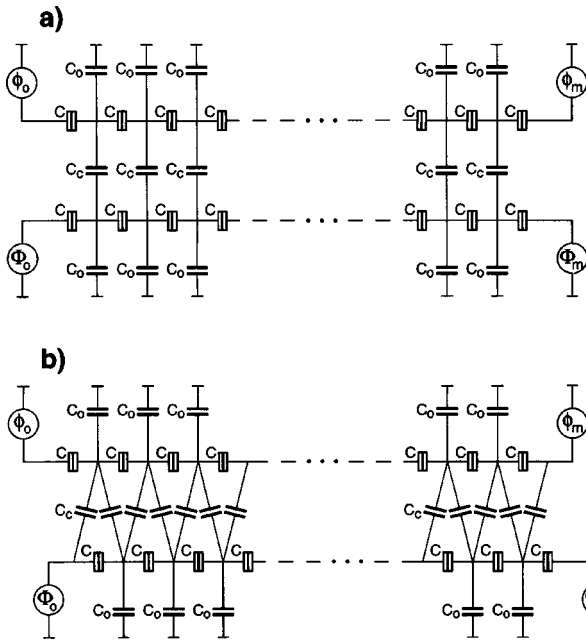


FIG. 1. Capacitively coupled two 1D arrays each with m small tunnel junctions, with equal junction capacitances C , equal stray capacitances C_o , and equal coupling capacitances C_c : (a) straight coupling; (b) slanted coupling.

the potential on each of the individual $m-1$ islands of the first and the second array by the column vectors $\bar{\Phi} = \{\Phi_1, \Phi_2, \dots, \Phi_{m-1}\}^T$ and $\bar{\varphi} = \{\varphi_1, \varphi_2, \dots, \varphi_{m-1}\}^T$, respectively. In addition, the number of excess electrons on each of the individual $m-1$ islands of the first and the second array is denoted by the column vectors $\bar{N} = \{N_1, N_2, \dots, N_{m-1}\}^T$ and $\bar{n} = \{n_1, n_2, \dots, n_{m-1}\}^T$, respectively.

In general, the Gibbs free energy for the biased double-coupled two 1D arrays contains two terms, the electrostatic energy E_S and the work done due to the charge redistribution associated with the change of the charge profile $\{\bar{N}, \bar{n}\}$ on the islands, which can be written as

$$F = E_S + W, \quad (1)$$

where the work done due to the charge redistribution associated with the change of the charge profile $\{\bar{N}, \bar{n}\}$ is given by

$$W = - \sum_{i=1}^{m-1} [\Phi_i(Q_i^s + Q_{iA}^c) + \varphi_i(q_i^s - Q_{iA}^c)] - \sum_{i=1}^m [\Phi_i Q_{iB}^c - \varphi_{i-1} Q_{iB}^c + V_i Q_i + v_i q_i], \quad (2)$$

where (Q_i^s, Q_i^c, Q_i) and (q_i^s, q_i^c, q_i) are the charges on the stray capacitor, the coupling capacitor, and the tunneling junction, respectively. In particular, one has $Q_{iA}^c = C_c(\Phi_i - \varphi_i)$, and $Q_{iB}^c = C_c(\Phi_i - \varphi_{i-1})$. Also, the local voltage for the upper array is defined as $V_i = \Phi_{i-1} - \Phi_i$, with $\Phi_0 = V$, and that of the lower array is $v_i = \varphi_{i-1} - \varphi_i$, with $\varphi_0 = U$. In addition, the electrostatic energy in Eq. (1) is defined as

$$E_S = E_1 + E_2 + E_{12} - e \sum_{i=0}^m (N_i \Phi_i + n_i \varphi_i), \quad (3)$$

with

$$E_1 = \frac{C_o}{2} \sum_{j=1}^{m-1} \Phi_j^2 + \frac{C}{2} \sum_{j=1}^m (\Phi_j - \Phi_{j-1})^2, \quad (4)$$

$$E_2 = \frac{C_o}{2} \sum_{j=1}^{m-1} \varphi_j^2 + \frac{C}{2} \sum_{j=1}^m (\varphi_j - \varphi_{j-1})^2, \quad (5)$$

$$E_{12} = \frac{C_c}{2} \left[\sum_{j=1}^{m-1} [\Phi_j - \varphi_j]^2 + \sum_{j=1}^m (\Phi_j - \varphi_{j-1})^2 \right], \quad (6)$$

representing the charging energy for the first array, the second array, and the coupling between the two arrays, respectively. We note that the use of Eq. (1) has at least two advantages. First, the important process of inner junction charge transfer, i.e., charge transfer between two neighboring islands without any other transfer on the other islands, can now be studied in detail. The other advantage is that it is now possible to obtain the electrostatic equations for the system directly from Eq. (1), which we discuss in the following.

Our goal here is to find out an explicit form of Eq. (1) at the fixed potential profile $\{\bar{\Phi}, \bar{\varphi}\}$ and changeable island charge number $\{\bar{N}, \bar{n}\}$. For this purpose, we take the derivative of Eq. (1) with respect to the potentials Φ_i and φ_i ($i = 1, 2, \dots, m-1$), after which we get $2(m-1)$ equilibrium conditions. These equilibrium conditions form $2(m-1)$ linear equations for the potential profile $\{\bar{\Phi}, \bar{\varphi}\}$ and the island charge number $\{\bar{N}, \bar{n}\}$, and they can be conveniently put into a simple matrix form (see Appendix)

$$\begin{pmatrix} \bar{M}_+ & \alpha \bar{Y} \\ -\alpha \bar{Y} & \bar{M}_- \end{pmatrix} \begin{pmatrix} \bar{\Phi} + \bar{\varphi} \\ \bar{\Phi} - \bar{\varphi} \end{pmatrix} = \frac{e}{C} \begin{pmatrix} \bar{N} + \bar{n} \\ \bar{N} - \bar{n} \end{pmatrix}, \quad (7)$$

where the coupling constant $\alpha = C_c/C$, and we have used a double bar to denote the $m-1$ by $m-1$ tridiagonal matrices \bar{M} and \bar{Y} . Specifically, \bar{M}_\pm is a symmetrical tridiagonal matrix having the same diagonal elements $D_\pm = -2 - C_o/C - 2\alpha \pm \alpha$, and the same off-diagonal elements $1 \pm \alpha/2$, while \bar{Y} is an antisymmetrical tridiagonal matrix having the same diagonal elements 0, and the same off-diagonal elements $1/2$. In addition, the first and last elements of \bar{N} and \bar{n} in Eq. (7) are understood to be $N_1 - C\Phi_0/e$ and $N_{m-1} - C\Phi_m/e$, $n_1 - C\varphi_0/e$ and $n_{m-1} - C\varphi_m/e$, respectively, to accommodate the effects of the bias voltages. Our formalism is quite general. In the case of a symmetric bias voltage, we have $\Phi_0 = V/2$, $\Phi_m = -V/2$, and $\varphi_0 = \varphi_m = 0$. For the experiments of Ref. 4, where both arrays are biased, one has $\Phi_0 = V_1$, $\varphi_0 = -V_2$, and $\Phi_m = \varphi_m$. We note that Eq. (7) is also known as the electrostatic equation for the system, the 1D version of which has been obtained by the consideration of the charge conservation law.⁴

It is straightforward to show that the off-diagonal block matrix $\alpha \bar{Y}$ in Eq. (7) will contribute to the problem only to order α^2 . Thus, in the weak coupling ($\alpha \ll 1$) region, it is justified to neglect the off-diagonal block matrix $\alpha \bar{Y}$ and, as a result, Eq. (7) can be solved analytically. By an extension of the techniques used in Ref. 4, for matrix inversion of a symmetric tridiagonal matrix, we obtain a solution of Eq. (7) in the weak coupling region as

$$\begin{pmatrix} \bar{\Phi} \\ \bar{\varphi} \end{pmatrix} = -\frac{e}{C} \begin{pmatrix} \bar{A}' & \bar{B}' \\ \bar{B}' & \bar{A}' \end{pmatrix} \begin{pmatrix} \bar{N} \\ \bar{n} \end{pmatrix}, \quad (8)$$

where the elements of the symmetric matrix \bar{A}' and \bar{B}' are given by, respectively,

$$A'_{ij} = \frac{1}{2} [R_{ij}(\lambda'_+) + R_{ij}(\lambda'_-)], \quad (9)$$

$$B'_{ij} = \frac{1}{2} [R_{ij}(\lambda'_+) - R_{ij}(\lambda'_-)], \quad (10)$$

with λ'_\pm defined by

$$-2ch\lambda'_\pm = D \pm \alpha/2. \quad (11)$$

Also, α is the same as in Eq. (7), $D = -2 - C_o/C - 2\alpha$, and

$$R_{ij}(\lambda'_\pm) = \frac{ch(m - |j - i|)\lambda'_\pm - ch(m - i - j)\lambda'_\pm}{2sh\lambda'_\pm shm\lambda'_\pm}. \quad (12)$$

We note that in Eqs. (9)–(12), one can check that in general $\lambda'_+ < \lambda'_-$ and $R_{ij}(\lambda'_+) > R_{ij}(\lambda'_-) > 0$, so that $A'_{ij} > B'_{ij} > 0$. In the $\alpha \rightarrow 0$ limit, B'_{ij} tends to zero ($\lambda'_- \rightarrow \lambda'_+$) and the two arrays becomes decoupled. Also, Eq. (8) can be explicitly written as

$$\begin{aligned} \phi_i &= \phi_i^0 + (A'_{i1} + \delta_{i0})\phi_0 + (A'_{i,m-1} + \delta_{im})\phi_m + B'_{i1}\varphi_0 \\ &\quad + B'_{i,m-1}\varphi_m, \end{aligned} \quad (13)$$

$$\begin{aligned} \varphi_i &= \varphi_i^0 + (A'_{i1} + \delta_{i0})\varphi_0 + (A'_{i,m-1} + \delta_{im})\varphi_m + B'_{i1}\phi_0 \\ &\quad + B'_{i,m-1}\phi_m, \end{aligned} \quad (14)$$

where the bias independent terms in Eqs. (13) and (14) are defined, respectively, as

$$\phi_i^0 = -\frac{e}{C} \sum_{j=1}^{m-1} (A'_{ij}N_j + B'_{ij}n_j), \quad (15)$$

$$\varphi_i^0 = -\frac{e}{C} \sum_{j=1}^{m-1} (B'_{ij}N_j + A'_{ij}n_j). \quad (16)$$

We note that in writing Eqs. (13) and (14), we have added Kronecker delta (δ) terms to ensure that the equations are correct for $i=0,1,\dots,m$ (recall in Eq. (8) that $i=1,2,\dots,m-1$).

Equation (8), supplemented by Eqs. (9)–(11), is a key result of this article. Once a charge profile $\{\bar{N}, \bar{n}\}$ is known, we can use Eq. (8) to determine the potential profile $\{\bar{\Phi}, \bar{\varphi}\}$. Before pursuing this subject, let us analyze our result of Eq. (8) for the simplest case, where no net charge exists in any of the island ($N_i = n_i = 0$) and the arrays act like a network of capacitors, and where one can easily check the results. In this case, at the bias voltage $\Phi_0 = V$ and $\Phi_m = \varphi_0 = \varphi_m = 0$, Eq. (8) reduces to,

$$\Phi_j = (A'_{j1} + \delta_{j0})V, \quad (17)$$

$$\varphi_j = B'_{j1}V. \quad (18)$$

Two limiting cases of Eqs. (17) and (18) are of particular interest. First, when $C_c \rightarrow 0$, one finds $B'_{ij} \rightarrow 0$, and $\varphi_j \rightarrow 0$. If one also has $C_o \rightarrow 0$ then $\Phi_j \rightarrow V(1 - j/m)$, and all the junctions have the same voltage V/m .

Next, we use Eqs. (7) and (8) to evaluate Eq. (1) exactly, and the result is

$$\begin{aligned} F &= E_0 + \frac{e^2}{2C} \sum_{i,j=1}^{m-1} \{N_i A'_{ij} N_j + n_i A'_{ij} n_j + n_i B'_{ij} N_j \\ &\quad + N_i B'_{ij} n_j\} - \phi_0 Q_0 - \phi_m Q_m - \varphi_0 q_0 - \varphi_m q_m, \end{aligned} \quad (19)$$

where E_0 is a quantity independent of the charge profile $\{\bar{N}, \bar{n}\}$, and

$$Q_0 = N_0 e + C(\phi_0 - \phi_1), \quad Q_m = N_m e + C(\phi_m - \phi_{m-1}), \quad (20a)$$

$$q_0 = n_0 e + C(\varphi_0 - \varphi_1), \quad q_m = n_m e + C(\varphi_m - \varphi_{m-1}). \quad (20b)$$

Equation (19) is a general expression for the Gibbs free energy F of double-coupled two 1D arrays with changeable charge $e\{\bar{N}, \bar{n}\}$ on the islands, at the fixed bias voltage $\{\Phi_0, \Phi_m, \varphi_0, \varphi_m\}$ and the fixed potential profile $\{\bar{\Phi}, \bar{\varphi}\}$ on the islands. By means of Eq. (19), one can directly determine the change of ΔF due to some charge transfer event. To be definite, here we discuss the case $\{N_k, N_{k'}, n_k, n_{k'}\}$ to $\{N'_k, N'_{k'}, n'_k, n'_{k'}\}$, i.e., the charge transfer between two islands k and k' , while the charges on the other islands are unchanged. We denote the net transferred charge as Q (Q can be electron-hole, single electron, or the combined electron-hole and single electron case, which will be discussed later). Thus, we obtain from Eq. (19) the $\Delta F^Q(k, k')$ due to the charge transfer $\{N_k, N_{k'}, n_k, n_{k'}\}$ to $\{N'_k, N'_{k'}, n'_k, n'_{k'}\}$,

$$\begin{aligned} \Delta F^Q(k, k') &\equiv F(\{N', n'\}) - F(\{N, n\}) \\ &= \Delta E^Q(k, k') + W^Q(k, k'), \end{aligned} \quad (21)$$

where the detailed form of the change of the charging energy $\Delta E^Q(k, k')$ and the work done $W^Q(k, k')$ in Eq. (21) can directly be worked out from Eq. (19).

One can now determine the tunneling threshold voltage by means of Eq. (21). At $T=0$, the tunneling of charge Q from the k to the k' island (restricting our discussion to the case of $k' = k \pm 1$) is energy favorable when $\Delta F^Q(k, k')$ of Eq. (21) is less than zero. Thus, the threshold energy V_t for the injection of a charge from the k th island onto the neighboring k' island can be obtained by equating $\Delta F^Q(k, k') = 0$. Here we study three cases of particular interest: (i) the *electron-hole pair* (e-h) case, where an electron is transferred in the upper array from the k island to the k' island, and at the same time in the lower array an electron from the k' island to the k island is transferred, i.e., $N'_k - N_{k'} = 1$, $n'_{k'} - n_k = -1$, $N'_k - N_k = -1$, $n'_k - n_{k'} = 1$; (ii) the *single charge* (e) case, where an electron is transferred from the k island to the k' island in the upper array, i.e., $N'_k - N_{k'} = 1$, $n'_{k'} - n_{k'} = 0$, $N'_k - N_k = -1$, $n'_k - n_k = 0$; (iii) the *combined* (e-h,e) case, where, in addition to (i) a single electron is transferred from k' to k'' , with the results $N'_{k''} - N_{k''} = 1$, $N'_{k'} - N_{k'} = 0$, $n'_{k'} - n_{k'} = -1$, $N'_k - N_k = -1$, $n'_k - n_k = 1$. It turns out that the final form of V_t depends on the setup of the bias voltage. In the case of both arrays biased with $\Phi_0 = V_1$, $\varphi_0 = -V_2$, and $\Phi_m = \varphi_m = 0$, we obtain from Eqs. (19) and (21),

$$V_2^{e-h}(k,k') = \frac{e}{C} \frac{R'_{k'k'} - R'_{k,k}}{\delta_{k,0} - R'_{1,k'} + R'_{1,k}} - V_1, \quad (22a)$$

$$V_2^e(k,k') = \frac{e'}{2C} \frac{A'_{k'k'} - A'_{k,k}}{\delta_{k,0} - A'_{1,k'} + A'_{1,k}} - V_1 \frac{B'_{1k'} - B'_{1k}}{\delta_{k,0} - A'_{1,k'} + A'_{1,k}}, \quad (22b)$$

$$V_2^{e-h,e}(k;k',k'') = \frac{e}{2C} \frac{A'_{k'k'} + A'_{k''k''} - 2R'_{kk'} + 2B'_{k'k''}}{\delta_{k,0} - A'_{1k'} + B'_{1k''}} - V_1 \frac{\delta_{k,0} - A'_{1k'} + B'_{1k'}}{\delta_{k,0} - A'_{1k'} + B'_{1k''}}, \quad (22c)$$

where A'_{ij} , B'_{ij} , and $R'_{ij} \equiv R_{ij}(\lambda_{\pm}')$ are defined by Eqs. (8)–(11), respectively. Equations (22a)–(22c) are interesting results. First, by using them one immediately finds that $dV_i/dm > 0$ and $dV_i/dk < 0$. This implies two facts: (i) for fixed values of C , C_o , and C_c , an array of larger m will generally have larger V_i ; (ii) once a charge is injected into the array, it will have no difficulty in traveling through (with increasing k) the array. This is to say that from Eq. (22) one can directly check that $V_i^Q(0,1) > V_i^Q(1,2)$ so that the actual threshold voltage value of the system is $V_i^Q = V_i^Q(0,1)$. Thus, the threshold energy V_i for the injection of a single charge can be deduced from Eq. (22) as

$$V_2^{e-h} = V_2^{e-h}(0,1) = \frac{e}{C} \frac{R'_{1,1}}{1 - R'_{1,1}} - V_1, \quad (23a)$$

$$V_2^e = V_2^e(0,1) = \frac{e}{2C} \frac{A'_{11}}{1 - A'_{11}} - V_1 \frac{B'_{11}}{1 - A'_{11}}, \quad (23b)$$

$$V_2^{e-h,e} = V_2^{e-h,e}(0;1,2) = \frac{e}{2C} \frac{2B'_{12} - A'_{22} - A'_{11}}{1 - A'_{12} + B'_{11}} - V_1 \frac{1 - A'_{11} + B'_{12}}{1 - A'_{12} + B'_{11}}. \quad (23c)$$

By using Eqs. (23a)–(23c), we can study the C_o and C_c effects, the number m effect, as well as the bias voltage V_1 effects on the threshold voltages for the various tunneling cases.

First, we examine the capacitor's (C_o and C_c) effects. For simplicity, we set $V_1 = 0$ in Eq. (23). It is straightforward to deduce from Eqs. (9)–(12) that B_{ij} tends to zero, while A'_{ij} and $R'_{ij}(\lambda_{\pm})$ both approach the same value, $(C_o/C)^{i-j-1}$ for $C_o/C \gg 1$ and $i(m-j)/m$ for $C_o/C \ll 1$, respectively. It follows that in the case of $C_o/C \ll 1$, (23) reduces to

$$V_2^{e-h} = \frac{e}{C} (m-1) - V_1, \quad (24a)$$

$$V_2^e = \frac{e}{2C} (m-1), \quad (24b)$$

$$V_2^{e-h,e} = \frac{e}{2C} (3m-5) - 2V_1. \quad (24c)$$

Also, when $C_o/C \gg 1$, Eq. (23) reduces to

$$V_2^{e-h} = \frac{e}{C_o} - V_1, \quad (25a)$$

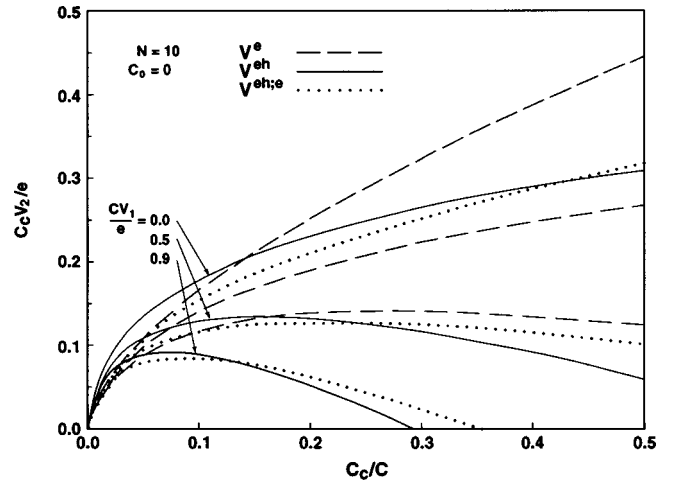


FIG. 2. Threshold voltage V_i (in units of e/C_o) for injecting a charge into the first island of a slanted coupled two 1D array, each with ten small junctions and no bias voltage for the other array, as a function of C_o/C , at three different values of $C_c/C = 0, 0.1, 0.2$ (from top to bottom). Full curves are for electron-hole pair, dotted curves are for single electron, and dashed curves are for combined electron-hole pair and single electron case. Also, C_c , C , and C_o , are the coupling capacitances, junction capacitances, and stray capacitances, respectively.

$$V_2^e = \frac{e}{2C_o}, \quad (25b)$$

$$V_2^{e-h,e} = \frac{e}{C_o} - V_1. \quad (25c)$$

Equations (24) and (25) show that the stray capacitors play an important role in determining the threshold voltage V_i . In the small stray capacitance limit [see Eq. (24)], the threshold voltage is proportional to $1/C$, while in the large stray capacitance limit, the threshold voltage becomes independent of C . These features of the threshold voltage V_i are further illustrated in Fig. 2, where we plot V_i (in units of e/C_o) vs C_o/C , for $\alpha = 0, 0.1$ and 0.2 , for a double-coupled two 1D arrays. Also seen from the figure, in the small α limit single electron tunneling always has a lower threshold voltage than that of the e-h and mixed tunneling.

Next, we study the bias voltage V_1 effects on the threshold voltages. For this purpose, in Fig. 3 we plot the threshold voltage V_2 (in units of e/C_c) for injecting a charge into the first island of a slanted coupled two 1D array each with ten small junctions and bias voltage V_1 for the other array, as a function of C_c/C , at three different values of $CV_1/e = 0, 0.5, 0.9$ (from top to bottom). As can be seen from the figure, overall the presence of V_1 will down shift the threshold voltage of V_2 for any kind of charge injection. Nevertheless, it has a more dramatic effect on the e-h tunneling threshold voltage than on that of the single electron tunneling threshold voltage. For example, in the region of $C_c/C \sim 0.1$, $V^e < V^{e-h}$ at $CV_1/e = 0$, and it changes to $V^e > V^{e-h}$ at $CV_1/e = 0.5$. In other words, at some favored value of C_c/C the presence of V_1 can switch the transport of the system from single electron to electron-hole transport.

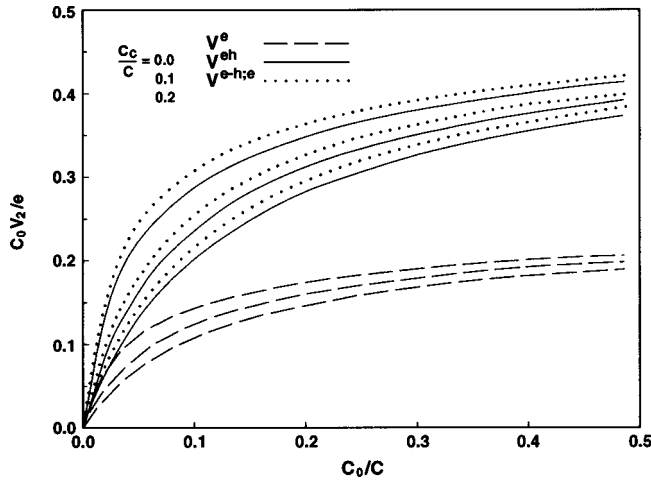


FIG. 3. Threshold voltage V_2 (in units of e/C_c) for injecting a charge into the first island of a slanted coupled two 1D array each with ten small junctions and bias voltage V_1 for the other array, as a function of C_c/C , at three different values of $CV_1/e = 0, 0.5, 0.9$ (from top to bottom). Full curves are for electron-hole pair, dotted curves are for single electron, and dashed curves are for combined electron-hole pair and single electron case. Also, C_c , C , and C_0 , are the coupling capacitances, junction capacitances, and gate capacitances, respectively.

III. SUMMARY

In summary, in this article we have studied the electrostatic problem of the slanted coupled two 1D arrays, and investigated the charge profiles and threshold voltages of various charge solitons. In particular, in the weak coupling limit we have obtained an analytic solution of Eq. (8) for the potential profile and the corresponding Gibbs free energy of Eq. (19). By means of Eq. (19), we have derived the threshold voltages Eq. (22) for various cases of charge transport in the slanted coupled 1D arrays with both arrays biased. Our results show that the stray capacitor C_0 the coupling capacitor C_c , and the bias voltage V_1 all play important roles in determining the threshold voltage of the system. In the small stray capacitance limit [see Eq. (24)], the threshold voltage is proportional to $1/C$, while in the large stray capacitance limit, the threshold voltage becomes independent of C . Also, in the small α limit, single electron tunneling always has a lower threshold voltage than that of the e-h and mixed tunneling cases. For the effects of bias voltage V_1 , we find that it has a more dramatic effect on the e-h tunneling threshold voltage than that of the single electron tunneling, i.e., at some favored value of C_c/C the presence of V_1 can switch the transport of the system from single electron to electron-hole transport.

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APPENDIX: DERIVATION OF EQ. (7)

The $2(m-1)$ equilibrium conditions as deduced from Eq. (1) are

$$\begin{aligned} \Phi_{i+1} + \Phi_{i-1} + D\Phi_i + \alpha(\varphi_i + \varphi_{i-1}) \\ = \frac{e}{C} N_i \quad (i=1,2,\dots,m-1), \end{aligned} \tag{A1}$$

$$\begin{aligned} \varphi_{i+1} + \varphi_{i-1} + D\varphi_i + \alpha(\Phi_i + \Phi_{i+1}) \\ = \frac{e}{C} n_i \quad (i=1,2,\dots,m-1), \end{aligned} \tag{A2}$$

where $D = -2 - Cg/C - 2\alpha$, and the coupling constant $\alpha = C_c/C$.

Next, we introduce a pair of new variables defined by

$$S_i = \Phi_i + \varphi_i, \tag{A3}$$

$$T_i = \Phi_i - \varphi_i, \tag{A4}$$

and use Eqs. (A3) and (A4) to put Eqs. (A1) and (A2) into the following form:

$$\begin{aligned} S_{i+1} + S_{i-1} + DS_i + \alpha \left(S_i + \frac{S_{i-1} - T_{i-1}}{2} + \frac{S_{i+1} + T_{i+1}}{2} \right) \\ = \frac{e}{C} (N_i + n_i), \end{aligned} \tag{A5}$$

$$\begin{aligned} T_{i+1} + T_{i-1} + DT_i + \alpha \left(-T_i + \frac{S_{i-1} - T_{i-1}}{2} - \frac{S_{i+1} + T_{i+1}}{2} \right) \\ = \frac{e}{C} (N_i - n_i). \end{aligned} \tag{A6}$$

It is not difficult to see that the matrix form of Eqs. (A5) and (A6) is Eq. (7) of the text.

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