

## Cotunneling in single-electron devices: Effects of stray capacitances

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An analytic treatment of the effects of stray capacitances on cotunneling in single-electron devices with equal junction capacitances is presented. By using analytical forms of the Gibbs free energy and by extending the Jensen-Martini approximation [Phys. Rev. B **46**, 13 407 (1992)], we have calculated the threshold voltages and currents for cotunneling in one-dimensional arrays, single-electron traps, and single-electron turnstiles. Our results show that the main effect of the stray capacitances on the cotunneling is to reduce the threshold voltages, whereas it has very little effect on the magnitude of the tunneling current. In general, when the stray capacitances increase, the current-voltage curve of the single-electron device is shifted towards the low-voltage side. The relevance of our theoretical results to some experiments are also discussed.

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### I. INTRODUCTION

In recent years some small circuits, "Single-Electron" devices,<sup>1</sup> cooled below 1 K have been devised where the electrons can be transferred one at a time in a stepwise fashion. In these single-electron devices, small tunnel junctions have been used as switches that control the charge flow in the circuits. The basic operating principle of the small tunnel junction is the Coulomb blockade effect, where it is found that in a single small tunnel junction, having tunnel resistance  $R_T$  and capacitance  $C$ , such that  $R_T \gg R_k = h/e^2 = 25.6$  k $\Omega$  and the charging energy  $e^2/2C$  exceeds the characteristic energy  $k_B T$  of thermal fluctuations, a suppression of single-charge tunneling dramatically reduces the current at voltages  $V < e/2C$ . One of the most important features of the existing single-electron devices is that they all contain one-dimensional (1D) long arrays<sup>2-8</sup> of small tunnel junctions. The use of long arrays<sup>2</sup> has at least three advantages: (i) it is easier to fabricate high-quality devices with high  $R_T$  and low  $C$  in the form of long arrays; (ii) the electromagnetic environment influence, which may smear out the Coulomb blockade effect, can be kept at a minimum; and (iii) it possesses some unique features of electronic transfer such as space correlation.

The most common case of single-charge tunneling is that of a single-electron tunneling across one junction at a time. However, in this paper we are concerned with cotunneling in which several tunneling events across different junctions occur at the same time.<sup>1-4</sup> Inclusion of such a process can significantly affect the accuracy of single-electron devices. In the Coulomb blockade region of single-junction tunneling, cotunneling across a long array of  $N$  junctions with a bias voltage  $V$  ( $N$ th-order cotunneling or simply  $N$  cotunneling) can occur because the change of electrostatic energy due to this process is  $-eV$ . By the same token, an  $m$  ( $m \leq N$ ) cotunneling process is always possible in the Coulomb blockade region of single-junction tunneling if one assumes that there is no time limit for observing the tunneling. Thus, in order to determine the accuracy of the single-electron devices, it is essential to understand what the rate of transition of  $m$  cotunneling is. In the literature, many theoretical stud-

ies have devoted to this subject. Averin and Odintsov<sup>2</sup> pioneered the study by proposing a rate formula for  $m$  cotunneling in the first-order approximation. Lafarge and Esteve<sup>4</sup> carried out calculations beyond the lowest-order perturbation. In a detailed study, Jensen and Martinis<sup>3</sup> have developed a better approximation for calculating the rate formula for  $m$  cotunneling, which avoids the divergence problem in the original work of Averin and Odintsov in a natural way. While the actual devices always have some stray capacitances, all the previous works have been based on the use of ideal 1D array with no stray capacitances. It is known that the presence of the stray capacitance enhances effectively the junction capacitance and thereby reduces the threshold voltage of the Coulomb blockade of single-electron tunneling. On the other hand, it is unclear how the stray capacitance will affect the rate of tunneling. In this paper we study the cotunneling phenomenon in single-electron devices in a more realistic term by including the stray capacitances in the calculation.

In a series of works,<sup>6-8</sup> we have obtained exact solutions to the electrostatics and the associated Gibbs free energy for various long-array systems with stray capacitances (the 1D arrays, the single-electron trap, and the single-electron turnstile). In a single-electron device, although the bias voltage controls the average value of the current passing through the system, the dynamics of an electron in the system at  $T=0$  is, in principle, solely determined by the Gibbs free energy. The net transfer of an electron from one island to another through the  $m$  tunnel junctions ( $m$  cotunneling) between them is favorable if the Gibbs free energy decreases in this process and vice versa. Thus the essence of the dynamics is the evaluation of the Gibbs free energy, which consists of a charging energy term and a work done term. Here we implement the analytical form of the Gibbs free energy and use the Jensen-Martini approximation to study the cotunneling rate of the single-electron devices. The paper is organized as follows. In Sec. II, we describe the basic formulation, where we extend the Jensen-Martini formula to the general case of the cotunneling rate for single-electron devices with stray capacitance, which include 1D arrays, single-electron traps, and single-electron turnstiles. Our results are summarized in Sec. III.

## II. FORMULATION

Consider a single-electron device with a long array of junctions. Following Averin and Odintsov,<sup>2</sup> an  $m$  cotunneling transition that transfers one electron through  $m$  ( $m \leq N$ ) tunnel junctions can be described by an arbitrary sequence of  $m$  single tunneling events  $\{j_1, \dots, j_k, \dots, j_m\}$ , where  $j_k$  denotes the position in the array of the  $k$ th tunneling. After  $k$  steps in the sequence  $\{j_1, \dots, j_k, \dots, j_m\}$ , the system is in the state  $s_k$ . Its intermediate energy  $E_k(m)$  is given by the sum of the Gibbs energy  $F_k$  relative to the energy of the initial state and the energies of the tunneling (electron-hole) excitons created by previous tunneling events on junctions  $j_1, \dots, j_k$ . In general, the rate of the  $m$  cotunneling process, which was worked out by Averin and Odintsov, has a complicated multidimensional integral form. For a 1D array of  $N$  small junctions, with equal junction capacitances  $C_1 = C_2 = \dots = C_N = C$  and without stray capacitances  $C_0$ , it can be calculated analytically subject to some approximations with the intermediate electron-hole excitation energies. The procedure we adopt is based on use of the Jensen-Martinis (JM) formula,<sup>3</sup> where the current due to  $m$  cotunneling in a 1D long array with equal junction tunnel resistances  $R_T$  and capacitances  $C$  and no stray capacitance takes the form

$$I = e \binom{N}{m} \Gamma_m, \quad (1)$$

where the  $m$  cotunneling rate

$$\Gamma_m = \frac{2\pi}{\hbar} \left[ \frac{R_k}{4\pi^2 R_T} \right]^m S_m^2 \frac{(-\Delta F_m)^{2m-1}}{(2m-1)!}, \quad (2)$$

with

$$\begin{aligned} S_m &= \sum_{\text{perm}\{j_1, \dots, j_m\}} \left( \prod_{k=1}^{m-1} \frac{1}{E_k(m)} \right) \\ &= \sum_{\text{perm}\{j_1, \dots, j_m\}} \left( \prod_{k=1}^{m-1} \frac{1}{\Delta F_k - \frac{k}{m} \Delta F_m} \right). \end{aligned} \quad (3)$$

The first equality in (3) is a general result,<sup>2,3</sup> whereas the second equality incorporates the JM approximation, which relies on the fact that the main contribution to multidimensional integral over the energy  $\omega_k$  of electron-hole excitations, implicitly contained in (1), comes from the point where  $\omega_1 = \omega_2 = \dots = \omega_{m-1}$ . Also in (3), the change of Gibbs free energy due to the  $m$  cotunneling is given by

$$\Delta F_m = \frac{e^2}{2C} \frac{m}{N} \left( N - m - \frac{2CV}{e} \right). \quad (4)$$

We have presented the JM formula only in its  $T=0$  form, which is the subject of this paper. The extension of the present work to finite temperatures can be worked out without difficulty. We note that from (4) one observes that the threshold voltage of the  $m$  cotunneling process is  $V_m = (N-m)e/2C$ , which indicates that a higher  $m$  cotunneling process should have lower  $V_m$ . In the following, we restrict our discussion to the application of (1)–(3) to a class of problems where in the single-electron devices (the 1D

array, the single electron trap, and the single-electron turnstile) one has equal junction capacitances  $C$  and equal stray capacitances  $C_0$  and where the analytical forms of (1)–(3) are available.

For a 1D array ( $A$ ) having equal junction capacitances  $C$  and equal stray capacitances  $C_0$ , the change of Gibbs free energy appearing in (2) and (3) can be written as<sup>6</sup>

$$\Delta F_k^A = \frac{e^2}{2C} R_{kk}^A - eV(1 - R_{1k}^A), \quad (5)$$

where

$$R_{1k}^A = \sinh(N-k)\lambda / \sinh N\lambda, \quad (6)$$

$$R_{kk}^A = R_{1k}^A \sinh k\lambda / \sinh \lambda, \quad (7)$$

and

$$\frac{C_0}{C} + 2 = 2 \cosh \lambda. \quad (8)$$

In addition, the intermediate energy appearing in (3) can be written as

$$\begin{aligned} E_k^A(m) &= \frac{e^2}{2C} \left( R_{kk} - \frac{k}{m} R_{mm}^A \right) \\ &\quad - eV \left( 1 - R_{1k}^A - \frac{k}{m} (1 - R_{1m}^A) \right). \end{aligned} \quad (9)$$

Equations (5)–(9) are interesting results. First, from (5) one immediately obtains the threshold voltage of the  $m$  cotunneling process in a 1D array as

$$V_m^A = \frac{e}{2C} \frac{R_{mm}^A}{1 - R_{1m}^A}. \quad (10)$$

Equation (10) is further illustrated in Fig. 1, where we plot  $V_m^A$  as a function of  $m$  for a 1D array with  $N=7$  junctions at different values of  $C_0/C=0, 0.005, 0.01, 0.05, 0.1, 0.2, 0.5$ . The figure shows that in general  $V_m^A$  decreases with increase of  $m$  or  $C_0/C$ . Second, in the no stray capacitance limit ( $\lambda \rightarrow 0$ ), one has  $R_{1k}^A = (N-k)/N$  and  $R_{kk}^A = kR_{1k}^A$ , and (9) reduces to the JMs results  $E_k^A(m) = e^2 k(m-k)/2CN$ . Third, substituting (5) and (9) into (2) and (3), respectively, one can evaluate the rate of the  $m$  cotunneling process in a 1D array analytically. A typical result is shown in Fig. 2, where we plot the tunneling current  $I$  (in units of  $e/CR_k$ ) as a function of the bias voltage at different values of  $C_0/C=0, 0.001, 0.005, 0.01$ . The figure indicates that the main effect of the stray capacitance on the tunneling of electrons is a shift of the threshold voltages and that there is no significant change of the tunneling rate. In fact, the  $I$ - $V$  curve of systems with different  $C_0/C$  values but at a fixed  $m$  have almost identical shapes except for a shift due to the different threshold voltages.

Next, we study the single-electron trap, where the end of the 1D array is connected to a well capacitor  $C_w$ . For a

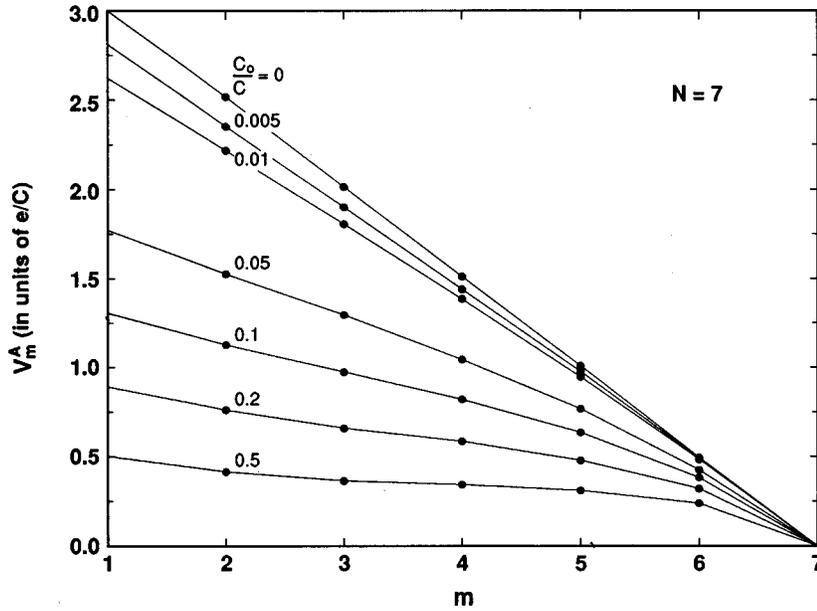


FIG. 1. Threshold voltage  $V_m^A$  (in units of  $e/C$ ) for the  $m$  cotunneling process in a 1D array with  $N=7$  junctions as a function of  $m$  at different values of  $C_0/C=0,0.005,0.01,0.05,0.1,0.2,0.5$ . Here  $C$  and  $C_0$  are the junction capacitances and stray capacitances, respectively.

single-electron trap ( $T$ ) having equal junction capacitances  $C$  and equal stray capacitances  $C_0$ , the change of Gibbs free energy can be written as<sup>7</sup>

$$\Delta F_k^T = \frac{e^2}{2C} R_{kk}^T - eV(1 - R_{1k}^T), \quad (11)$$

where

$$R_{1k}^T = \frac{\sinh(N-k-1)\lambda - \left(1 + \frac{C_w}{C}\right) \sinh(N-k)\lambda}{\sinh(N-1)\lambda - \left(1 + \frac{C_w}{C}\right) \sinh N\lambda}, \quad (12)$$

$$R_{kk}^T = R_{1k}^T \sinh k\lambda / \sinh \lambda, \quad (13)$$

and  $\lambda$  is given by (8). In addition, the intermediate energy can be written as

$$E_k^T(m) = \frac{e^2}{2C} \left( R_{kk}^T - \frac{k}{m} R_{mm}^T \right) - eV \left( 1 - R_{1k}^T - \frac{k}{m} (1 - R_{1m}^T) \right). \quad (14)$$

One can now use (11)–(14), together with (1)–(3), to study the threshold voltages and the current due to the  $m$  cotunneling process in a single-electron trap with equal stray capacitances and junction capacitances.

The threshold voltage can be obtained from (11) by the condition  $\Delta F_m^T = 0$ , with the result

$$V_m^T = \frac{e}{2C} \frac{R_{mm}^T}{1 - R_{1m}^T}. \quad (15)$$

Equation (15) is further illustrated in Fig. 3, where we plot  $V_m^T$  as a function of  $m$  for a single-electron trap with  $N=7$

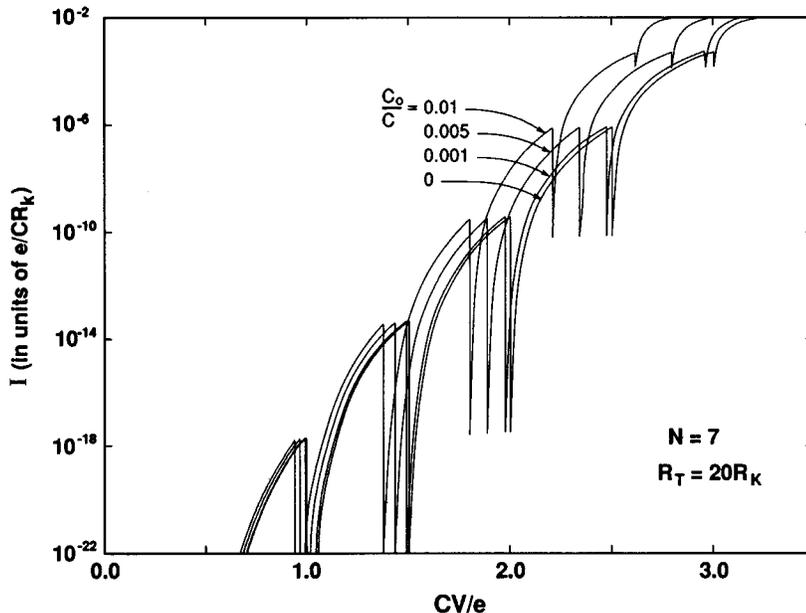


FIG. 2. Tunneling current  $I$  (in units of  $e/CR_K$ ) as a function of the bias voltage  $V$  for a 1D array with  $N=7$  junctions at  $R_T=20R_K$  and different values of  $C_0/C=0,0.001,0.005,0.01$ . Here  $R_T$  is the tunneling resistance and  $C$  and  $C_0$  are the junction capacitances and stray capacitances, respectively.

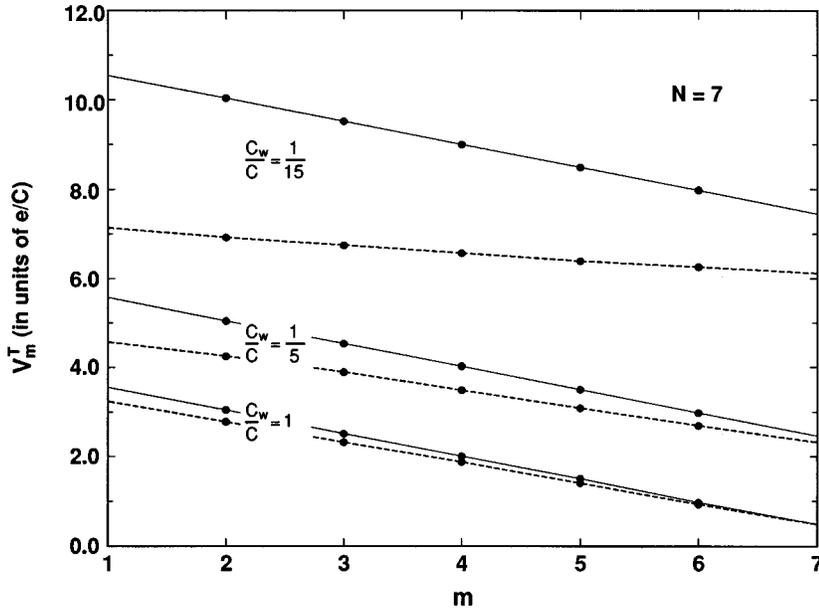


FIG. 3. Threshold voltage  $V_m^T$  (in units of  $e/C$ ) for the  $m$  cotunneling process in a 1D trap with  $N=7$  junctions as a function of  $m$  at different values of  $C_0/C=0$  (full lines), 0.005 (dotted lines), and  $C_w/C=\frac{1}{15}, \frac{1}{5}, 1$ . Here  $C$ ,  $C_0$ , and  $C_w$  are the junction capacitances, stray capacitances, and well capacitance, respectively.

junctions at different values of  $C_0/C=0, 0.005$  and  $C_w/C=1/15, 1/5, 1$ . The figure shows that, in general,  $V_m^T$  decreases with an increase of  $m$  or  $C_0/C$ . In addition, it decreases with increasing  $C_w/C$ . The current due to the  $m$  cotunneling process in a single-electron trap can be calculated analytically by substituting (11) and (14) into (2) and (3), respectively. A typical result is shown in Fig. 4, where we plot the tunneling current  $I$  (in units of  $e/CR_K$ ) as a function of the bias voltage at different values of  $C_w/C=0.1, 0.5, 1$  for a  $N=7$  trap with  $C_0/C=0.005$ . The figure indicates that the main effect of the well capacitance on the tunneling of electrons is a shift of the threshold voltages and there is no significant change of the tunneling rate. In fact, the  $I$ - $V$  curve of systems with different  $C_0/C$  values but at a fixed  $m$  have almost identical shapes, except for a shift due to the different threshold voltages.

We now study the single electron ‘‘turnstile,’’ where a gate electrode controlled by a rf signal is capacitively

coupled to the center of a long array. Using the control of the gate voltage, one can make a single-electron enter the island from the left junction, hold it in the island for an arbitrary time, and finally make it leave the island from the right junction. Here we consider a  $2N$  turnstile, consisting of a 1D array of  $2N$  equal junction capacitances  $C$ , and equal stray capacitances  $C_0$ , where the bias voltage of the left edge is  $V/2$ , while that of the right edge is  $-V/2$ . The gate voltage  $V_g$  is connected to the middle electrode of the arrays via a coupling capacitance  $C_g$ .

The change of the Gibbs free energy  $\Delta F$  due to the single-junction charge transfer, for a  $2N$  turnstile with stray capacitances, has been previously calculated.<sup>8</sup> Our study shows that for the single-junction charge transfer, in order to pull an electron into the empty turnstile from the left-hand side, one should have  $\Delta F(0,1)<0$  and  $\Delta F(2N,2N-1)>0$ . In addition, one also needs to ensure that only one electron can be pulled-in and that the pulled in electron is trapped on

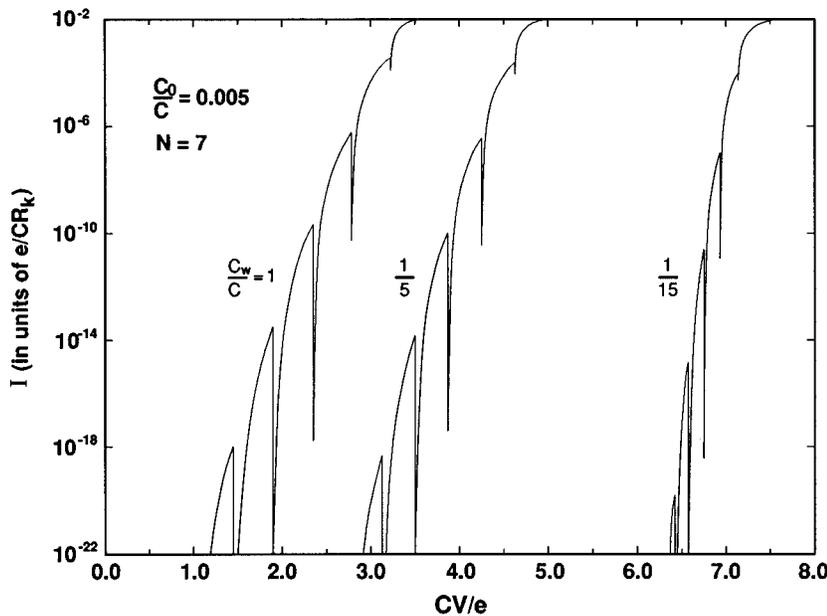


FIG. 4. Tunneling current  $I$  (in units of  $e/CR_K$ ) as a function of the bias voltage  $V$  for a 1D trap with  $N=7$  junctions at  $R_T=20R_K$  and different values of  $C_w/C=\frac{1}{15}, \frac{1}{5}, 1$  for a  $N=7$  trap with  $C_0/C=0.005$ . Here  $R_T$  is the tunneling resistance and  $C$ ,  $C_0$ , and  $C_w$  are the junction capacitances, stray capacitances, and well capacitance, respectively.

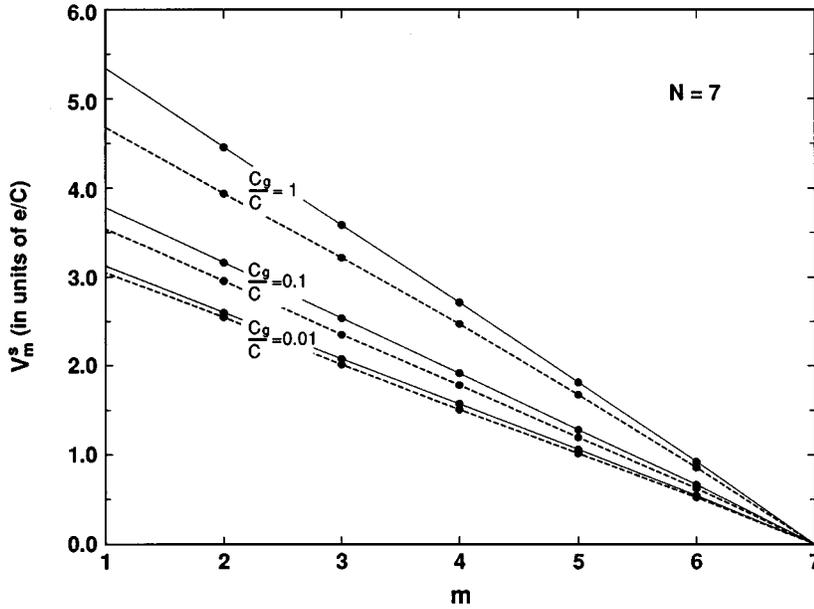


FIG. 5. Threshold voltage  $V_m^S$  (in units of  $e/C$ ) for the  $m$  cotunneling process in a single electron turnstile with  $N=7$  junctions as a function of  $m$  at different values of  $C_0/C=0$  (full lines), 0.01 (dotted lines), and  $C_g/C=0.01, 0.1, 1.0$ . Here  $C$ ,  $C_0$  and  $C_g$  are the junction capacitances, stray capacitances, and gate capacitance, respectively.

the central electrode. As a result of all these considerations, there exist many border lines that encircle each of the operating region of the turnstile. These border lines for single-junction tunneling can be identified by the condition that the relevant change of the Gibbs free energy equals zero. Naturally, all the calculations in Ref. 8 can be extended to the multijunction cotunneling cases. As an example, here we consider pulling an electron into the empty turnstile from the left-hand side by the  $k$  cotunneling process and discuss only the change of Gibbs free energy  $\Delta F(0,k)$ . Our treatment

here can easily be applied to the other border lines due to the  $k$  cotunneling process.

For a  $2N$  turnstile with stray capacitances, the change of the Gibbs free energy due to the  $k$  cotunneling process can be written as

$$\Delta F_k^S = \frac{e^2}{2C} R_{kk}^S - eV_g \frac{C_g}{C} R_{kN}^S - \frac{eV}{2} (1 - R_{1k}^S + R_{k,2N-1}^S), \quad (16)$$

where

$$R_{ij}^S = \frac{\sinh i \lambda \left[ \sinh(2N-j)\lambda + \theta(N-j) \frac{C_g - C_0}{C} \sinh N \lambda \sinh(N-j)\lambda / \sinh \lambda \right]}{\sinh \lambda \sinh 2N \lambda + \frac{C_g - C_0}{C} \sinh^2 N \lambda}, \quad i < j < N, \quad j < 2N-1, \quad (17)$$

with  $\lambda$  defined by (8) and  $\theta(x)$  being the Heaviside step function, which equals 1 for  $x > 0$  and 0 for  $x \leq 0$ . Also, the matrix element  $R_{ij}^S$  (17) has the symmetric properties

$$R_{ij}^S = R_{ji}^S, \quad R_{2N-i, 2N-j}^S = R_{ij}^S, \quad (18)$$

which is due to the symmetric structure of the turnstile with equal junction capacitances. In addition, the intermediate energy can be written as

$$E_k^S(m) = \frac{e^2}{2C} \left[ R_{kk}^S - \frac{k}{m} R_{mm}^S \right] - eV_g \frac{C_g}{C} \left[ R_{kN}^S - \frac{k}{m} R_{mN}^S \right] - \frac{eV}{2} \left[ 1 - R_{1k}^S + R_{k,2N-1}^S - \frac{k}{m} (1 - R_{1m}^S + R_{m,2N-1}^S) \right]. \quad (19)$$

The threshold voltage can be obtained from (16) by the condition  $\Delta F_m^T = 0$ , with the result

$$V_m^S = \frac{e}{2C} \frac{R_{mm}^S - 2C_g V_g R_{mN}^S / e}{1 - R_{1m}^S + R_{m,2N-1}^S}, \quad (20)$$

Equation (20) is further illustrated in Fig. 5, where we plot  $V_m^S$  as a function of  $m$  for a single-electron turnstile with  $N=7$  junctions at different values of  $C_0/C=0, 0.01$  and  $C_g/C=0.01, 0.1, 1.0$ . The figure shows that, in general,  $V_m^S$  decreases with an increase of  $m$  or  $C_0/C$ . In addition, it decreases with a decrease of  $C_g/C$ . The current due to the  $m$  cotunneling process in a single-electron trap can be calculated analytically by substituting (16) and (19) into (2) and (3), respectively. A typical result is shown in Fig. 6, where we plot the tunneling current  $I$  (in units of  $e/CR_K$ ) as a function of the bias voltage at different values of  $C_g/C=0.01, 0.1, 1.0$ ,  $C_0/C=0.005$ , and  $V_g = e/2C_g$ . The fig-

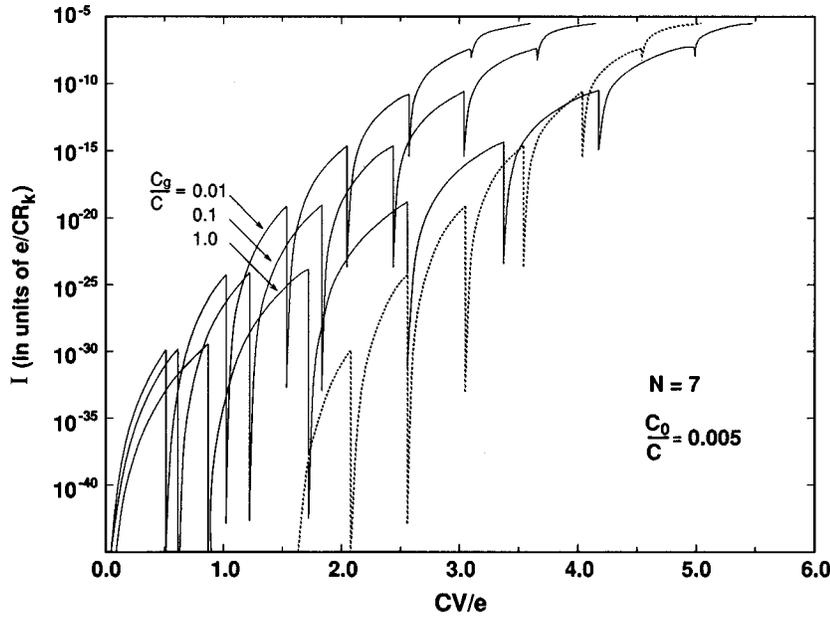


FIG. 6. Tunneling current  $I$  (in units of  $e/CR_K$ ) as a function of the bias voltage  $V$  for a single electron trap with  $N=7$  junctions at  $R_T=20R_K$  and different values of  $C_g/C=0.01, 0.1, 1.0$ ,  $C_0/C=0.005$  and gate voltage  $V_g=e/2C_g$ . Here  $R_T$  is the tunneling resistance, and  $C$ ,  $C_0$ , and  $C_g$  are the junction capacitances, stray capacitances, and gate capacitance, respectively. The dotted line is for the  $I$ - $V$  curve at  $C_g/C=0.01$ ,  $C_0/C=0.005$ , and  $V_g=e/4C_g$ .

ure indicates that the main effect of the gate capacitance to the tunneling of electrons is a shift of the threshold voltages and there is no significant change of the tunneling rate. In fact, the  $I$ - $V$  curve of systems with different  $C_g/C$  values but at a fixed  $m$  have almost identical shapes, except for a shift due to the different threshold voltages. Also, formulas (16) and (19) demonstrate that  $V_g$  is a crucial factor in determining the tunneling. For comparison, in Fig. 6 we plot the  $I$ - $V$  curve at  $V_g=e/4C_g$ , where it shows that the effect of  $V_g$  is to shift the  $I$ - $V$  curve.

### III. CONCLUSIONS

In this paper we have presented a general formalism for calculating the current due to cotunneling in single-electron devices with equal junction capacitances and stray capacitances. By using the analytical forms of the Gibbs free energies and by applying the Jensen-Matinis approximation, we have calculated the threshold voltages and currents for the cotunneling in 1D arrays, single-electron traps, and single-electron turnstiles, respectively. Our results show that the main effect of the stray capacitances on the cotunneling is to reduce the threshold voltages, whereas it has very little effect on the magnitude of the tunneling current. In general, when the stray capacitances increase, the  $I$ - $V$  curve of the single-electron device is shifted towards the low-voltage side.

The theoretical results presented in this paper are also

relevant to experimental results for single-electron devices. As an example, here we discuss the implication of our theoretical results for explaining the experimental results pertaining to a single-electron trap.<sup>10</sup> As discussed in Ref. 9, in order to understand the experimental results, it is crucial to consider the higher-order cotunneling process and  $C_0$ . Also, in order to determine which order of the cotunneling is effective, one needs to consider both the precision of the relevant experiments and the rate of the cotunneling. In the trap experiments of Ref. 10, an electron transition is observable when its rate is greater than of order 1/sec to  $10^{-2}$ /sec. Assuming that the stray capacitances have very small effect on the magnitude of the tunneling rate, in Ref. 9 we have used the JM results for the 1D array with no stray capacitance and concluded that the three-junction tunneling process is responsible for the experimental data of Ref. 10. Our results, presented in Fig. 4, confirm the assumption we made in Ref. 9. It is thus clear that the main effect of stray capacitances is not on the magnitude of the tunneling current but instead on the threshold voltage. For example, by means of (15) one can predict the effects of stray capacitances on the hysteresis voltage gap of a single-electron trap<sup>9</sup> and compare directly with the experiments of Ref. 10.

### ACKNOWLEDGMENT

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