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The blackbody reservoir and the Planck spectrum

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Abstract

We present a simple criterion for assessing the validity of various reservoir models viz. the results for the random force correlation should lead to the Planck spectrum. We note that the usual results associated with the RWA do not meet this criterion whereas the corresponding results for the independent-oscillator model do.

Dissipative processes are pervasive in many areas of physics. The generic problem treats a system which consists of a subsystem of interest interacting with a heat bath (reservoir). In many situations the heat bath is the radiation field and since this has been studied so much one might reasonably expect agreement in the literature on the various associated results, in particular the result for the random force correlation. However, this is not so.

Here, we point out a simple criterion which can be used for assessing the validity of various models viz. the results for the random force autocorrelation should lead to the Planck spectrum for the reservoir. In this sense, the radiation field reservoir can be regarded as a "Rosetta-stone" for assessing the validity of heat bath models.

Most investigations which treat the radiation field as a reservoir use the RWA (rotating wave approximation). The RWA is an integral part of the foundations of quantum optics and is discussed in both the old and the more modern textbooks. It concerns the interaction of a reservoir, consisting of an infinite number of oscillators, with either a two-level atom or a cavity mode or a charged harmonic oscillator. Here we concentrate on the case of a harmonic oscillator of frequency ω_0 but our observations will hold in general. In this case, it is found [1–3] that the spectrum of the autocorrelation of the random force is a function of ω_0, in contrast to the Planck spectrum.

It should also be noted that a variety of other problems have been found with the RWA model: it leads to an incorrect expression for the Lamb shift [4], the spectrum has no lower bound [5] and it is inconsistent with the Ehrenfest theorem [6]. Thus, we are motivated to look for a more exact model than the RWA in the hope that the corresponding results lead to the Planck spectrum. In this context, we revisit the independent-oscillator (IO) model [5], in which the quantum particle is surrounded by a large (eventually infinite) number of heat-bath particles,

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each attached to it by a spring. Among the advantages of this model are:

(a) It leads to an expression for the force autocorrelation which is independent of the potential (in contrast to the results which exist for the RWA model) and

(b) It can be shown to incorporate the model of a charged particle in an arbitrary potential and interacting with the radiation field.

In previous papers, we explored some of the ramifications of (b) and obtained an equation for the *oscillator* which incorporates radiation reaction effects and which does not lead to runaway solutions [7]. In this note, we wish to explore some of the ramifications for the *reservoir*.

Our starting point was the Hamiltonian H for a very general model of a heat bath viz. the IO model [5]. Then, using the Heisenberg equations of motion, we derived the equation of motion for a quantum particle of mass m moving in a one-dimensional potential $V(x)$ and linearly coupled to the passive heat bath. The equation takes the form of a generalized quantum Langevin equation,

$$m\ddot{x} + \int_{-\infty}^{t} \mathrm{d}t' \, \mu(t-t')\dot{x}(t') + V'(x) = F(t) + f(t), \tag{1}$$

where the dot denotes the derivative with respect to t and where $V'(x) = \mathrm{d}V(x)/\mathrm{d}x$ is the negative of the time-independent external force and $\mu(t)$ is the so-called memory function. In addition, $F(t)$ is the random (fluctuation) force and $f(t)$ is a c-number position-independent external force. Explicit expressions for $\mu(t)$ and $F(t)$ were written down in terms of the heat bath parameters [5].

In particular, we showed that the (symmetric) autocorrelation of $F(t)$ is

$$C_F(t-t') \equiv \tfrac{1}{2}\langle F(t)F(t') + F(t')F(t)\rangle = \frac{1}{\pi}\int_0^{\infty} \mathrm{d}\omega \, \mathrm{Re}[\tilde{\mu}(\omega + \mathrm{i}0^+)]\hbar\omega \times \coth(\hbar\omega/2kT)\cos[\omega(t-t')], \tag{2}$$

which is a form of the fluctuation–dissipation theorem. Here $\tilde{\mu}(\omega)$ is the Fourier transform of $\mu(t)$.

It should be emphasized that $F(t)$ is the field-free operator. Also, $\mu(t)$ does not depend on the potential. As a consequence, $C_F(t)$ is independent of the potential. Actually, this conclusion can be deduced from very general principles. The essential point is that when an oscillator (or an atom) is coupled to a reservoir the properties of the reservoir (correlations of the reservoir variables, reservoir energy density, etc.) are unchanged. This is true not because the coupling is weak, but because the reservoir is infinite; a single oscillator placed in the blackbody field of a cavity at temperature T clearly cannot change the correlations of the electric field at the position of the oscillator. However, we should stress that $C_F(t)$ is strongly dependent on the nature of the interaction between the oscillator and all of the reservoir oscillators (as made manifest by the $\mathrm{Re}\,\tilde{\mu}(\omega)$ term in (2)). For the RWA model this interaction is only an approximation to the exact interaction; this explains why $C_F(t)$ for the RWA model does not correspond exactly to the Planck spectrum. However, the reason why it also depends on the potential (ω_0 dependence) requires a further explanation, the essence of which is that the integrals involved in force correlation calculations in most of the existing literature are handled incorrectly [8].

Here, we wish to consider the particular case where the reservoir is the radiation field and the quantum particle is an electron of charge $-e$. Then our H becomes the well-known H of non-relativistic quantum electrodynamics but generalized to incorporate an electron form factor (Fourier transform of the electron charge distribution). As a result, we found that the fluctuation force is $-e$ times the field-free electric field smoothed over a finite electron radius. We also obtained the corresponding result for $\mu(t)$ and we used these results in (1) to investigate the properties of the oscillator [7]. Now we make use of (2) to investigate the properties of the reservoir. For this purpose we need consider only a *point* electron and in this limit we have

$$F(t) = -eE(t), \tag{3}$$

where $E(t)$ is the fluctuating field-free electric field. In addition (see Eq. (5.14) of Ref. [5] with the form factor taken to be unity)

$$\mathrm{Re}\,\tilde{\mu}(\omega) = \frac{2e^2}{3c^3}\omega^2. \tag{4}$$

It follows that we can write, for each of the component directions,

$$C_E(t-t') \equiv \tfrac{1}{2}\langle E(t)E(t') + E(t')E(t)\rangle = \int_0^\infty \mathrm{d}\omega P(\omega) \cos \omega(t-t'), \tag{5}$$

where

$$P(\omega) = \frac{\hbar\omega}{\pi e^2} \mathrm{Re}\ \tilde{\mu}(\omega) \coth(\hbar\omega/2kT) = \frac{2}{3\pi c^3} \hbar\omega^3 \coth(\hbar\omega/2kT). \tag{6}$$

Next, we turn to an evaluation of the total energy of the field, W say. Since the energy associated with the magnetic field $B(t)$ is the same as that for the electric field and recalling that (5) holds for the x, y and z components, we obtain

$$W = \frac{1}{8\pi}\langle E(t)^2 + B(t)^2\rangle = \frac{3}{4\pi} C_E(0) = \int_0^\infty \mathrm{d}\omega\ u(\omega), \tag{7}$$

where

$$u(\omega) = \frac{3}{4\pi} P(\omega) = (\hbar\omega^3/2\pi^2 c^3) \coth(\hbar\omega/2kT), \tag{8}$$

which is the Planck spectrum, plus the zero-point contribution, for the energy-density of the electromagnetic field (the reservoir).

We conclude that the IO model incorporates as a special case the problem of a charged harmonic oscillator interacting with the radiation field and that the associated reservoir displays the Planck spectrum, in contrast to the case of the RWA model. More generally, we have presented a useful criterion for assessing the validity of heat bath models. Plans for the future include use of the IO model in applications where the RWA model was formerly used. In this context, we have already obtained a master equation for the IO model, a special case of which leads to a master equation for a charged oscillator coupled to the radiation field, without any approximations being made to the Hamiltonian [9].

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