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Total power radiated by an accelerated charge

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We propose a modification in the familiar Larmor formula for the total power radiated by a non-relativistic electron.

We have recently presented a new approach to the problem of radiation reaction in non-relativistic electrodynamics and obtained an equation of motion which does not exhibit the problems associated with the well-known Abraham–Lorentz equation [1]. But since the latter result can be derived using only energy conservation and the Larmor formula (for the total power radiated by a non-relativistic electron) one might suspect that the latter needs modification. This suspicion is borne out, as we shall now demonstrate.

Our starting-point (ref. [1], eq. (8)) is the classical equation of motion for an electron of mass M and charge e, moving in one dimension, in an external c-number force $f(t)$:

$$M\frac{d^2x(t)}{dt^2}=f(t)+\tau_e\frac{df(t)}{dt}\,, \tag{1}$$

where $\tau_e=2e^2/3Mc^3=6\times10^{-24}$ s. Consider now a force that is switched on in the distant past and off in the distant future, i.e.

$$f(\pm\infty)=0\,. \tag{2}$$

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It follows that the solution of our equation of motion (1) is

$$\begin{aligned} v(t)&\equiv\dot{x}(t)\\ &=v(-\infty)+\frac{1}{M}\int_{-\infty}^{t}dt'\,f(t')+\frac{\tau_e}{M}f(t)\,, \end{aligned} \tag{3}$$

and, in particular,

$$v(\infty)=v(-\infty)+\frac{1}{M}\int_{-\infty}^{\infty}dt'\,f(t')\,. \tag{4}$$

Hence, the final velocity is exactly what it would be without radiation. This is not as surprising as it might seem at first glance, since, in the dipole approximation, the radiation is symmetrical about the particle displacement, as a result of which there is no net momentum exerted by the electromagnetic field on the particle. Furthermore, (4) is in essence an expression of momentum conservation since the integral therein is exactly the total impulse given by the external field to the particle.

It is also of interest to re-write (3) in the form

$$v(t)=V(t)+\frac{\tau_e}{M}f(t)\,, \tag{5}$$

where $V(t)$ is the velocity which the electron would have at time t if there were no radiation:

$$V(t)\equiv v(-\infty)+\frac{1}{M}\int_{-\infty}^{t} dt'\, f(t')\,. \tag{6}$$

It follows, using (4) and (6), that

$$V(\pm\infty)=v(\pm\infty)\,. \tag{7}$$

Also

$$\frac{dV(t)}{dt}=\frac{1}{M}f(t)\,. \tag{8}$$

We are now ready to consider energy conservation. The work done by the force $f(t)$ is

$$W=\int_{-\infty}^{\infty} dt\, f(t)v(t)\,. \tag{9}$$

Hence, using (5) in (9), we get

$$W=T+W_R\,, \tag{10}$$

where

$$W_R\equiv\frac{\tau_e}{M}\int_{-\infty}^{\infty} dt\, f^2(t)\,, \tag{11}$$

and, using (7) and (8),

$$T\equiv\int_{-\infty}^{\infty} dt\, f(t)V(t)=\tfrac{1}{2}M\int_{-\infty}^{\infty} dt\frac{d}{dt}V^2(t)$$

$$=E(\infty)-E(-\infty)\,, \tag{12}$$

where

$$E=\tfrac{1}{2}Mv^2\,. \tag{13}$$

Therefore, T is to be identified as the gain in kinetic energy due to the action of the force. It follows that W_R is the radiated energy. We may also write

$$W_R\equiv\int_{-\infty}^{\infty} dt\, P(t)\,, \tag{14}$$

where $P(t)$, the power radiated, is given by

$$P(t)=\frac{\tau_e}{M}f^2(t)=\frac{2e^2}{3c^3}[f(t)/M]^2\,, \tag{15}$$

which differs from the Larmor formula in that $f(t)/M$ appears instead of $\ddot{x}(t)$, and, of course, these quantities are not equal, because of the radiative reaction term appearing in (1).

Thus, the question arises as to why the usual derivation of the Larmor formula [2] does not lead to (15). The answer is simply because it assumes that the electron is a point whereas the essence of our previous work [1] is that one must assume, in order to avoid problems with causality, that the electron has structure (a point which is also contained in the papers of various other authors, for example ref. [3]). In fact, if one modifies the usual derivation to incorporate electron structure then it may be demonstrated that again (15) emerges.

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