$$\lim_{k \to 0} F(k,E) = \int dt \exp iEt \langle [S_z(t), S_z(0)] \rangle \equiv 0 \quad (13)$$

and therefore eq.(4) contains no relevant information about gapless modes.

The second flaw in the proof is that even if one works with the proper non-trivial spectral functions pertaining to the $S^+(k)S^-(-k)$ or $S^x(k)S^y(-k)$ Green function, and arrives at a formula like

$$\lim_{k \to 0} E(k) \propto \lim_{k \to 0} \langle [[H, S^+(k,0)] S^-(-k,0)] \rangle, \quad (14)$$

and even though this double commutator is zero *at* $k = 0$, a careful analysis [2] shows that such double commutators do not necessarily approach their $k = 0$ value in the *limit* of zero k.

Such limits are only well behaved if the Hamiltonian contains no excessively long range forces [3].

The proof suggested by Crisan is therefore incorrect. The use of Green function notation does not make unnecessary the analysis found in previous proofs of the theorem.

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MOTION OF A RELATIVISTIC ELECTRON WITH AN ANOMALOUS MAGNETIC MOMENT IN A CONSTANT MAGNETIC FIELD

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Received 4 July 1968

The anomalous magnetic moment of the electron is accounted for by adding a phenomenonological Pauli term to the Dirac equation. The resultant eigenvalues are applied to (a) the non-linear Lagrangian of the E-M field and (b) spontaneous pair production.

The Dirac equation for an electron of mass m with an anomalous magnetic moment, μ say, in a constant homogeneous magnetic field H (directed along the z axis, say) takes the form [1,2] (in our units $\hbar = c = 1$ and $\alpha = e^2 = 1/137$)

$$i \frac{\partial \psi}{\partial t} = \{\boldsymbol{\alpha} \cdot (\boldsymbol{P} + e\boldsymbol{A}) + \gamma_4 m + \mu \gamma_4 \boldsymbol{\Sigma} \cdot \boldsymbol{H}\} \quad (1)$$

where the term containing μ (the so-called Pauli anomalous interaction term) is an addition to the usual Dirac Hamiltonian. Since different values for the energy eigenvalues derived from this equation appear in the literature [1,2], we have re-derived the result in a conventional manner, and we find, in agreement with ref. 2, that the energy eigenvalues E are given by

$$E = \pm \{p_z^2 + [m\{1 + \frac{H}{H_c}(2n + \xi + 1)\}^{\frac{1}{2}} + \xi \mu H]^2\}^{\frac{1}{2}} \quad (2)$$

where $n = 0, 1, 2, \ldots$ is the principal quantum number, $\xi = \pm 1$ refers to spin up and spin down, p_z is the momentum of the particle along the Z axis and $H_c = m^2/e = 4.4 \times 10^{13}$ gauss. For our purposes it is sufficient to take $\mu = (\alpha/2\pi)\mu_B$ where μ_B is the Bohr magneton and so we can write

$$E = \pm \{p_z^2 + m^2 [\{1 + \frac{H}{H_c}(2n + \xi + 1)\}^{\frac{1}{2}} + \xi \frac{\alpha}{4\pi} \frac{H}{H_c}]^2\}^{\frac{1}{2}} \quad (3)$$

Let us now consider applications of eq. (3).

(a) An exact expression for the non-linear Lagrangian of the electromagnetic field has been derived [3,4] in the case where $\mu = 0$ (i.e. neglect of the anomalous magnetic moment). Using the more general eigenvalue given by eq. (3) we have derived an additional non-linear correction, L_2 say, to the Lagrangian of the magnetic field (the electric field will be considered later) as follows ($H^* \equiv H/H_c$)

$$L_2 = \frac{m^2}{32\pi^2}\left(\frac{\alpha}{2\pi}\right)^2 H^{*2} \times \int_0^\infty \frac{d\eta}{\eta^2} \exp(-\eta)\{\eta H^* \coth(\eta H^*) - 1\} \qquad (4)$$

In the weak and strong field limits we find

$$L_2 = \frac{m^4}{96\pi^2}\left(\frac{\alpha}{2\pi}\right)^2 H^{*4} \qquad \text{for } H^* \ll 1 \qquad (5)$$

$$L_2 = \frac{m^4}{32\pi^2}\left(\frac{\alpha}{2\pi}\right)^2 H^{*2} \ln(H/H_c) \text{ for } H^* \gg 1 \qquad (6)$$

(b) It will be noticed from eq. (3) that, for values $p_z = 0$, $n = 0$ and $\xi = -1$, we get a minimum value for $|E|$ given by

$$|E|_{min} = m\{1 - (\alpha/4\pi)H^*\} \qquad (7)$$

Thus the minimum separation between positive and negative energy states, ΔE say, is $2m(1 - \alpha H^*/4\pi)$ and so we can conclude that, for values of H equal to $4\pi\alpha^{-1}H_c$, the minimum separation is zero and thus spontaneous pair production may occur. This has important astrophysical implications particularly with respect to the expanding universe if a primordial magnetic field [5] exists. A detailed exposition of the above work will be published elsewhere.

Part of this research was accomplished while the author held a National Research Council Senior Research Associateship supported by the National Aeronautics and Space Administration; he would also like to thank Dr. Robert Jastrow for his hospitality at the Institute for Space Studies.

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