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QUANTUM FLUCTUATION EFFECTS IN A SINGLE ELECTRON BOX

G. Y. Hu, R. F. O'Connell, and Jai Yon Ryu
Department of Physics and Astronomy, Louisiana State University
Baton Rouge, Louisiana 70803-4001, USA

Quantum fluctuation effects in a single electron box are analyzed by considering the gate voltage fluctuations with the use of the quantum Langevin model. At T = 0, we derive an analytic expression for the junction charge as a function of the gate voltage and its fluctuations. It is shown that, at T = 0, the sawtooth shape of the junction charge, as predicted by existing theories, is rounded off by the quantum fluctuation effects. At finite temperature, the theory is compared with experiments, and a good fit is obtained at all the relevant temperatures.

I. INTRODUCTION

In a single electron box[1,2], a junction of capacitance C and tunnel Resistance R_T, is connected in series with a true capacitance C_s and a voltage source U. The intermediate electrode between the junction and the capacitor forms an "island" which contains an integer number n of extra electrons. Lafarge et al.[1,2] have measured the average junction charge $<Q>$, which is simply related to the average value $<n>$, in a single electron box. Their experimental results for the $<Q>$ versus U curves at different temperatures, ranging from 20 to 312 mK, are in agreement with the theoretical predictions (which considers thermal effects only) above 100 mK, but not at lower temperatures. Esteve has pointed out the importance of the quantum fluctuations of n in explaining the experimental data at the lowest temperature. Also, he showed that a perturbation theory [1] in R_k/R_T for the quantum corrections to $<Q>$ does not settle the problem, and a complete non-perturbative theory is needed. Here we study the quantum fluctuations effects in a single electron box by a non-perturbative theory, the quantum Langevin model. [3]

II. FORMULATION

When the environmental effect is neglected, the theory for the single electron box is well developed.[1,2] At $T = 0$, there are n electrons on the island for which the electrostatic energy is minimal. At finite temperature, one has to evaluate the thermodynamic average $<n>$, and the average junction charge $<Q>$ is related to $<n>$ by[1]

$$<Q> = \frac{C}{C+C_S}(\tilde{Q} - <n> e) \quad . \tag{1}$$

where $\tilde{Q} \equiv C_S U$. In the low temperature region $(k_B T << e^2/2(C_S + C))$, (1) can be evaluated and the result is [1,4]

$$<Q> = \frac{eC}{C+C_S}\left(\{x\} - \tfrac{1}{2}e^{\gamma(\{x\}-\frac{1}{2})}/\cosh[\gamma(\{x\}-\tfrac{1}{2})]\right) \quad , \tag{2}$$

where $x = C_S U/e = \tilde{Q}/e$, $\gamma = e^2/[2(C_s + C)k_B T]$, and $\{x\} \equiv x - n$ is the fractional part of x. From (2), one predicts that $<Q>$ varies with x in a sawtooth pattern which is progressively more rounded as the temperature increases. It is found that the experimental results are in agreement with the theoretical predications of (2) above 100 mK, but not at the lowest experimental temperature (20 mK). Thus, we are motivated to extend the above theory by including the effects of the environment on U. In other words, U is now subject to quantum fluctuations. Correspondingly, there are fluctuations in $\tilde{Q}$ which we designate as q.

As before[3] we take the probability distribution $P(q)$ to be gaussian:

$$P(q) = \frac{1}{\sqrt{2\pi\langle q^2\rangle}} e^{-q^2/2\langle q^2\rangle} \quad , \tag{3}$$

where the mean square fluctuation can be calculated via a quantum Langevin equation.[3] Thus, denoting the average over the fluctuations by a bar, (1) becomes

$$\langle\bar{Q}\rangle = \frac{C}{C+C_S}\{\tilde{Q} - \langle\bar{n}\rangle e\} \quad , \tag{4}$$

where, in particular,

$$<n(\tilde{Q})> = \int_{-\infty}^{\infty} dq P(q) \frac{\sum_n n e^{-\gamma[n^2 - 2n(x + \frac{q}{e})]}}{\sum_n e^{-\gamma[n^2 - 2n(x+\frac{q}{e})]}} \quad , \tag{5}$$

Similarly, using (2) we obtain

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$$\langle\tilde{Q}\rangle = \frac{eC}{C+C_S}\left(\{x\} - \frac{1}{2}\int_{-\infty}^{\infty} dq\, P(q) e^{\gamma Z(q)}/\cosh[\gamma Z(q)]\right), \quad (6)$$

with $Z(q) = \{x\} - \frac{1}{2} + \frac{q}{e}$. Eq. (6) is the main result of this paper. In fact, when $T = 0$, (6) can be evaluated analytically and the result is

$$\langle\tilde{Q}\rangle_{T=0} = \frac{eC}{C+C_S}\left(\{x\} - \frac{1}{2}\,\mathrm{erfc}\left(\frac{\frac{1}{2}-\{x\}}{\sqrt{2\langle q^2\rangle}/e}\right)\right), \quad (7)$$

where $\mathrm{erfc}(z) = 1 - \frac{2}{\sqrt{\pi}}\int_0^z dt e^{-t^2}$. Eq. (7) shows that at $T = 0$, $\langle\tilde{Q}\rangle$ is a periodic oscillating function of x having a period of one, and zero values at $x = n/2$. In addition, the amplitude of $\langle\tilde{Q}\rangle$ strongly depends on the magnitude of the quantum fluctuations $\langle q^2\rangle$ · This is illustrated in Fig. 1, where $\langle\tilde{Q}\rangle$ has a linear behavior as a function of $\{x\}$ near the $\{x\} = 1/2$ region.

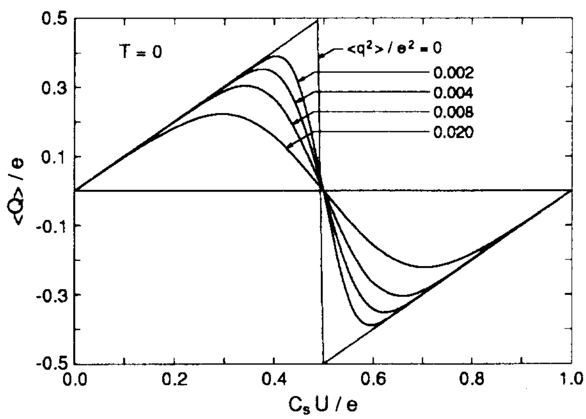

Figure 1 Average charge $<Q>$ vs. gate voltage U, at various values of charge fluctuations.

III. CAMPARE WITH EXPERIMENTS

We have evaluated (6) at the temperatures $T = 20$, 85, 149, 199, 312 mK, capacitances$C = 0.6$ fF, and $C_S = 85\ aF$, for various values of$\langle q^2\rangle$. The results are presented in Fig. 2, where we plot the best fit theoretical (dotted) curves of (6) at $\langle q^2\rangle/e^2 = 0.006$ to the experimental data (full curves). The good fit between (6) and the experimental data at all the relevant temperatures suggests that the rounded shape of the averaged charge is mainly due to the quantum fluctuations of the gate voltage.

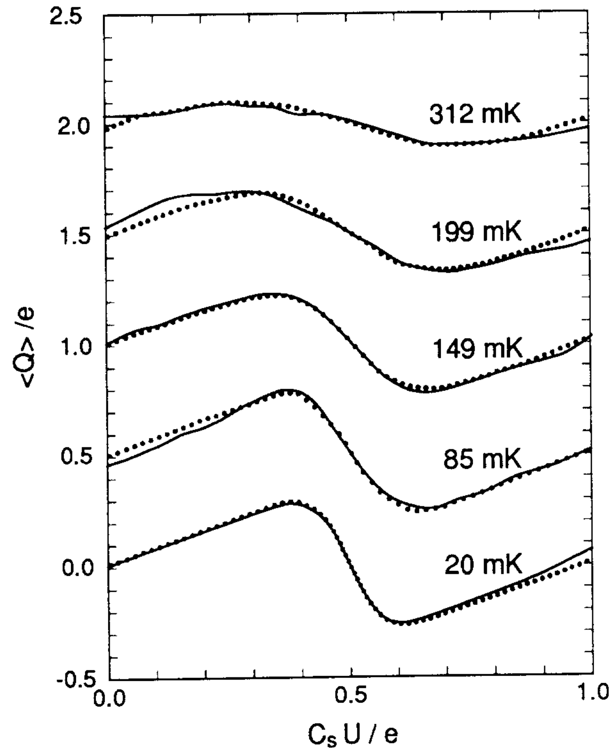

Figure 2 Average charge $<\tilde{Q}>$ vs. gate voltage U , at various temperatures.

In summary, in this paper we have studied the quantum fluctuation effects in a single electron box. By using the quantum Langevin model (3)-(5), we have obtained (6) for the average charge in the low temperature limit. Our $T = 0$ result (7) shows that the sawtooth-shaped average charge in the conventional theory is rounded off by the quantum fluctuations. A very good fit is obtained between the theory and the experiments of Lafarge et al.

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