Charge fluctuations and zero-bias resistance in small-capacitance tunnel junctions

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The zero-bias resistance in small capacitance tunnel junctions was recently measured by Cleland, Schmidt, and Clarke and these authors used a quantum Langevin model to explain the data [Phys. Rev. B **45**, 2950 (1992)]. Their model takes into account Nyquist noise generated in the leads but here we extend it to include all sources of Nyquist noise with emphasis on the noise generated in the junction itself. This enables us to get better agreement with the experimental data, particularly for junctions with high tunnel resistance and high lead resistance.

In recent years, the study of ultrasmall tunnel junctions, with capacitance C_J such that the charging energy $e^2/2C_J$ of a single electron exceeds the characteristic energy k_BT of thermal fluctuations, has attracted both theoretical and experimental attention.[1-3] It has been shown that Coulomb blockade, a suppression of single-electron tunneling, greatly reduces the current at voltages $V<e/2C_J$. Also, the I-V curve is offset by the Coulomb gap $e/2C_J$ at higher voltages. One of the most interesting features of the Coulomb blockade and Coulomb gap in small capacitance junctions is that these effects are found to depend strongly on the nature of the environment coupled to the junctions.[3-10] Several groups[4-6] have observed Coulomb blockade in a single junction. In particular, it is found[6] that (i) the Coulomb gap is much more clearly visible for those junctions with higher resistance leads than those of lower resistance leads, and (ii) the zero-bias resistance R_0 of the junctions increases as the temperature is lowered and flattens off at the lowest temperatures.

Two theories, the quantum Langevin model[6] and the phase correlation theory,[7-10] have been applied to explaining the above experimental results. The studies show that the phase correlation theory yields an I-V characteristic in good agreement with the experimental data in many aspects, but it fails to predict the flattening off associated with the zero-bias resistance R_0 at the lowest temperatures.[6-10] The quantum Langevin model[6] proposed by Cleland, Schmidt, and Clarke (CSC) captures the basic physics of quantum smearing (finite I at $V<e/2C_J$) of the Coulomb blockade by taking into account the zero-point fluctuations of the instantaneous charge on the junction, and thus provides a qualitatively correct description of the data. However, the CSC model is restricted to a particular RCL circuit model for the transmission line. In particular, it does not include the quantum fluctuations due to the finite tunnel resistance R_J and thus underestimates the actual value of the charge fluctuations. Since a low charge fluctuation will cause a relatively weak quantum smearing of the Coulomb blockade, the predicted values of R_0 by the CSC model is, as expected, too high in comparison with the experimental data. The purpose of this paper is to enlarge the scope of the CSC model by using a generalized quantum Langevin equation (QLE) theory to treat not only a general type of environment (transmission lines, etc.), but to also include all dissipative sources (tunnel resistance, transmission lines, etc.) on the same footing with respect to contributions to quantum charge fluctuations. We will show that when both the transmission line and tunnel resistance effects are included, the QLE theory agrees very well with the experimental data.

Consider a small tunnel junction with capacitance C_J coupled with a dissipative environment, which is represented by an external circuit of impedance $Z(\omega)$. Our main interest here is to study the quantum charge fluctuation in the small capacitance, and all the dissipative sources are thus included in the environmental part of the circuit. Since there are two sources which produce fluctuations, the transmission lines and the tunnel resistance, we will include both of them in parallel in the effective impedence $Z(\omega)$. The model circuit in the present network analysis is drawn as the Thevenin equivalent circuit[3,11] Fig. 1(a), where $V(t)$ is a voltage source and $Z(\omega)$ is an effective impedence which includes the effects of the environment. For the experiments of CSC, the impedence $Z(\omega)$ consists of R_J (tunnel resistance), C_s (stray capacitance), R_s (lead resistance), and L (lead inductance), as shown in Fig. 1(b). We note that the main difference between our model [Figs. 1(a) and 1(b)] and the CSC model is that R_J is now included as part of the effective impedence $Z(\omega)$ [we will show later that when the effect of R_J is neglected ($R_J\rightarrow\infty$), our model reduces to the CSC model]. Also, we note that Flensberg and Jonson[9] have studied the important distinction between the continuous charge fluctuations due to the transmission lines and the discrete charge fluctuation due to the charge transfer. Their work gives support to our assumption that the effective impedence due to the addition of the fluctuation across the junction and in the leads simply add in parallel. Also, the replacing of the tunnel resistance by an Ohmic resistance R_J is expected to be a good approximation in the low damping regime.[9,10] Next, by the theory of the Thevenin-Norton equivalent circuit,[3,11] the voltage bias circuit Fig. 1(a) can be equivalently drawn as the current bias circuit Fig. 1(c),

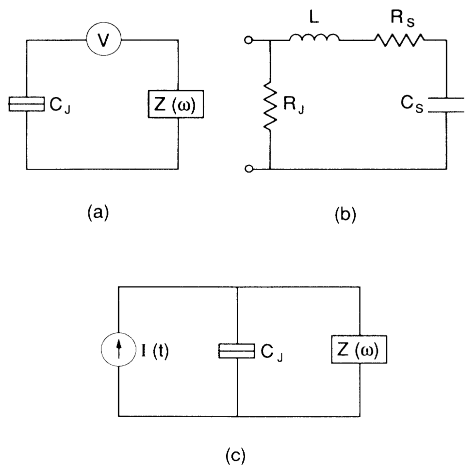

FIG. 1. Circuit model for a small tunnel junction of capacitance C_J coupled to an effective impedence $Z(\omega)$: (a) voltage bias (Thevenin) circuit; (b) configuration of $Z(\omega)$ for the experiment of Ref. 6 with tunnel resistance R_J, lead resistance R_s, inductance L, and stray capacitance C_s; (c) current bias (Norton) circuit equivalent to (a).

where $I(t)$ is the current source. Figure 1(c) is convenient for the study of charge fluctuations by means of the QLE, as we will show in the following.

The quantum Langevin equation for the Fourier transform $q(\omega)$ of the charge fluctuation on the junction in the circuit of Fig. 1(c) is

$$i\omega q(\omega)+\frac{1}{C_J Z(\omega)}q(\omega)=I_F(\omega)\ , \tag{1}$$

where $I_F(\omega)$ is the Fourier transform of the current fluctuation. The current fluctuation is related to the dissipation [here the impedence $Z(\omega)$] by the well-known fluctuation-dissipation theorem[12]

$$S_I(\omega)=\frac{\hbar\omega}{\pi}\mathrm{Re}[Z^{-1}(\omega)]\coth(\hbar\omega/2k_BT)\ , \tag{2}$$

where $S_I(\omega)$ is the Fourier transform of the symmetric autocorrelation function of the current fluctuation $I_F(t)$. Solving (1) for $q(\omega)$, and using (2), we obtain the following mean-square charge fluctuation:

$$\langle q^2(t)\rangle=\int_0^\infty d\omega\frac{\hbar\omega C_J^2}{\pi}\coth\frac{\hbar\omega}{2k_BT}\mathrm{Re}\left[\frac{1}{i\omega C_J+Z^{-1}(\omega)}\right] \tag{3}$$

Equation (3) is an exact result for (1) in the linear-response regime, which is valid in the case of the CSC experiments. In addition, it is in a general form applicable to any kind of environmental structure attached to the small junction. An interesting observation is that if the Ohmic model [$Z^{-1}(\omega)$=const] is applied to (3), at $T=0$ one gets a divergent $\langle q^2\rangle$, which reflects the fact that there is a f-sum rule divergence for the Ohmic model. Thus, Eq. (3) suggests the need to go beyond the Ohmic model[2,3,6–8] for the study of the environmental effect on single-electron tunneling. In the following we apply (3) to a non-Ohmic model to study the charge fluctuation and the zero-bias resistance, and we compare with the experiments of CSC.[6]

The impedence of the dissipative environment for the CSC experiments [see Fig. 1(b)] is readily obtained as

$$Z^{-1}(\omega)=R_J^{-1}+\left[R_s+i\omega L-\frac{i}{C_s\omega}\right]^{-1}. \tag{4}$$

Substituting (4) into (3), we obtain

$$\langle q^2(t)\rangle=\frac{\hbar C_J^2}{\pi}\int_0^\infty d\omega\,\omega\coth\frac{\hbar\omega}{2k_BT}R(\omega)\ , \tag{5}$$

where

$$R(\omega)\equiv R_s\frac{a_1+a_2\omega^2+a_3/\omega^2}{[b_1-\omega^2/\omega_{LC}^2]^2+[b_2\omega+b_3/\omega]^2}\ , \tag{6}$$

and $\omega_{LC}^2=1/LC_J$,

$$a_1=1+\frac{R_s}{R_J}+\frac{2L}{C_sR_JR_s},\quad a_2=\frac{L^2}{R_JR_s},\quad a_3=\frac{1}{C_s^2R_sR_J}\ ,$$
$$b_1=1+\frac{R_s}{R_J}-\frac{C_J}{C_s},\quad b_2=C_JR_s+\frac{L}{R_J},\quad b_3=\frac{1}{C_sR_J}\ . \tag{7}$$

We note that (5)–(7) are a direct extension of the CSC model.[6] When $R_J\to\infty$ and $C_s\to\infty$, from (7) we have $a_1=b_1=1$, $a_2=a_3=b_3=0$, and (6) reduces to

$$R^{\mathrm{CSC}}(\omega)=R_s\Big/\left\{\left[1-\frac{\omega^2}{\omega_{LC}^2}\right]^2+C_J^2R_s^2\omega^2\right\}. \tag{8}$$

Equations (5) and (8) give the results for $\langle q^2\rangle$ in the CSC model, where the R_J and the C_s effects are neglected. Our expressions (5)–(7) afford a convenient way to discuss the R_J and C_s effects. Here we concentrate our discussion on the case where the basic assumption that $R_J\to\infty$ and $C_s\to\infty$ adopted by CSC is still valid, except that the ratio R_s/R_J takes a finite value. In this case, from (7) we have $a_1=b_1=1+(R_s/R_J)\equiv b_0$, $a_2=a_3=b_3=0$, and (6) reduces to an expression similar to (8):

$$R(\omega)=R_sb_0\Big/\left\{\left[b_0-\frac{\omega^2}{\omega_{LC}^2}\right]^2+C_J^2R_s^2\omega^2\right\}\ , \tag{9}$$

where $b_0=1+R_s/R_J$. Substituting (9) into (5), we obtain the quantum charge fluctuation for a small capacitance tunnel junction ($C_J\ll C_s$) in the weak damping limit (both R_J and R_s are large compared to $R_Q\equiv h/4e^2$). At $T=0$ the calculation can be carried out analytically, and the result is

$$\frac{\langle q^2\rangle}{e^2}=\frac{\hbar\omega_{\mathrm{LC}}}{4\pi E_c}b_0f(\alpha)\ , \tag{10}$$

where $E_c=e^2/2C_J$ and

$$f(\alpha)=\begin{cases}\dfrac{2\alpha}{\sqrt{-\Delta}}\left[\dfrac{\pi}{2}-\tan^{-1}\dfrac{\alpha^2-2b_0}{\sqrt{-\Delta}}\right], & \Delta<0\\[2ex] \dfrac{\alpha}{\sqrt{\Delta}}\ln\dfrac{\alpha^2-2b_0+\sqrt{\Delta}}{\alpha^2-2b_0-\sqrt{\Delta}}\ , & \Delta>0\end{cases} \tag{11}$$

with $\alpha = R_s\sqrt{C_J/L}$ and $\Delta = \alpha^2(\alpha^2 - 4b_0)$. We note that in the limit $R_J \gg R_s$, $b_0 \equiv 1 + R_s/R_J \rightarrow 1$, and our results (10) and (11) reduce to the corresponding expressions for the quantum charge fluctuations solely due to the transmission lines, strictly speaking, first derived in CSC [Phys. Rev. Lett. **64**, 1565 (1990)]. In addition, (10) and (11) demonstrate that the tunnel resistance R_J is another important contribution to the quantum charge fluctuation. The fact that R_s/R_J is finite makes the $\langle q^2 \rangle$ evaluated from (10) and (11) significantly higher than that of the CSC model. In fact, the value of $\langle q^2 \rangle / e^2$ calculated by (10) and (11) for junctions 5 and 7 with the corresponding measured values of C_J, L, R_s, R_J in Ref. 6, increases from 0.10 and 0.65 (values obtained by the CSC model), respectively, to 0.46 and 0.89. In other words, the presence of a finite tunnel resistance increases the quantum charge fluctuation and so significantly smears the Coulomb blockade. Therefore we expect the zero-bias resistance as obtained from the present model should be significantly lower than that of the CSC model, and tends to be a better agreement with the experiments. This is the subject of the following discussion.

We are now in a position to calculate the zero-bias resistance R_0 of the quantum smearing of the Coulomb blockade in small capacitance tunnel junctions. For this purpose, we follow the heuristic argument in Ref. 6 that the value of $\langle q^2 \rangle$ is the spread of the distribution $P(q)$, i.e., the charge fluctuation evaluated from the quantum Langevin equation controls the probability distribution $P(q)$ in a way that

$$P(q) = \frac{1}{\sqrt{2\pi\langle q^2 \rangle}} e^{-q^2/2\langle q^2 \rangle} . \qquad (12)$$

Also, after accommodating the spread in values of q in the calculation of current-voltage (I-V) characteristics, the effective tunneling rate is written as[6]

$$\langle \Gamma(Q) \rangle = \int_{-\infty}^{\infty} \Gamma(Q+q) P(q)\, dq , \qquad (13)$$

where $P(q)$ is defined by (12), and the rate for Q to go to $Q \pm e$ is

$$\Gamma^{\pm}(Q) = -\frac{e/2 \pm Q}{eR_J C_J} [\exp(\Delta E^{\pm}(Q)/k_B T) - 1]^{-1} , \qquad (14)$$

with $\Delta E^{\pm}_{(Q)} = (1 \pm 2Q/e)E_c$. Since the dc current is related to the $\Gamma^{\pm}$ by the relation $I = e\{\langle \Gamma^{+}(Q) \rangle - \langle \Gamma^{-}(Q) \rangle\}$, it is straightforward to obtain the zero-bias resistance R_0 from (12)–(14), from which we have

$$\frac{R_0}{R_J} \equiv \frac{1}{R_J} \frac{dV}{dI}\bigg|_{V=0} = \left\{ \int_{-\infty}^{\infty} dq\, P(q) \left[\frac{1}{1 - e^{\Delta E^{+}/k_B T}} \left[1 + \frac{\Delta E^{+}/k_B T}{e^{-\Delta E^{+}/k_B T} - 1} \right] + \frac{1}{1 - e^{\Delta E^{-}/k_B T}} \left[1 + \frac{\Delta E^{-}/k_B T}{e^{-\Delta E^{-}/k_B T} - 1} \right] \right] \right\}^{-1} , \qquad (15)$$

with $\Delta E^{\pm} = (1 \pm 2q/e)E_c$. In the $T=0$ limit, (15) can be evaluated analytically and the result is

$$\frac{R_0}{R_J} = \left[\operatorname{erfc} \left[\frac{e}{\sqrt{8\langle q^2 \rangle}} \right] \right]^{-1} , \qquad (16)$$

where $\operatorname{erfc}(x) \equiv 1 - (2/\sqrt{\pi}) \int_0^x e^{-t^2} dt$. Equation (16) shows that at $T=0$, a nonzero charge fluctuation $\langle q^2 \rangle$ will result in a finite value for R_0. In addition, the larger the value of $\langle q^2 \rangle$ the smaller R_0 is. In other words, the quantum smearing of the Coulomb blockade by the zero-point fluctuations of the instantaneous charge produces a finite value for R_0. For example, for junctions 8 and 9 in Ref. 6, we obtain, from (5), (9), and (16), $R_0/R_J = 3.75$ and 3.08, respectively. These numbers are useful when we compare our results with the experiments.

In the CSC experiments[6] the zero-bias resistance R_0 is seen to depend on R_J (see the full dots in Fig. 2). In using the theory of Brown and Simanek,[13] which calculates the R_0 of a single small junction as a function of temperature and junction resistance R_J in the absence of environmental effects, CSC demonstrated that the theoretical values of R_0 have a strong dependence on R_J (see the dotted curve in Fig. 2), but with a noticeable quantitative deviation from the experimental data. Our Eqs. (5), (9), (12), and (15) provide an improved theory for R_0 which combines the effects of both finite R_J and the environment. In Fig. 2 we plot the predicted value of R_0/R_J as a function of R_J calculated from (5), (9), (12), and (15) for the junctions with Ni-Cr leads at $E_c/k_B T = 10$, $L = 4.5$ nH, $R_s = 130$ kΩ, and $C_J = 2.5$, 4.5, 6.5, fF. As can be seen from the figure the experimental data points of high resistance $R_J = 82$ and 133 (kΩ) (the corresponding mea-

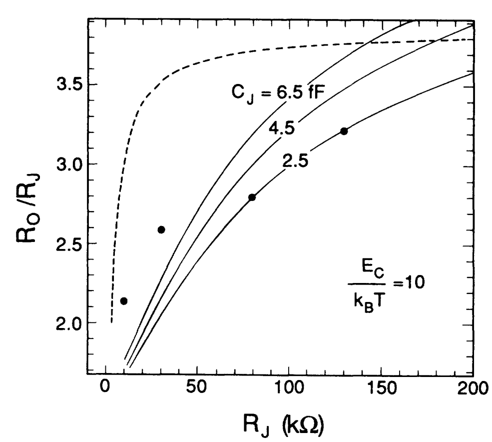

FIG. 2. Zero-bias resistance R_0/R_J (R_J is the tunnel resistance) calculated from the generalized quantum Langevin theory for values of the capacitance $C_J = 2.5$, 4.5, 6.5 fF and $E_c/k_B T = 10$, where $E_c = e^2/2C_J$ is the charging energy. Dashed lines and full dots are, respectively, the theoretical values of Brown and Simánek (Ref. 13) and the experimental data at $E_c/k_B T = 10$ from Fig. 17 of Ref. 6.

sured values of C_J being 3.5±1 and 3±1 fF) fall closely on the theoretical curve of $C_J=2.5$ fF. This excellent agreement suggests that when $R_J \gg R_Q$ our theory works very well. On the other hand, the experimental data points of low resistance $R_J=8.8$, 29.4 kΩ (the corresponding measured values of C_J are 6.5±1, 6±1 fF) are a little bit above the theoretically predicted curves. Since a smaller R_0 value corresponds to a larger value of $\langle q^2 \rangle$, the disagreement between theory and experiments in the low R_J region implies that our theory overestimates the charge fluctuation $\langle q^2 \rangle$, and needs some improvement in the low R_J region.

Another important feature for the values of R_0 identified by the CSC experiments is that R_0 approaches R_J at the high-temperature limit, it increases with decreasing T and flattens out as T is further reduced so that $k_B T \ll E_c$ (see Fig. 3). The quantum Langevin model of CSC, which neglects the R_J contribution to the fluctuations, reflects the above trend of R_0 but with a clear quantitative difference. Our Eqs. (5), (9), (12), and (15) extend the CSC model to include the R_J effects on the charge fluctuations and on the zero-bias resistance. In Fig. 3, we plot R_0/R_J vs $E_c/k_B T$ as calculated from (5), (9), (12), and (15) for five different values of $R_J=133$, 82, 29.4, 23, 11.3 kΩ. The corresponding values for C_J and R_s used are, respectively, $C_J=2.5$, 2.5, 6, 4, 3 fF, and $R_s=130$, 130, 130, 9, 9 kΩ, which are comparable to the measured values for the junction numbers 8, 9, 7, 5, 3 in Ref. 6. As can be seen from the figure, at low temperatures $(E_c/k_B T \gg 1) R_0$ flattens off to finite values as given by (16), and the theoretical values (lines) matches the experimental data (symbols) almost perfectly for junctions with high lead resistances (Nos. 8 and 9) even though the one with lower junction resistance (No. 7) has a relatively higher measured R_0 value than predicted at $T \to 0$. Also, the agreement between theory and experiment for the temperature dependence of R_0 for junctions (Nos. 8, 9, and 7) with high lead resistance is quite good. This shows again that the quantum Langevin equation approach presented here works very well for weak damping small tunneling junctions. On the other hand, for low lead resistance junctions (Nos. 5 and 3) our theory still deviates from the experimental data, and does not improve the CSC model very much.

In conclusion, in this paper we have generalized the CSC quantum Langevin model for the charge fluctuations and zero-bias resistance of the small tunnel junction. When the tunnel resistance and the transmission line effects are both considered, we have used (5), (9), (12), and (15) to calculate the zero-bias resistance R_0 as a function of tunnel resistance R_J as well as temperature, and compared with the experiments of Ref. 6. For those junctions with high lead resistance, the agreement between our theory and experiments is excellent, while for low lead resistance junctions the theory fails to predict values of R_0 in the range of experimental data. To our knowledge, in the literature no other theory has ever produced results quantitatively comparable to the experimental data on the flattening off of R_0 at low temperatures. Since no parameter is introduced in our calculation, the results of the good comparison between the theory and experiments suggests that the generalized quantum Langevin model is a good tool to study the physics of weak damping small tunnel junctions, but that the theory needs to be further improved if applied to strongly damped junctions.

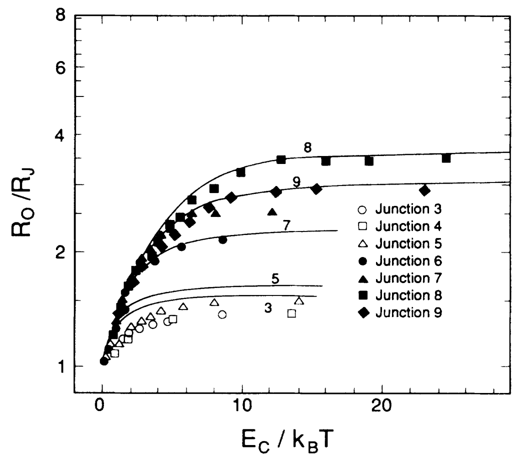

FIG. 3. Zero-bias resistance R_0/R_J (R_J is the tunnel resistance) vs $E_c/k_B T$ ($E_c=e^2/2C_J$ is the charging energy and C_J is the capacitance) calculated by the generalized quantum Langevin theory (full lines). Symbols (solid for Ni-Cr leads and open for Cu-Au leads) are experimental data from Ref. 6.

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