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Environmental effects on a single electron box

G.Y. Hu, R.F. O'Connell

Department of Physics and Astronomy, Louisiana State University, Baton Rouge, LA 70803-4001, USA

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Abstract

Lafarge et al. (Zeitschrift für Physik B 85 (1991) 327) have measured the average junction charge in a single electron box. Their results are in agreement with theoretical predictions at temperatures T above 100 mK but not at lower T values. We explain the lower T experimental results by incorporating quantum charge fluctuations due to the environment by use of a quantum Langevin equation model. It is shown that, at $T = 0$, the sawtooth shape of the junction charge, as predicted by existing theories, is rounded off by the quantum fluctuation effects. At finite temperature, the theory is compared with the experiments of Lafarge et al., and a good fit is obtained at all the relevant temperatures.

Recently, a big effort has been made toward developing single charge transfer devices [1, 2], where the transfer of charge through small capacitance tunnel junction devices is controlled at the single electron level. These devices are potentially useful in meteorological applications such as the accurate measurement of the fine structure constant α and the realization of a current standard. The simplest such device and the building block of more sophisticated devices [1, 2], is a tunnel junction of capacitance C and resistance R_T, connected to a voltage source U via a capacitance C_S (see Fig. 1), nicknamed the 'single electron box'. Lafarge et al. [1, 2] have measured the average junction charge $\langle Q \rangle$, which is simply related to the average value $\langle n \rangle$, in a single electron box. Their results demonstrate that a single electron box allows a direct observation of the charge variables associated with single electron tunneling across a junction, based on the quantization of the island charge. In particular, their experimental results for the $\langle Q \rangle$ versus U curves at different temperatures T, ranging from 20 to 312 mK, are in agreement with the theoretical predictions (which considers thermal effects only) above 100 mK, but not at lower temperatures. Esteve [2] has pointed out the importance of the quantum fluctuations of n in explaining the experimental data at low T. This is because no matter how small T is, the tunneling probability is in fact finite, as a result of which the number of electrons in the electron box is not strictly conserved. However, it has also been shown that an approach based

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on a perturbation theory [2–5] in R_k/R_T (where the resistance quantum $R_k = h/e^2 \approx 25.8\ \mathrm{k}\Omega$) for the quantum corrections to $\langle Q \rangle$ does not settle the problem, and a complete non-perturbative theory is clearly needed. To date, no theoretical calculation of the quantum fluctuation effects on the single electron box gives results in agreement with the experiments.

Our purpose here is to provide a theoretical framework which is capable of incorporating quantum charge fluctuations, due to the environment, in a single electron box, and gives good agreement with the low-T experimental results. The theory we adopt here is the quantum Langevin (QLE) model [6–9], which has been previously applied to the single [6, 7] and double [8] junction systems. It has been demonstrated the QLE model captures the basic physics of quantum smearing of Coulomb blockade (finite I at $V < e/2C$, where C is the junction capacitance) by taking into account the zero-point fluctuations of the instantaneous charge on the junction. Also, it has been shown [9] that in the weak coupling limit ($R_k/R_T < 1$, which is certainly the case for the single electron box [1, 2]), the QLE model is consistent with the well known Landauer formalism. The advantage of the QLE model is that the quantum charge fluctuations $\langle q^2 \rangle$ are calculable quantities [6] once the set up of the experimental system is known (through the impedence of the environment, see discussion below). In addition, for those junctions where the environmental setup is not exactly known, and a direct calculation of $\langle q^2 \rangle$ is not possible, the QLE theory can still be used as a one parameter ($\langle q^2 \rangle$) theory to study the quantum charge fluctuation effects, which is mainly due to the feedback between the electrometer and the single electron box [1, 4].

We start with a brief review of the main aspects of the QLE model for the environmental effects on single electron tunneling. First, one solves the quantum Langevin equation [6, 7] for the Fourier transform $q(\omega)$ of the charge fluctuation $q(t)$. Then, using the fluctuation dissipation theory, one obtains the mean-square charge fluctuation [6, 7]

$$\langle q^2(t) \rangle = \int_0^\infty d\omega \frac{\hbar\omega C^2}{\pi} \coth\frac{\beta\hbar\omega}{2} \mathrm{Re}\left(\frac{1}{i\omega C + Z^{-1}(\omega)}\right), \tag{1}$$

where $\beta = 1/k_B T$, and $Z(\omega)$ is the impedence of the environment (including the contribution from the tunnel junction). We note that, as indicated in Ref. [6], the Ohmic model, which replaces the $Z(\omega)$ in (1) by a frequency independent resistance R, diverges at the $T = 0$ limit and is not suitable for the study of the $T = 0$ quantum fluctuations.

To calculate the effect of quantum smearing of Coulomb blockade in small tunnel junctions, we use the QLE model which incorporates the fact that the charge fluctuations q obey a Gaussian distribution (justified by our analysis in Ref. [9] since we are dealing with the weak coupling limit). After accommodating the spread in values of q, and in terms of the effective tunneling rates $\langle \Gamma^{\pm} \rangle$, the tunneling current is [6, 7]

$$I = e(\langle \Gamma^-(Q)\rangle - \langle \Gamma^+(Q)\rangle) \equiv e \int_{-\infty}^{\infty} \mathrm{d}q\, [\Gamma^-(Q+q) - \Gamma^+(Q+q)]\, P(q)\,, \qquad (2)$$

where the distribution function for the charge fluctuation is

$$P(q) = \frac{1}{\sqrt{2\pi\langle q^2\rangle}}\, \mathrm{e}^{-q^2/2\langle q^2\rangle}. \qquad (3)$$

In addition, the tunneling rate in Eq. (2) is given by

$$\Gamma^{\pm}(Q) = \frac{\frac{1}{2}e \pm Q}{eCR_{\mathrm{T}}} \frac{1}{\exp[\beta \Delta E^{\pm}(Q)] - 1}, \qquad (4)$$

where $\Delta E^{\pm}(Q) = (1 \pm 2Q/e)E_c$, $E_c = e^2/2C$, and $Q = CV$, with V the voltage applied across the tunnel junction.

We now use the QLE model to study the single electron box, coupling to a dissipative environment being represented by an external circuit of impedance $Z(\omega)$ (see Fig. 1). The device is generally operated at the high tunnel resistance $R_{\mathrm{T}} \gg R_{\mathrm{k}}$, and at low temperatures where the typical thermal energy $k_{\mathrm{B}}T$ is much less than the energy required to charge the island with one electron, $e^2/2(C + C_{\mathrm{S}})$. This is the case where the number of electrons n on the island is a well defined quantity with small quantum and thermal fluctuations and the use of the QLE theory is justified.

When the environmental effect is neglected, the theory for the single electron box is well developed [1, 2]. At $T = 0$, there are n electrons on the island for which the electrostatic energy is minimal. At finite temperature, one has to evaluate the thermodynamic average $\langle n\rangle$, and the average junction charge $\langle Q\rangle$ is related to $\langle n\rangle$ by [2]

$$\langle Q\rangle = \frac{C}{C + C_{\mathrm{S}}}(\tilde{Q} - \langle n\rangle e)\,, \qquad (5)$$

where $\tilde{Q} = C_{\mathrm{S}}U$. In the low temperature region ($k_{\mathrm{B}}T \ll e^2/2(C_{\mathrm{S}} + C)$), Eq. (5) can be evaluated and the result is [10]

$$\langle Q\rangle = \frac{eC}{C + C_{\mathrm{S}}} \left\{ \{x\} - \frac{1}{2}\, \mathrm{e}^{\gamma(\{x\} - \frac{1}{2})}/\cosh\left[\gamma\left(\{x\} - \frac{1}{2}\right)\right] \right\}, \qquad (6)$$

where $x = C_{\mathrm{S}}U/e$, $\gamma = e^2/[2(C_{\mathrm{S}} + C)k_{\mathrm{B}}T]$, and $\{x\} \equiv x - n$ is the fractional part of x. In the limit of $T \to 0$ one obtains [10]

$$\langle Q\rangle_{T=0} = \frac{eC}{C + C_{\mathrm{S}}}\, [\{x\} - \theta(\{x\} - \tfrac{1}{2})]\,. \qquad (7)$$

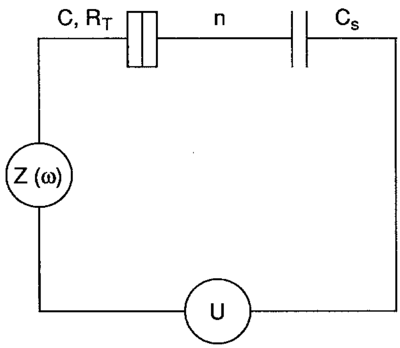

Fig. 1. A single electron box consisting of a junction of capacitance C in series with a true capacitor C_S coupled to a voltage source U via the external impedance $Z(\omega)$. The island between the junction and capacitor contains an integer number n of excess electrons.

From (6), one predicts that $\langle Q\rangle$ varies with x in a sawtooth pattern which is progressively more rounded as the temperature increases. It is found that the experimental results are in agreement with the theoretical predications of (6) above 100 mK, but not at the lowest experimental temperature (20 mK). Thus, we are motivated to extend the above theory by including the effects of the environment on U by the QLE theory, i.e., U is now subject to quantum fluctuations described by (1)–(3). Correspondingly, there are fluctuations in $\tilde{Q}$ which we designate as q, and these obey the Gaussian probability distribution $P(q)$ as given by (3). Thus, denoting the average over the fluctuations by a bar, (5) becomes

$$\langle \bar{Q}\rangle = \frac{C}{C+C_S}\{\tilde{Q} - \langle \bar{n}\rangle e\}\,, \tag{8}$$

where, in particular,

$$\langle n(\tilde{Q})\rangle = \int_{-\infty}^{\infty} \mathrm{d}q\, P(q)\, \frac{\sum_n n \exp\left\{-\gamma\left[n^2 - 2n\left(x + \frac{q}{e}\right)\right]\right\}}{\sum_n \exp\left\{-\gamma\left[n^2 - 2n\left(x + \frac{q}{e}\right)\right]\right\}}\,. \tag{9}$$

Also, in Eq. (8) there is no bar over $\tilde{Q}$ since the average over $\tilde{Q}$ is the same as the case with no fluctuations. Similarly, using the low temperature limit result (6), we obtain

$$\langle \bar{Q}\rangle = \frac{eC}{C+C_S}\left\{\{x\} - \tfrac{1}{2}\int_{-\infty}^{\infty} \mathrm{d}q\, P(q) \exp\left[\gamma\left(\{x\} - \tfrac{1}{2} + \frac{q}{e}\right)\right]\right.$$
$$\left. \Big/ \cosh\left[\gamma\left(\{x\} - \tfrac{1}{2} + \frac{q}{e}\right)\right]\right\} \tag{10}$$

Eq. (10) is a key result of this paper. When $T = 0$, (10) can be evaluated analytically and the result is

$$\langle \bar{Q} \rangle_{T=0} = \frac{eC}{C + C_S}\left[\{x\} - \tfrac{1}{2}\,\mathrm{erfc}\left(\frac{\frac{1}{2} - \{x\}}{\sqrt{2\langle q^2 \rangle}/e}\right)\right], \tag{11}$$

where $\mathrm{erfc}(z) = 1 - (2/\sqrt{\pi}) \int_0^z \mathrm{d}t\, \mathrm{e}^{-t^2}$. Eq. (11), which includes environmental effects, should be contrasted with (7), which is the corresponding result in the absence of environmental effects. It shows that in general at $T = 0$, $\langle \bar{Q} \rangle$ is a periodic oscillating function of x having a period of one and zero points at the values of $x = n/2$. In addition, the amplitude of $\langle \bar{Q} \rangle$ strongly depends on the magnitude of the quantum fluctuations $\langle q^2 \rangle$. This is illustrated in Fig. 2, where we plot (11) in one period $0 < x \leqslant 1$ at the values of $\langle q^2 \rangle / e^2 = 0, 0.002, 0.004, 0.008, 0.02$. The figure indicates that $\langle \bar{Q} \rangle$ has a linear behavior as a function of $\{x\}$ near the $\{x\} = \frac{1}{2}$ region. In fact, from (11) one can obtain directly a linear relation in the $\{x\} \approx \frac{1}{2}$ region as

$$\langle \bar{Q} \rangle \approx \frac{eC}{C + C_S}(\{x\} - \tfrac{1}{2})\left(1 - \frac{e}{\sqrt{2\pi\langle q^2 \rangle}}\right), \qquad \text{for } \{x\} \to \tfrac{1}{2}\,. \tag{12}$$

Eq. (12) tells us that in the absence of quantum fluctuations ($\langle q^2 \rangle = 0$) the $\langle \bar{Q} \rangle$ versus $\{x\}$ curve will have infinite slope at $\{x\} = \frac{1}{2}$, which is the known result of previous theory [1, 2]. On the other hand, for finite values of $\langle q^2 \rangle$ this slope will take a finite value. For example, if $\langle q^2 \rangle / e^2 = 0.006$ then

$$\langle \bar{Q} \rangle \approx -4.2 \frac{eC}{C + C_S}(\{x\} - \tfrac{1}{2})\,.$$

Next, we compare our theoretical result (10) for the averaged charge in a single electron box as a function of the gate voltage, with the experimental data of Lafarge

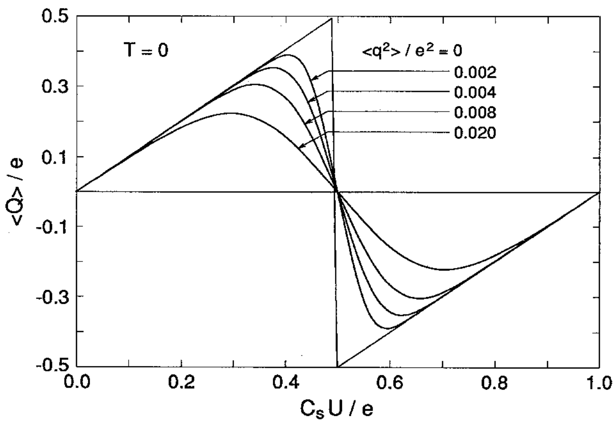

Fig. 2. The $T = 0$ average charge $\langle \bar{Q} \rangle$ in a single electron box as a function of gate voltage U, at various values of charge fluctuations in the quantum Langevin model.

et al. [1]. For this purpose we have evaluated (10) at the temperatures $T = 20, 85, 149, 199, 312$ mK, capacitances $C = 0.6$ fF, and $C_S = 85$ aF. The results are presented in Fig. 3, using the best fit value of $\langle q^2 \rangle / e^2 = 0.006$, and it will be noted that the very good agreement is obtained between theory (dotted curves) and experiment (full curves). Due to lack of knowledge of the details of the environmental parameters of the experiment, it is not possible to calculate $\langle q^2 \rangle$ directly but we note that the number $\langle q^2 \rangle / e^2 = 0.006$ used in our fit is consistent with our previous calculation [7] for the single junction system. This suggests that the rounded shape of the averaged charge as measured in the experiments [1], may be explained as mainly due to the quantum charge fluctuations. Also, the fact that a single temperature independent $\langle q^2 \rangle$ can explain the data for different temperatures T follows from Eq. (1) since it is clear that $\langle q^2 \rangle$ is very weakly dependent on T for the very low T values (20 mK to 312 mK) under discussion here. This is also evident from Fig. 9b of Ref. [6].

In summary, by using the quantum Langevin model (1)–(4), we have obtained (10) for the average charge in the low temperature limit. Our $T = 0$ result (11) shows that the sawtooth-shaped average charge in the conventional theory is rounded off by the quantum fluctuations through the explicit expression (12) in the sawtooth region. At finite temperature, our theoretical result (10) is compared with the experiments of Lafarge et al., and a reasonably good fit is obtained. A more accurate comparison cannot be made because lack of detailed information on the environmental set-up

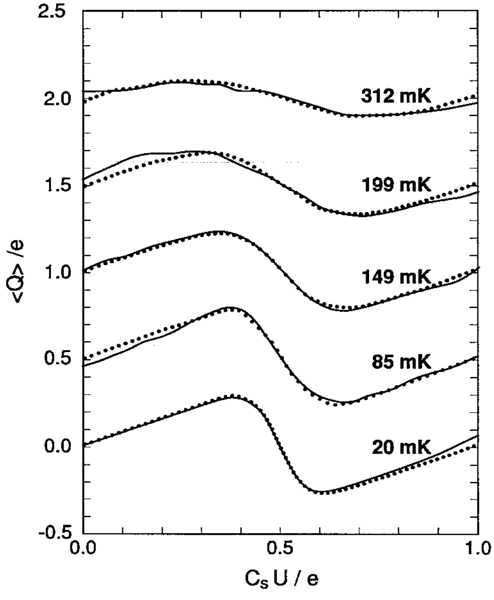

Fig. 3. Average charge $\langle \bar{Q} \rangle$ in a single electron box as a function of gate voltage U at several different temperatures. Solid lines are data taken from Fig. 9 of Ref. [2]. Dashed lines are theoretical predictions (10) (supplemented by (3)) from the quantum Langevin theory at the charge fluctuation value $\langle q^2 \rangle / e^2 = 0.006$.

used by the experiments prevents us from calculating $\langle q^2 \rangle$ ab initio. This points out the desirability of more sophisticated experiments where the environmental set-up is known and controllable. On the theoretical side, it would also be desirable to complement the present approach with a calculation based on the system Hamiltonian which would include from the start both electron tunneling and charge fluctuation terms.

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