On the relationship between the quantum Langevin model and the Landauer formula

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Abstract

We show that, in the weak coupling limit, the main formalism of the phenomenological quantum Langevin model for single electron tunneling in small tunnel junctions is derivable within the framework of the rigorous generalized Landauer formula.

The study of Coulomb blockade, a suppression of single electron tunneling (SET) (for a review see Ref. [1]) in small capacitance junctions, is attracting much experimental and theoretical interest. Likharev et al. [1] proposed the semiclassical orthodox theory, according to which the tunneling current is totally suppressed at zero temperature and voltages $V < V_0 = e/2C$, where C is the capacitance. It turns out that Coulomb blockade is most pronounced if the influence of the environment is weak. More recent studies show that for the Coulomb blockade of single electron tunneling [2,3], there are two sources, the leads connecting the junction to the external circuit (the environment) and the discrete charge transfer across the junction, which reduce the effective Coulomb barrier i.e., partially smear off the Coulomb blockade. It has been demonstrated that the quantum Langevin (QLE) model [2,3] captures the basic physics of quantum smearing of Coulomb blockade (finite I at $V < e/2C$, where C is the junction capacitance) by taking into account the zero-point fluctuations of the instantaneous charge on the junction [2,3]. Nevertheless, the QLE model is phenomenological in nature. Thus, our first goal will be to show

that, at least in the weak coupling limit, it leads to results which are identical to those obtained by use of the rigorous generalized Landauer formula [4–6].

We start with a brief review of the main aspects of the quantum Langevin equation (QLE) model for the environmental effects on single electron tunneling. First, one solves the quantum Langevin equation [2,3] for the Fourier transform $q(\omega)$ of the charge fluctuation $q(t)$. Then, using the fluctuation dissipation theorem, one obtains the mean-square charge fluctuation [2,3]

$$\langle q^2(t) \rangle = \int_0^\infty d\omega \, \frac{\hbar\omega C^2}{\pi} \coth(\tfrac{1}{2}\beta\hbar\omega)$$

$$\times \mathrm{Re}\left(\frac{1}{i\omega C + Z^{-1}(\omega)} \right), \tag{1}$$

where $\beta = k_B T$, and $Z(\omega)$ is the impedance of the environment (including the contribution from the tunnel junction). An interesting observation is that if the Ohmic model ($Z(\omega) = $ constant) is applied to (1), at $T = 0$ one gets a divergent $\langle q^2 \rangle$, and at $T \to \infty$ (classical limit) one gets $\langle q^2 \rangle = k_B TC$. The above

analysis indicates that the Ohmic model is not applicable to (1) in the low temperature limit since there is an f-sum rule divergence problem.

To calculate the effect of quantum smearing of Coulomb blockade in small tunnel junctions, it is assumed that the charge fluctuations q obey a Gaussian distribution. After accommodating the spread in values of q, and in terms of the effective tunneling rates $\langle \Gamma^{\pm} \rangle$, the tunneling current is [2,3]

$$I = e[\langle \Gamma^-(Q) \rangle - \langle \Gamma^+(Q) \rangle]$$

$$\equiv e \int_{-\infty}^{\infty} \mathrm{d}q \, [\Gamma^-(Q+q) - \Gamma^+(Q+q)] P(q) , \quad (2)$$

where the distribution function for the charge fluctuation is

$$P(q) = \frac{1}{\sqrt{2\pi\langle q^2 \rangle}} \exp(-q^2/2\langle q^2 \rangle) . \quad (3)$$

In addition, the tunneling rate in (2) is given by

$$\Gamma^{\pm}(Q) = \frac{\frac{1}{2}e \pm Q}{eCR_{\mathrm{T}}} \frac{1}{\exp[\beta\Delta E^{\pm}(Q)] - 1} , \quad (4)$$

where $\Delta E^{\pm}(Q) = (1 \pm 2Q/e)E_{\mathrm{c}}$, $E_{\mathrm{c}} = e^2/2C$, and $Q = CV$, with V the applied voltage.

Next, we demonstrate that, in the weak coupling limit, the QLE theory is related to the well known generalized Landauer formula [4–6], where the tunneling current in the tunneling resistance approximation can be written as

$$I = \frac{1}{eR_{\mathrm{T}}} \int_{-\infty}^{\infty} \mathrm{d}E \, P(E) \left(\frac{eV-E}{1-\exp[-\beta(eV-E)]} \right.$$

$$\left. + \frac{eV+E}{1-\exp[\beta(eV+E)]} \right), \quad (5)$$

where $P(E)$, the probability distribution function, is given by

$$P(E) = \frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} \mathrm{d}t \exp[-f(t) + iEt/\hbar] . \quad (6)$$

Also, the phase fluctuation function $f(t)$ is given by

$$f(t) = E_{\mathrm{c}} \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{\pi\hbar\omega} \{ [1 - \cos(\omega t)]$$

$$\times \coth(\tfrac{1}{2}\beta\hbar\omega) - i \sin(\omega t) \}$$

$$\times \mathrm{Re}\left(\frac{C}{i\omega C + Z^{-1}(\omega)} \right). \quad (7)$$

In the weak coupling limit $(Z(\omega) \to \infty)$, one can show that the short time behavior dominates the contribution to the $f(t)$ of (7). In this limit, (7) can be evaluated analytically and the result is

$$f(t) = \frac{e^2}{2C^2\hbar^2} \langle q^2 \rangle t^2 - i\frac{E_{\mathrm{c}}}{\hbar} t , \quad (8)$$

where $\langle q^2 \rangle$ is given by (1). We note that (8) is a general result in the weak coupling limit. For the particular case of the Ohmic model in the high temperature limit, one has $\langle q^2 \rangle = k_{\mathrm{B}}TC$.

Using (8), the probability density (6) in the weak coupling limit can be evaluated explicitly as

$$P(E) = \frac{C}{e\sqrt{2\pi\langle q^2 \rangle}} \exp\left(- \frac{C^2(E_{\mathrm{c}}+E)^2}{2e^2\langle q^2 \rangle} \right). \quad (9)$$

By means of (9), one can make a variable change $E \to qe/C$, and rewrite (6) as the charge probability distribution function

$$\frac{e}{C} P(E - E_{\mathrm{c}}) \to$$

$$P(q) = \frac{1}{\sqrt{2\pi\langle q^2 \rangle}} \exp(-q^2/2\langle q^2 \rangle) . \quad (10)$$

Substituting (10) into (5), we obtain

$$I = \frac{1}{eR_{\mathrm{T}}} \int_{-\infty}^{\infty} \mathrm{d}E \, P(E - E_{\mathrm{c}}) \left(\frac{eV-E+E_{\mathrm{c}}}{1-\exp[\beta(eV-E+E_{\mathrm{c}})]} \right.$$

$$\left. + \frac{eV+E-E_{\mathrm{c}}}{1-\exp[-\beta(eV+E-E_{\mathrm{c}})]} \right)$$

$$= e \int_{-\infty}^{\infty} \mathrm{d}q \, P(q) [\Gamma^-(CV+q) - \Gamma^+(CV-q)] , \quad (11)$$

where $\Gamma^{\pm}$ is given by (4). Thus, the weak coupling result (11), which is based on the generalized Lan-

dauer formula, is equivalent to the QLE result (2). Therefore, we have shown that in the weak coupling limit, the QLE formalism is identical with the generalized Landauer formalism. This provides insight as to why the QLE gives very good agreement, in the weak coupling region, with the experimental results [3].

In summary, in this paper we have shown that the main formalism (1)–(4) of the quantum Langevin model is derivable within the framework of the rigorous generalized Landauer formula (5) in the weak coupling region.

As a final remark, we address the question of why the two approaches deviate for strong coupling. The answer is that the Langevin model for single electron tunneling assumes the Gaussian form (3) for the distribution function. The steps (5)–(10) demonstrate that this is in fact correct for the weak coupling case. However, when the coupling is strong, the derived distribution function in the Landauer formalism, given by (6) and (7), is clearly not Gaussian. For example, to next order beyond the weak coupling, the

real part of $f(t)$ will contain not only the terms on the right side of (8) but also a $t^4 \langle q^4 \rangle$ term, i.e. it will display a non-Gaussian form with contributions from higher order charge fluctuations. We conclude that the Gaussian assumption breaks down in the strong coupling limit.

References

[1] D.V. Averin and K.K. Likharev, in: Mesoscopic phenomena in solids, eds. B.L. Altshuler, P.A. Lee and R.A. Webb (Elsevier, Amsterdam, 1991) p. 173.
[2] A.N. Cleland, J.M. Schmidt and J. Clarke, Phys. Rev. Lett. 64 (1990) 1565; Phys. Rev. B 45 (1992) 2950.
[3] G.Y. Hu and R.F. O'Connell, Phys. Rev. B 46 (1992) 14219; Phys. Rev. B, in press.
[4] R. Landauer, Philos. Mag. 21 (1970) 863; M. Buttiker, Phys. Rev. Lett. 57 (1986) 1761.
[5] S. Feng, Phys. Lett. A 143 (1990) 400.
[6] A. Odintsov, Physica B 189 (1993) 258.