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Does the Electron Have a Structure?

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"... it ain't likely to have a radius of exactly zero," is the conclusion of H. G. Dehmelt[1] *from his Nobel Prize (1989) winning observations on trapped electrons. There are small discrepancies between Dehmelt's observations and the theoretical predictions of quantum electrodynamics (QED), which assumes that the electron is a point particle. Here we present evidence in support of Dehmelt's contention that the electron has a structure. Essentially, we point out that the nonrelativistic limit of QED is at variance with a fundamental principle underlying all of physics, viz. the second law of thermodynamics.*

Asim Barut is one of the gurus of classical and quantum electrodynamics. His continual flow of ideas continues to enchant us, and he has taught us that there are many gold nuggets still waiting to be mined in this arena. Judging from his various papers on the subject, it is clear that the radiation reaction problem is a subject close to his heart, and here we demonstrate the close connection between this problem and the question of the structure of the electron.

Our remarks are based on recent work carried out by the present author, in collaboration with Ford and Lewis,[2,3] on the dipole interaction of a nonrelativistic quantum oscillator with the electromagnetic field. This is a well-defined physical problem for which an exact analysis was possible. Our starting point is the well-known Hamiltonian of quantum electrodynamics, generalized to include the possibility of electron structure, by using an electron form factor [see Eq. (15) of Ref. 2] of the form $\Omega/(\Omega^2+\omega^2)$, where ω refers to a typical photon frequency and Ω is a large cutoff frequency. Letting $\Omega \to \infty$ corresponds to the case of a point elec-

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tron, but we found that taking this limit leads to a violation of the fundamental principle of causality which demands that the response due to an external force cannot precede the force. More technically speaking, the principle of causality demands that the poles of the generalized susceptibility must all lie in the lower half of the complex plane, and our analysis in Ref. 3 showed that this demand is equivalent to demands associated with the second law of thermodynamics.

In Refs. 2 and 3 we presented a new approach to the overall problem by writing down the equation of the motion of the radiating electron in the form of a generalized quantum Langevin equation (GLE) where the dissipative term is in the form of an integral over time from $-\infty$ to t. This equation includes an external potential but, for our present purposes, this is an unessential generalization and so we confine out attention here to the case of a free electron subject to a time-dependent external force. Next, by taking the Fourier transform of the GLE, it was possible to write down an exact solution of the equation of motion in the form

$$\tilde{x}(\omega)=\alpha(\omega)\{\tilde{f}(\omega)+\tilde{F}(\omega)\} \tag{1}$$

where $\tilde{x}(\omega)$, $\tilde{f}(\omega)$, and $\tilde{F}(\omega)$ are the Fourier transforms of the electron coordinate $x(t)$, the external force $f(t)$, and the fluctuation force $F(t)$, respectively. Here $\alpha(\omega)$ is the generalized susceptibility (a c-number) and is given by

$$\alpha(\omega)=[-m\omega^2-i\omega\tilde{\mu}(\omega)]^{-1} \tag{2}$$

where m is the bare mass of the electron and $\tilde{\mu}(\omega)$ is the Fourier transform of the memory function $\mu(t)$ appearing in the GLE. In fact, $\tilde{\mu}(\omega)$ is a function of Ω and has the important property (which follows from the second law of thermodynamics, as shown in Section III of Ref. 4) that $\mathrm{Re}\,\tilde{\mu}(\omega)\geqslant 0$. This enabled us to conclude that Ω can be no larger than $\tau_e^{-1}\equiv 3c/2r_0=1.6\times 10^{23}\ \mathrm{s}^{-1}$, where r_0 is the classical radius of the electron. This ensures that all the poles of $\alpha(\omega)$ must lie in the lower half of the complex plane, in conformity with the principle of causality.[5]

Thus, from very general considerations, we have the striking result that the assumption of a point electron ($\Omega\to\infty$) leads to conflicts with the second law of thermodynamics and the principle of causality.

If we choose $\Omega=\tau_e^{-1}$, the largest value allowed, then we obtain for the equation of motion of the radiating electron [using Eqs. (12), (17), and (23) of Ref. 3 in the case where the oscillator frequency ω_0 is taken to zero]:

$$-M\omega^2\tilde{x}(\omega)=(1-i\omega\tau_e)\{\tilde{f}(\omega)+\tilde{F}(\omega)\} \tag{3}$$

where M is the renormalized electron mass. For our present purposes, we will take the classical limit of this equation by regarding $\tilde{x}(\omega)$ as a c-number and by dropping the fluctuation force. Next, taking the inverse Fourier transform of this equation leads to the equation of motion for the radiating electron:

$$M\frac{d^2x(t)}{dt^2}=f(t)+\tau_e\frac{df(t)}{dt} \tag{4}$$

This equation is superior to the Abraham–Lorentz (AL) equation in that it is second-order, it incorporates radiation effects, and it has well-behaved solutions. If we treat the last term on the right-hand side as a perturbation, then we obtain the AL equation. In other words, the AL equation is only correct to first order in τ_e. It is the use of such equations outside the realm of their validity that has led to runaway solutions and violations of causality.

Another striking feature of (4) is that it possesses a property whose desirability has been emphasized by Barut,[6] viz., in the absence of an external force $f(t)$ it reduces to Newton's equation for a free particle (a property lacking in the AL equation).

We emphasize that, within the framework of the electric dipole approximation and assuming that $\Omega=\tau_e^{-1}$, Eq. (4) is exact. In fact, Ford and the present author have written down a more general equation [Eq. (5) of Ref. 7] which contains Ω explicitly and which reduces to the AL equation for the choice $\Omega\to\infty$ and to Eq. (4) above in the case $\Omega=\tau_e^{-1}$. In fact, the latter choice is the only one which leads to an equation which fulfills Barut's demand[6] discussed above.

We emphasize that Eq. (4) results from assuming an Ω value equal to τ_e^{-1}, where τ_e is 2/3 of the time for a photon to traverse the classical electron radius $r_0=e^2/Mc^2=2.818\times10^{-15}$ m. It is striking that we are predicting a value for the electron radius of the order of the *classical* value using a rigorous *quantum* approach. It should be stressed that (if one takes account of Lorentz contraction effects) this result is not inconsistent with the fact that electron–positron colliding beam experiments demand no modifications in conventional QED down to distances less than 10^{-15} cm.[8]

It might be argued that, even though QED theory starts with the assumption of a point electron, structure effects come into play indirectly via relativistic,[9] spin,[10] and radiative[11] effects. However, it is our contention that there should be a rigorous self-consistent theory of a nonrelativistic nonspinning electron. Such a theory demands that the electron has structure, in conformity with Dehmelt's speculation, based on his

Nobel Prize winning observations on trapped electrons, that "... it ain't likely to have a radius of exactly zero."[1]

It is of interest to note that Dirac himself was also motivated (apparently from considerations other than those discussed above) to consider in detail a relativistic equation describing a particle with nonzero mass and positive energy,[12] and we note that the exclusion of negative energy necessarily implies that we are dealing with states which cannot be localized to a point.[9] After an initial flurry of activity and some generalizations of Dirac's work,[13,14] interest waned. Our goal here is to argue that Dehmelt's experimental results, as well as our own theoretically based conclusions, demand a serious reconsideration of the relativistic theory of an electron with structure. Finally, we note that we have recently presented[15] a relativistic generalization of Eq. (4). The author is pleased to acknowledge that the remarks made in this contribution are based on results obtained in collaboration with Professors G. W. Ford and J. T. Lewis. He also benefited from discussions with Professors A. O. Barut and L. H. Chan. We conclude by wishing Asim a Happy Birthday and hope that we will enjoy the benefit of his wisdom for many years to come. This research was partially supported in part by the U.S. Office of Naval Research under Grant No. N00014-90-J-1124.

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