

Effect of stray capacitances on single electron tunneling in a turnstile

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Based on the exact solution for the potential profile of the $2N$ turnstile with equal junction capacitances C , equal stray capacitances C_0 , and a coupling capacitance C_c , we obtain explicit expressions for the Gibbs free energy as well as the corresponding charging energy and the barrier height. In particular, we analyze the effects of the stray capacitances on the turnstile operation. In the $C_0=0$ case, our results for the turnstile operation reduce to those of D. V. Averin, A. A. Odintsov, S. V. Vyshenskii [J. Appl. Phys. **73**, 1297 (1993)]. In general, when C_0/C is increased, the operable region of the turnstile decreases. Thus, in order to have a high quality turnstile, it is necessary to keep the stray capacitances small. © 1996 American Institute of Physics. [S0021-8979(96)06215-9]

I. INTRODUCTION

Recent advances in nanoscale fabrication techniques^{1,2} have enabled one to design devices based on the controlled transfer of single electrons due to the Coulomb blockade effect. These devices are, in particular, potentially useful for metrological applications such as fundamental standards of dc current and for digital devices. The most remarkable candidates for such standards are the single-electron turnstile,³ where a gate electrode controlled by an rf signal is capacitively coupled to the center of the array, and the single electron pump,⁴ where two gate electrodes controlled by two rf signals are capacitively coupled to the electrodes inside the array. Using the gate voltage V_g in these devices, one can make a single electron enter the island from the left junction, hold it in the island for an arbitrary time, and finally make it leave the island through the right junction. In the literature, Averin, Odintsov, and Vyshenskii have analyzed the dynamics of single electron tunneling in the turnstile and presented a detailed diagram illustrating the turnstile operation in the ‘bias voltage–gate voltage’ plane (see Fig. 2 in Ref. 5). Nevertheless, their study is restricted to the simplified turnstile with no stray capacitances, while the actual experimental systems⁶ have stray capacitances. The aim of this article is to perform a general study of the dynamics of single electron tunneling in the turnstile. In particular, we analyze the effects of the stray capacitances on the turnstile operation.

II. FORMULATION

Let us consider a $2N$ turnstile, consisting of a one-dimensional (1D) array of $2N$ equal junction capacitances C , and equal stray capacitances C_0 , as shown in Fig. 1, where the bias voltage of the left edge is $\Phi_0 = V/2$, while that of the right edge is $\Phi_{2N} = -V/2$. The gate voltage V_g is connected to the middle electrode of the arrays via a coupling capaci-

tance C_c . We denote the potential and the number of excess electrons on each of the individual $2N-1$ islands between the junctions in the array by the column vectors $\bar{\Phi} = \{\Phi_1, \Phi_2, \dots, \Phi_N, \dots, \Phi_{2N-1}\}^T$ and $\bar{n} = \{n_1 - CV/2e, n_2, \dots, n_N - \alpha U, n_{N+1}, \dots, n_{2N-1} + CV/2e\}^T$, respectively, where $U = CV_g/e$ and $\alpha = C_c/C$. The equations giving the relations between the island potentials $\{\Phi_i\}$ and the number of the excess electrons $\{n_i\}$ on the islands are derived from the charge conservation laws, which are expressed as

$$\Phi_{i-1} + D\Phi_i + \Phi_{i+1} = n_i e / C \quad (1)$$

$(i = 1, 2, \dots, N-1, N+1, \dots, 2N-1),$

$$\Phi_{N-1} + D'\Phi_N + \Phi_{N+1} = (n_N - \alpha U) e / C, \quad (2)$$

where $D = -2 - \beta$ with $\beta = C_0/C$ and $D' = -2 - \alpha$. These equations can be conveniently written in its matrix form

$$\mathbf{M}\bar{\Phi} = \bar{n}e/C, \quad (3)$$

where \mathbf{M} is a $2N-1 \times 2N-1$ symmetric matrix having submatrices as follows:

$$\mathbf{M} = \begin{pmatrix} \mathbf{S} & \bar{\mathbf{1}} & \mathbf{0} \\ \bar{\mathbf{1}}^T & D' & \bar{\mathbf{1}}' \\ \mathbf{0} & \bar{\mathbf{1}}'^T & \mathbf{S} \end{pmatrix}. \quad (4)$$

Here \mathbf{S} is an $(N-1) \times (N-1)$ symmetric tridiagonal matrix, having the same diagonal elements $-2 - \beta$ and the same off-diagonal elements 1, the column vectors $\bar{\mathbf{1}} = \{0, 0, \dots, 1\}^T$ and $\bar{\mathbf{1}}' = \{1, 0, \dots, 0\}$ have all $N-1$ elements, and $\mathbf{0}$ is an $(N-1) \times (N-1)$ null matrix. By using the method presented in Ref. 7 for inversion of a symmetric matrix \mathbf{M} , we obtain from Eq. (3)

$$\bar{\Phi} = \mathbf{M}^{-1} \bar{n}e/C = -\mathbf{R}\bar{n}e/C, \quad (5)$$

where the elements of the symmetric matrix \mathbf{R} are given by

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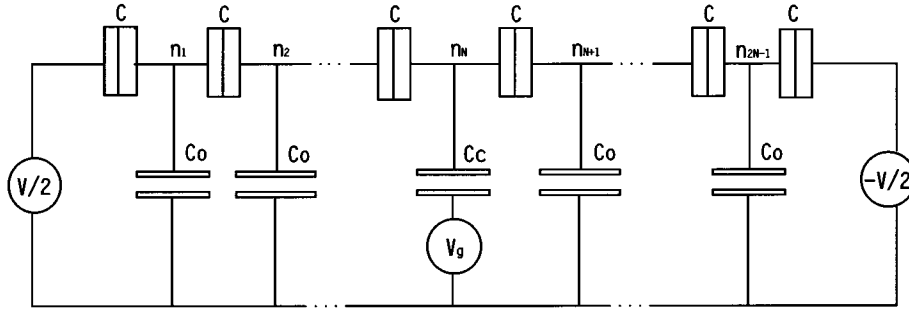


FIG. 1. Schematic of a $2N$ turnstile, which consists of $2N$ small tunnel junctions in series, with equal junction capacitances C , equal stray capacitances C_0 , and a coupling capacitance C_c . The bias voltages of the left edge and right edge are $V/2$ and $-V/2$, respectively. The gate voltage V_g is connected to the middle electrode of the arrays via the coupling capacitor.

$$R_{ij} = \frac{\sinh i\lambda [\sinh(2N-j)\lambda + (D-D')\sinh N\lambda \sinh(N-j)\lambda \theta(N-j)/\sinh \lambda]}{\sinh \lambda \sinh 2N\lambda + (D-D')\sinh^2 N\lambda} \quad \text{for } i \leq j, \quad i \leq N, \quad \text{and } j \leq 2N-1 \quad (6)$$

with λ defined by $-2 \cosh \lambda = D$, $\theta(x)$ being the Heaviside step function, which equals 1 for $x > 0$ and 0 for $x \leq 0$. The symmetric matrix \mathbf{R} in Eq. (5) has the following symmetric properties:

$$R_{ji} = R_{ij}, \quad R_{2N-i, 2N-j} = R_{ij}, \quad (7)$$

which is due to the symmetric structure of the turnstile with equal junction capacitances. Equation (5), supplemented by Eq. (6), is the main result of this article. We see that the potential profile $\{\Phi_i\}$ can be determined from Eq. (5) if the charge profile $\{n_i\}$ is given.

Now we evaluate the Gibbs free energy of the $2N$ turnstile, which is a crucial quantity in determining the rate of tunneling through the small junctions. The Gibbs free energy of the $2N$ turnstile is the sum of the electrostatic energy E_s and the work done W due to the charge redistribution associated with the change of the charge profile $\{\bar{n}\}$ on the island:

$$F = E_s + W, \quad (8)$$

where the electrostatic energy E_s is defined as

$$E_s = \frac{C}{2} \left[\sum_{i=1}^{2N} (\Phi_i - \Phi_{i-1})^2 + \beta \left(\sum_{i=1}^{N-1} \Phi_i^2 + \sum_{i=N+1}^{2N-1} \Phi_i^2 \right) + \alpha (V_g - \Phi_N)^2 \right] - e \sum_{i=0}^{2N} n_i \Phi_i. \quad (9)$$

Here the first term on the right-hand side of Eq. (9) is the total charging energy for the junctions, the second and third terms are the charging energies for the stray capacitors and the coupling capacitor, respectively, and the last term is the electrostatic energy of the excess electrons in the islands between every two nearest-neighbor junctions connected in series. The work done due to the charge redistribution associated with the change of the charge profile $\{\bar{n}\}$ is given by

$$W = - \sum_{i=1}^{2N} V_i Q_i - \left(\sum_{i=1}^{N-1} \Phi_i Q_i^s + \sum_{i=N+1}^{2N-1} \Phi_i Q_i^s \right) - (V_g - \Phi_N) Q^c, \quad (10)$$

where the first, second, and last terms on the right-hand side of Eq. (10) are, respectively, the work done by the contribution of the $2N$ junctions, the stray capacitors, and the coupling capacitor. Also, $V_i = \Phi_{i-1} - \Phi_i$ while $\Phi_0 = V/2$ and $\Phi_{2N} = -V/2$ denote the local voltages, and Q_i , Q_i^s , and Q^c are the charges on the i th junction, on the i th stray capacitor,

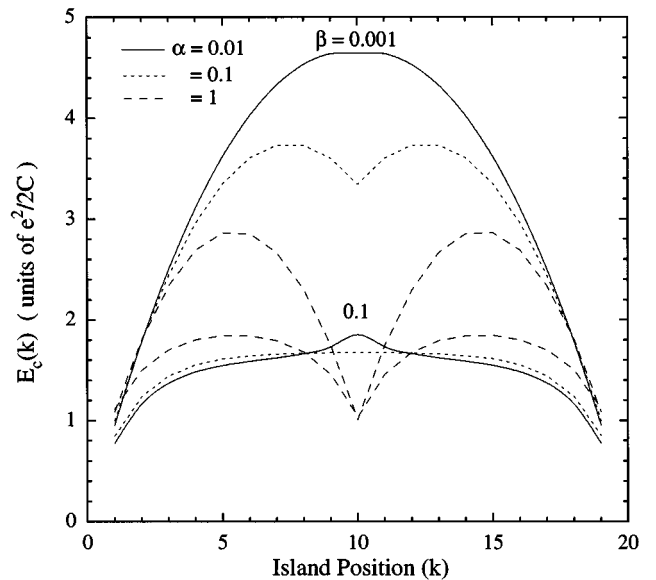


FIG. 2. Charging energy $E_c(k)$ (in units of $e^2/2C$) for a $2N$ turnstile with the number of junctions $N=10$, with $U = CV_g/e = 1$ and $CV/e = 0.5$, and with an excess electron at the k th island, as a function of k at three different values of $\alpha = 0.01$ (full curves), 0.1 (dotted curves), and 1 (dashed curves) for $\beta = 0.001$ and 0.1 , where $\alpha = C_c/C$ and $\beta = C_0/C$ with C_c , C_0 , and C being the coupling capacitance, the stray capacitance, and the junction capacitance, respectively.

and on the coupling capacitor, respectively. As seen from Eqs. (9) and (10), the Gibbs free energy of Eq. (8) is now expressed in terms of the potential profile $\{\Phi\}$ and charge profile $\{\bar{n}\}$. With Eqs. (5)–(7) we obtain explicitly

$$F = E_c - \frac{V}{2} (Q_0 - Q_{2N}) - V_g Q_N^c, \quad (11)$$

where

$$E_c = E_0 + \frac{e^2}{2C} \sum_{i,j=1}^{2N-1} n_i R_{ij} n_j, \quad (12)$$

$$E_0 = \frac{1}{4} C V^2 (1 - R_{11} + R_{1,2N-1}) + \frac{1}{2} C_c V_g^2 (1 - \alpha R_{NN}), \quad (13)$$

$$Q_0 = n_0 e + C(\Phi_0 - \Phi_1), \quad (14)$$

$$Q_{2N} = n_{2N} e + C(\Phi_{2N} - \Phi_{2N-1}), \quad (15)$$

$$Q_N^c = C_c (V_g - \Phi_N). \quad (16)$$

Equation (11) is a general expression for the Gibbs free energy of a $2N$ turnstile with bias voltage $\{\Phi_0, \Phi_{2N}\}$, charge $\{\bar{n}\}$, and potential profile $\{\Phi\}$ on the islands.

Next, we calculate the charging energy E_c of the system, where there is an excess electron on the k th island. In this case, one has $n_i = \delta_{ik}$, and the charging energy term in Eq. (12) reduces to

$$E_c(k) = E_0 + \frac{e^2}{2C} R_{kk}, \quad (17)$$

where E_0 is given by Eq. (13) and is independent of the charge profile $\{\bar{n}\}$. Using Eq. (6), the charging energy can be rewritten as

$$E_c(k) = \begin{cases} E_0 + \frac{e^2}{2C} \frac{\sinh k\lambda [\sinh(2N-k)\lambda + (\alpha - \beta) \sinh N\lambda \sinh(N-k)\lambda / \sinh \lambda]}{\sinh \lambda \sinh 2N\lambda + (\alpha - \beta) \sinh^2 N\lambda} & \text{for } 0 < k \leq N \\ E_0 + \frac{e^2}{2C} \frac{\sinh(2N-k)\lambda [\sinh k\lambda + (\alpha - \beta) \sinh N\lambda \sinh(k-N)\lambda / \sinh \lambda]}{\sinh \lambda \sinh 2N\lambda + (\alpha - \beta) \sinh^2 N\lambda} & \text{for } N \leq k < 2N. \end{cases} \quad (18)$$

Based on a numerical evaluation of Eq. (18), we present in Fig. 2 the dependence of the charging energy $E_c(k)$ on the island position k for values of $\alpha=0.01, 0.1, \text{ and } 1$ and $\beta=0.001$ and 0.1 for a fixed $N=10, U=1, \text{ and } CV/e=0.5$. As shown in the figure, $E_c(k)$ has exactly a symmetric form about the middle island ($k=10$). When α and β become zero, $E_c(k)$ has its the maximum value on the middle island (N). As the value α increases, the positions of the maximum values of the $E_c(k)$ move from the middle island (N) to the $(N/2)$ th and $(3N/2)$ th islands. For large α , the $E_c(k)$ for the middle island approaches the minimum value, and hence the barrier height on the middle island will be a maximum.

To get the explicit expression for the barrier height of the trapped electron, we find, using Eq. (18), the position k_m , corresponding to the maximum value of the barrier height:

$$k_m = \begin{cases} N - \frac{1}{4\lambda} \ln \left(\frac{e^\lambda - e^{-\lambda} + (\alpha - \beta)(e^{2N\lambda} - 1)}{e^\lambda - e^{-\lambda} + (\alpha - \beta)(1 - e^{-2N\lambda})} \right) & \text{for } 0 < k \leq N \\ N + \frac{1}{4\lambda} \ln \left(\frac{e^\lambda - e^{-\lambda} + (\alpha - \beta)(e^{2N\lambda} - 1)}{e^\lambda - e^{-\lambda} + (\alpha - \beta)(1 - e^{-2N\lambda})} \right) & \text{for } N \leq k < 2N. \end{cases} \quad (19)$$

In the above evaluation, we have treated k_m as a continuous variable, whereas it is an integer. Thus, to obtain the position, we should take the closest integer to the value given by Eq. (19). In the $\beta \ll 1$ limit, Eq. (19) reduces to a simple form,

$$k_m = \begin{cases} \frac{N}{2} \left(1 + \frac{1}{1 + \alpha N} \right) & \text{for } 0 < k \leq N \\ \frac{N}{2} \left(3 - \frac{1}{1 + \alpha N} \right) & \text{for } N \leq k < 2N. \end{cases} \quad (20)$$

For very small α and β , all the k_m of Eqs. (19) and (20) tend to the value of N , which is the position of the middle island, while, in the $\alpha N \gg 1$ limit, the k_m of Eq. (20) approaches $N/2$ for $0 < k \leq N$ and $3N/2$ for $N \leq k < 2N$, respectively, as seen from Fig. 2. With Eqs. (18) and (19), we can obtain the value of the barrier height ΔE for an electron on the edge of the junction and on the middle island, respectively:

$$\Delta E^{(1)} \equiv (e^2/2C) \{R_{k_m k_m} - R_{11}\} = \begin{cases} \frac{e^2}{2C} \left(\frac{\tanh k_m \lambda}{2 \sinh \lambda} - \frac{\sinh \lambda \sinh(2N-1)\lambda + (\alpha - \beta) \sinh N\lambda \sinh(N-1)\lambda}{\sinh \lambda \sinh 2N\lambda + (\alpha - \beta) \sinh^2 N\lambda} \right) & \text{for } 0 < k_m \leq N \\ \frac{e^2}{2C} \left(\frac{\tanh k'_m \lambda}{2 \sinh \lambda} - \frac{\sinh \lambda \sinh(2N-1)\lambda + (\alpha - \beta) \sinh N\lambda \sinh(N-1)\lambda}{\sinh \lambda \sinh 2N\lambda + (\alpha - \beta) \sinh^2 N\lambda} \right) & \text{for } N \leq k_m < 2N \end{cases}, \quad (21)$$

$$\Delta E^{(N)} \equiv (e^2/2C)\{R_{k_m k_m} - R_{NN}\} = \begin{cases} \frac{e^2}{2C} \left(\frac{\tanh k_m \lambda}{2 \sinh \lambda} - \frac{\sinh^2 N \lambda}{\sinh \lambda \sinh 2N \lambda + (\alpha - \beta) \sinh^2 N \lambda} \right) & \text{for } 0 < k_m \leq N \\ \frac{e^2}{2C} \left(\frac{\tanh k'_m \lambda}{2 \sinh \lambda} - \frac{\sinh^2 N \lambda}{\sinh \lambda \sinh 2N \lambda + (\alpha - \beta) \sinh^2 N \lambda} \right) & \text{for } N \leq k_m < 2N \end{cases} \quad (22)$$

with $k'_m = 2N - k_m$. The barrier height $\Delta E^{(N)}$ for the trapped electron on the middle island increases when either the number of the junctions N or the value α increases and the value β decreases. However, the barrier height $\Delta E^{(1)}$ for an electron on the edge of the junction increases when the number of the junctions N increases or the value α decreases.

III. OPERATION CONDITIONS FOR TURNSTILE

Next, we calculate the change of the Gibbs free energy ΔF due to some charge transfer tunnel event by means of Eq. (11). For simplicity, we consider only the case where the charge transfer occurred between islands k and k' , while the charges on the other islands remain unchanged. We denote the charges on these two islands before and after the charge transfer, respectively, as $\{n_k, n_{k'}\}$ and $\{n'_k, n'_{k'}\}$, and the net transferred charges as Q . Under the above condition, the change of the Gibbs free energy $\Delta F^Q(k, k')$ due to the charge transfer $\{n_k, n_{k'}\}$ to $\{n'_k, n'_{k'}\}$ can be derived from Eq. (11). In particular, for the single electron transfer case with $n_i = \delta_{ik}$ and $n'_i = \delta_{i, k'}$, it reduces to

$$\Delta F^e(k, k') = \frac{e^2}{2C} \left[(R_{k'k'} - R_{kk}) - \frac{CV}{e} (\delta_{0, k'} - \delta_{0, k} + \delta_{2N, k} - \delta_{2N, k'} + R_{1k'} - R_{1k} + R_{2N-1, k} - R_{2N-1, k'}) - 2(\alpha U - \delta_{1, n_N}) \times (R_{Nk'} - R_{Nk}) \right]. \quad (23)$$

The tunneling of a charge soliton from the k th island to the k' th island in the turnstile takes place when the change of the Gibbs free energy $\Delta F^Q(k, k')$ is less than zero. Using Eq. (23) and following the original argument of Averin, Odintsov, and Vyshenskii,⁵ we now derive the operating conditions for an empty turnstile with capacitances. In order to pull an electron into the empty turnstile from the left-hand side, one should have $\Delta F^e(0, 1) < 0$ and $\Delta F^e(2N, 2N-1) > 0$, which give the conditions

$$u + \frac{v}{A} > B, \quad (24)$$

$$u - \frac{v}{A} < B, \quad (25)$$

where

$$u = N(2C_c V_g / e - 1), \quad v = (2 + \alpha N) CV / e, \quad (26)$$

$$A = \frac{(2 + N\alpha) R_{1N} / N}{1 - R_{11} + R_{1, 2N-1}}, \quad (27)$$

$$B = N(R_{11} - R_{1N}) / R_{1N}. \quad (28)$$

In addition to Eqs. (24) and (25), one also needs to ensure that only one electron can be pulled in, and that the pulled-in electron is trapped on the central electrode. Using Eq. (23), these conditions imply

$$u + \frac{v}{A} < B + 2N, \quad (29)$$

$$\frac{v}{A'} - u < B', \quad (30)$$

where

$$A' = \frac{(2 + N\alpha)(R_{NN} - R_{N, N+1}) / N}{R_{1, N-1} - R_{1, N+1}}, \quad (31)$$

$$B' = N(R_{N+1, N+1} - R_{N, N+1}) / (R_{NN} - R_{N, N+1}). \quad (32)$$

Similar to the conditions (24), (25), (29), and (30), one can obtain from Eq. (23) a set of conditions for the trapped electron in the central electrode to be pushed out through the right-hand branch of the turnstile:

$$\frac{v}{A'} - u > B', \quad (33)$$

$$- \frac{v}{A'} - u < B', \quad (34)$$

$$\frac{v}{A} - u < B + 2N, \quad (35)$$

$$\frac{v}{A} + u < B. \quad (36)$$

Equations (24), (25), (29), (30), and (33)–(36) define the regions in the parameter plane (v, u) , where the turnstile can be operated correctly by modulation of the gate voltage V_g between the pull-in and the push-out regions. This is further illustrated in Fig. 3, where we plot the pull-in conditions Eqs. (24), (25), (29), and (30) and push-out conditions Eqs. (33)–(36) in the (v, u) plane at three different values of stray capacitances: (a) $\beta=0$, (b) $\beta=0.05$, and (c) $\beta=0.2$. When $\beta=0$ (corresponding to zero stray capacitance), it is clear from Eq. (6) that Eqs. (27) and (28) reduce to, respectively,

$$A = A' = 1, \quad (37)$$

$$B = B' = (1 + \alpha N)(N - 1). \quad (38)$$

It follows that, in the case of zero stray capacitance, our results reduce to those of the Averin, Odintsov, and Vyshenskii [see Eqs. (5)–(7), and Fig. 2 in Ref. 5]. In this case, Eqs. (24), (25), (29), (30) and (33)–(36) form two rectangular regions in the (v, u) plane [see Fig. 3(a)], where the upper

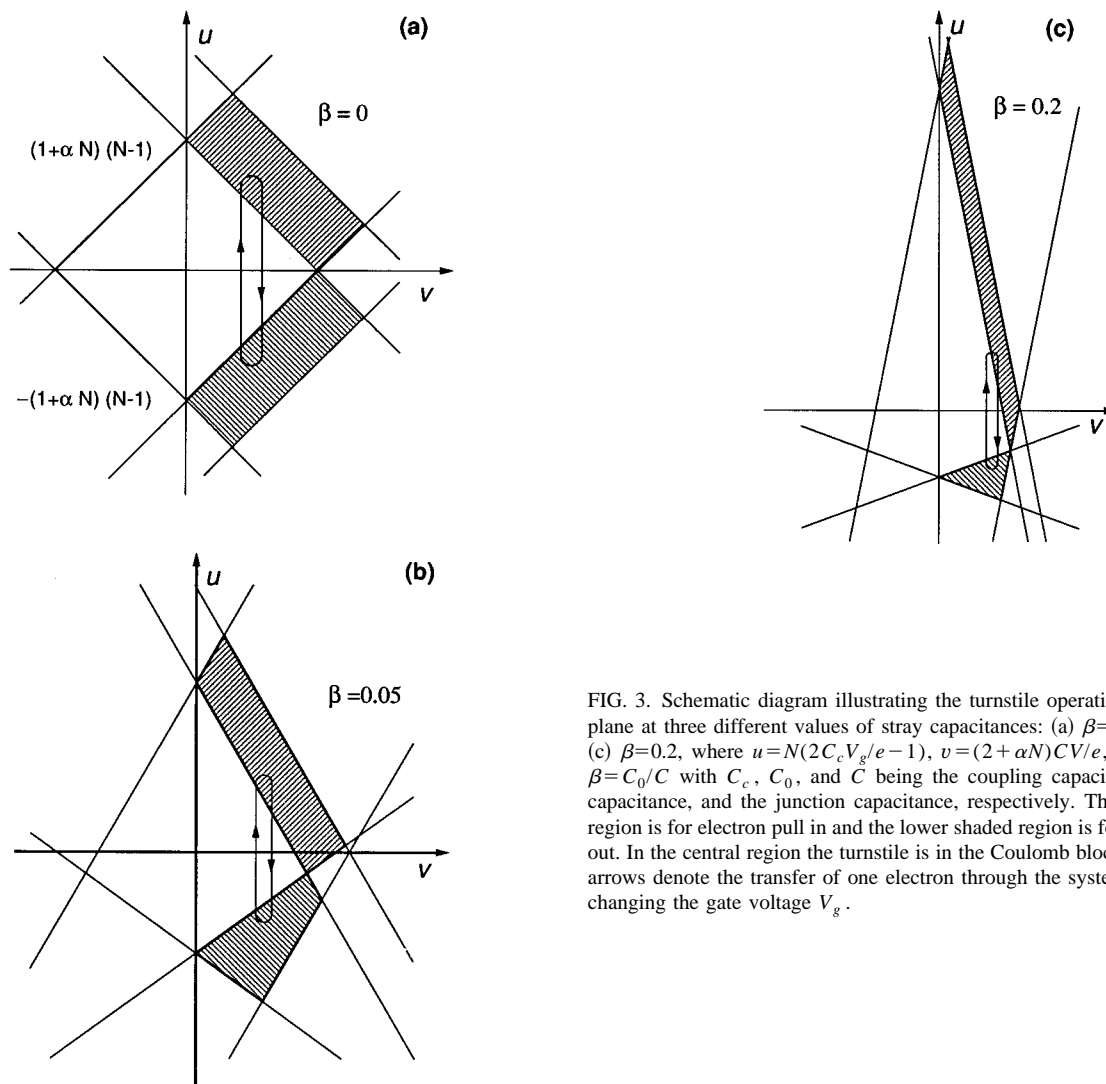


FIG. 3. Schematic diagram illustrating the turnstile operation in the (v, u) plane at three different values of stray capacitances: (a) $\beta=0$; (b) $\beta=0.05$; (c) $\beta=0.2$, where $u=N(2C_c V_g/e-1)$, $v=(2+\alpha N)CV/e$, $\alpha=C_c/C$, and $\beta=C_0/C$ with C_c , C_0 , and C being the coupling capacitance, the stray capacitance, and the junction capacitance, respectively. The upper shaded region is for electron pull in and the lower shaded region is for electron push out. In the central region the turnstile is in the Coulomb blockade state. The arrows denote the transfer of one electron through the system by means of changing the gate voltage V_g .

shaded region is for electron pull in and the lower region is for electron push out. These two regions are separated by a square-shaped Coulomb blockade region in which the current does not flow through the turnstile. In this way, when a small frequency of gate modulation V_g is applied to the system so that V_g is switched between the upper and lower dashed regions in Fig. 3 (as illustrated by arrows), exactly one electron is transferred through the turnstile per period of V_g modulation. Also, it is indicated by Figs. 3(b) and 3(c) that when $\beta \neq 0$, the operating conditions deviates from that of the $\beta=0$ case, dramatically. In general, when β is increased, the central region of Coulomb blockade in Fig. 3 shrinks, and the operable regions of the turnstile become smaller. Thus, in order to have a high quality turnstile, it is important to keep the stray capacitances small ($\beta \ll 1$).

IV. SUMMARY

In summary, in this article we have presented an exact analytical solution of Eq. (5) for the potential profiles of the $2N$ turnstile with equal junction capacitances, equal stray

capacitances, and a coupling capacitance. On the basis of Eq. (5), we obtained explicit expressions for the free energy, the charging energy and the barrier height for a designated charge soliton configuration. It is shown that the charging energy, the barrier height and the free energy are very sensitive to the values α and β . Our results show that for very small α and β , the charging energy has the maximum value on the middle island, and hence the barrier height on the middle island becomes zero. Also, we have derived the operating conditions, Eqs. (24), (25), (29), (30), and (33)–(36), for an empty turnstile with stray capacitances. Utilizing these conditions, we have presented a detailed diagram illustrating the turnstile operation in the (v, u) plane, as shown in Fig. 3. In the $\beta=0$ ($C_0=0$) case [see Fig. 3(a)], our results reduce to those of the Averin, Odintsov, and Vyshenskii.⁵ When β increases, the operable region of the turnstile decreases [see Figs. 3(b) and 3(c)]. Thus, in order to have a high quality turnstile, it is necessary to keep the stray capacitances very small. In conclusion, we have obtained results which give insight into the behavior of the $2N$ turnstile and should provide guideposts for future experiments.

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