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Quantum tunneling in a blackbody radiation field ☆

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It is generally considered that for a so-called normal system dissipation decreases tunneling rates. Here we show that at least one example of a normal heat bath, the blackbody radiation field, leads to an increase in tunneling. The reason for this exception to the general rule is the presence of mass renormalization.

The pioneering work of Caldeira and Leggett [1] on the effect of dissipation on quantum tunneling has generated an extensive literature. It is generally thought that tunneling rates decrease in the presence of so-called normal coupling to the environment. Although it is possible to construct models for which the tunneling rates increase [3,4], such models are generally unphysical, and Leggett has used the term "anomalous dissipation" to characterize them [4]. Our purpose here is to show that the blackbody radiation field is an example of a normal dissipative system for which the coupling increases the tunneling rate, contrary to the usual rule. The reason for the exception is the presence of mass renormalization.

Normal dissipation for a system of one degree of freedom linearly coupled to a passive heat reservoir can be characterized in terms of the quantum Langevin equation. This is the Heisenberg equation of motion for a dynamical variable $x(t)$ and takes the form [5]

$$m\ddot{x}+\int_{-\infty}^{t} \mathrm{d}t'\, \mu(t-t')\dot{x}(t')+V'(x) = F(t)+f(t)\,, \qquad (1)$$

where the dot indicates the time derivative and $V'=\partial V/\partial x$. On the right-hand side $F(t)$ is a random operator force with vanishing mean, $\langle F(t)\rangle=0$, and whose other properties, in particular the relation with the memory function $\mu(t)$, are described in ref. [5]. Finally, $f(t)$ is an applied c-number force. One assumes that the applied force vanishes in the distant past and future, so that at $t=\pm\infty$ the system is in thermal equilibrium. A system with normal dissipation is one such that the total work done by the applied force is [#1]

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[#1] This definition corresponds to that given in eqs. (3.8) and (3.9) of ref. [4].

$$W = \int_{-\infty}^{\infty} dt\, f(t) \langle \dot{x}(t) \rangle . \tag{2}$$

For a normal system the requirements of (i) causality, (ii) the second law of thermodynamics, and (iii) reality imply that the Fourier transform of the memory function,

$$\tilde{\mu}(z) = \int_0^{\infty} dt\, e^{izt} \mu(t) , \quad \mathrm{Im}(z) < 0 , \tag{3}$$

must be a positive real function [5]. In particular this means that on the real axis

$$\mathrm{Re}\{\tilde{\mu}(\omega + i0^+)\} \geqslant 0 , \tag{4}$$

and that in the upper half plane one has the Stieltjes formula

$$\tilde{\mu}(z) = -iCz + \frac{2iz}{\pi} \int_0^{\infty} d\omega \frac{\mathrm{Re}\{\tilde{\mu}(\omega + i0^+)\}}{z^2 - \omega^2} , \tag{5}$$

where C is a real positive constant which in our case can be absorbed into the mass m.

We frame our discussion in terms of the generalized susceptibility, $\alpha_0(\omega)$, for the free particle. This is defined in terms of the Fourier transform (indicated by a tilde) of the solution of the quantum Langevin equation (1) with $V=0$. One writes this solution in the form $\langle \tilde{x}(\omega) \rangle = \alpha_0(\omega) \tilde{f}(\omega)$. Then, from (1) we see that the reciprocal of the generalized susceptibility is given by

$$[\alpha_0(\omega)]^{-1} = -m\omega^2 - i\omega\tilde{\mu}(\omega) . \tag{6}$$

Now we specialize to the case of an electron coupled to the blackbody radiation field. There the dynamical variable $x(t)$ is the linear displacement of the electron and, according to the above definition, this is obviously a normal system [5]. The Fourier transform of the memory function can be expressed in the form [6]

$$\tilde{\mu}(z) = \frac{2e^2 \Omega_c^2 z}{3c^3 (z + i\Omega_c)} , \tag{7}$$

where Ω_c is a large cut-off frequency. Putting this in the general expression (6) for the free-particle susceptibility, we can write

$$[\alpha_0(\omega)]^{-1} = \frac{-m\omega - iM\Omega_c}{\omega + i\Omega_c} \omega^2 , \tag{8}$$

where M is the renormalized mass,

$$M = m + \frac{2e^2 \Omega_c}{3c^3} . \tag{9}$$

It will be convenient to use the bare mass m instead of Ω_c as a parameter, writing $\Omega_c = (M-m)/M\tau_e$, where

$$\tau_e = \frac{2e^2}{3Mc^3} = 6 \times 10^{-24}\ \mathrm{s} \tag{10}$$

is the electromagnetic time. We can then write (8) in the form

$$[\alpha_0(\omega)]^{-1} = -M\omega^2 \frac{M - m(1 + i\omega t_e)}{M - m - i\omega M \tau_e} . \tag{11}$$

Note that we must require that the bare mass be non-negative,

$$0 \leqslant m \leqslant M , \tag{12}$$

since for a negative bare mass $\alpha_0(z)$ has a pole in the upper half-plane, which would lead to a violation of the second law of thermodynamics. Zero bare mass corresponds to the maximum value of the cutoff, $\Omega_c = 1/\tau_e$ [#2].

Now we consider quantum tunneling at zero temperature for a system. We discuss this using two methods, which apply to different situations. The first is that appropriate to elastic tunneling at an energy near the top of a smooth barrier. What one does is to consider the tunneling through a parabolic barrier of the form

$$V(x) = -\tfrac{1}{2} M \Omega_0^2 x^2 . \tag{13}$$

In the absence of dissipation, the transmission probability of elastic tunneling is given by the well known exact expression [7]

$$D_0 = \frac{1}{1 + \exp(-2\pi E/\hbar\Omega_0)} , \tag{14}$$

#2 It is of interest that in this zero-bare-mass limit $[\alpha_0(\omega)]^{-1}$ is identical (aside from the sign convention of the Fourier transform) to $K(\omega)$ in eq. (3.13) of ref. [4]. There it is asserted that this is an example of anomalous dissipation, but we see here that it is not necessarily the case.

where E is the energy measured from the top of the barrier. In the presence of dissipation the corresponding transmission probability is given by this same expression with Ω_0 replaced with Ω, with Ω the real positive root of the equation [8]

$$[\alpha_0(\mathrm{i}\Omega)]^{-1}=M\Omega_0^2\,. \tag{15}$$

Using the expression (11) appropriate for the coupling to the radiation field, this becomes

$$M\Omega^2\frac{M-m+m\Omega\tau_e}{M-m+M\Omega\tau_e}=M\Omega_0^2\,. \tag{16}$$

It is clear that, on account of the condition (12), the factor multiplying $M\Omega^2$ on the right is always less than unity. It follows that the root Ω is always greater than Ω_0, corresponding to an increased transmission probability. For the case of maximum cutoff, corresponding to $m=0$, we have the explicit expression

$$\Omega=[\sqrt{1+(\Omega_0\tau_e/2)^2}+\Omega_0\tau_e/2]\Omega_0\,. \tag{17}$$

In the weak coupling limit, $\Omega_0\tau_e\ll 1$, this becomes $\Omega\approx(1+\Omega_0\tau_e/2)\Omega_0$.

Next we consider the method of Caldeira and Leggett [1,2], which was the first used in discussing this problem, and which is appropriate to the discussion of tunneling from a well near the bottom of a high barrier. In this case the transmission probability can be written in the form

$$D=\exp\{-S_{\mathrm{eff}}[x(\tau)]/\hbar\}\,, \tag{18}$$

where S_{eff} is the effective action in imaginary time. Leggett has shown that this can be written [4]

$$S_{\mathrm{eff}}[x(t)]=\frac{1}{2\pi}\int_0^\infty \mathrm{d}\omega\,[\alpha_0(\mathrm{i}\omega)]^{-1}|\tilde{x}(\omega)|^2 + \int_{-\infty}^{\infty}\mathrm{d}\tau\,V(x(\tau))\,, \tag{19}$$

where $x(\tau)$ is the classical trajectory in the inverted potential, as described, e.g., in ref. [2]. Again, since

$$[\alpha_0(\mathrm{i}\omega)]^{-1}=M\omega^2\frac{M-m+m\omega\tau_e}{M-m+M\omega\tau_e}\leqslant M\omega^2\,, \tag{20}$$

it is clear that the effect of the radiation damping is to reduce S_{eff}, leading to increased transmission probability.

What has gone wrong with the earlier proofs that normal dissipation always causes decreased tunneling? Thus, in ref. [4] it was shown in general that for a normal system the effect of dissipation is to reduce the tunneling probability. The method used was the second method described above, appropriate to tunneling near the bottom of a high barrier. Correspondingly, in ref. [8] we showed that elastic tunneling near the top of a smooth barrier is also reduced in the case of normal dissipation. The point is that in both these discussions it was considered that dissipation corresponds to the presence of the memory term in the quantum Langevin equation (1) and that no dissipation corresponds to setting $\mu(t)$ equal to zero. In this sense the theorems still hold and dissipation always slows tunneling. In the blackbody case this would correspond to replacing M by m in the barrier potential (13), defining dissipationless tunneling in terms of the bare mass m. But the bare mass is unobservable: the observed mass is the renormalized mass M and this is what should appear in (13). The point here is that the coupling to the radiation field has two kinds of effects: a dynamical effect, corresponding to an electromagnetic contribution to the mass, and a dissipative effect, corresponding to radiation reaction. Both effects appear in the memory term $\mu(t)$.

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