Magneto-optical effects from free electrons in magnetic white dwarfs

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Abstract. When calculating synthetic spectra and theoretical values for the polarization of magnetic white dwarfs, it is important to take the magneto-optical effects arising from free electrons into account. The hitherto used formalism of Pacholczyk breaks down at fields higher than 100 MG where the cyclotron resonance occurs in the optical region of the electromagnetic spectrum. In a new program for the analysis of magnetic white dwarfs a consistent approach is used that allows for damping by electron collisions. Inaccuracies originating from the discrete division of the stellar disc into a limited number of surface elements are reduced by taking into account the variation of the magnetic field over the individual elements.

1. Introduction

The magnetic field structure of magnetic white dwarfs can be determined by comparing the observed flux distribution and wavelength dependent polarization with detailed model calculations. For this purpose the radiative transfer equations (Unno, 1956; Beckers, 1969) must be solved for the Stokes parameters I, Q, U and V in a realistic model atmosphere at different parts of the stellar surface.

With the help of such models Martin and Wickramasinghe have analyzed five magnetic white dwarfs with polar field strengths between 5 and 36 MG (Wickramasinghe and Martin, 1979; Martin and Wickramasinghe, 1979). In the high field region Wickramasinghe and Ferrario (1988) obtained a good fit for the famous star Grw + 70°8247 between 3400 Å and 6000 Å by assuming a dipole model with a polar field strength of 320 ± 20MG. They found serious discrepancies between theory and observation for the polarization at all wavelengths and for the flux at $\lambda > 6000$ Å. This is also true for comparisons of polarization measurements of highly magnetic white dwarfs between 0.35 and 1.65μm and models published by West (1989). The probable reason is the lack of accurate continuum opacities, but this assumption can only be checked by ruling out other sources of uncertainty, e.g. the treatment of magnetic birefringence.

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Beside absorption, Faraday and Voigt effect have influence on the radiative transport in a magnetized plasma. Both, the change of the refractive index in the neighborhood of spectral lines and the anomalous dispersion arising from free electrons in the continuum, affect the emerging spectrum and polarization. This was demonstrated by Martin and Wickramasinghe (1981 and 1982) for fields lower than 10^8 G. Their main result was that — compared to the situation without magneto-optical effects taken into consideration — the lines are deepened (especially for transitions with a change of the magnetic quantum number by $\Delta m = 0$) and the linear polarization $\sqrt{Q^2 + U^2}/I$ is greatly reduced. The wavelength dependent profile of the circular polarization was slightly altered, but no large modification of its magnitude was found.

For the magneto-optical parameters of the continuum — their effect is larger than the contribution from the lines — Martin and Wickramasinghe (1982) used two formulae of Pacholczyk (1976) (equation (19) and (31) in this paper) that can be applied, provided the frequency of the radiation is much larger than the cyclotron frequency $\omega_c = eB/m_e c$ (e is the electron charge, m_e the electron mass and c the velocity of light). This assumption is not fulfilled for stars like Grw + 70°8247 , where the magnetic field B exceeds 100 MG and the cyclotron resonance occurs in the optical region. With the consistent approach of this paper the magnetic birefringence can be calculated at arbitrary fields and wavelengths.

2. The radiative transfer equations

When magneto-optical effects are neglected the radiative transfer in a magnetized plasma can be described by the three Stokes parameters I, Q and V (Unno, 1956). This simplification is no longer possible if the Faraday rotation leads to different azimuths of the plane of polarization at various optical depths. Then the fourth Stokes parameter U is necessary and four partial differential equations have to be solved (Becker, 1969; Hardorp et al., 1976):

$$\mu \frac{dI}{d\tau} = \eta_I(I - B) + \eta_Q Q + \eta_V V, \tag{1}$$

$$\mu \frac{dQ}{d\tau} = \eta_Q(I - B) + \eta_I Q + \rho_R U, \tag{2}$$

$$\mu \frac{dU}{d\tau} = \rho_R Q + \eta_I U - \rho_W V, \tag{3}$$

$$\mu \frac{dV}{d\tau} = \eta_V (I - B) + \rho_W U + \eta_V V. \tag{4}$$

We used the Rosseland absorption coefficient κ_R to define the optical depth scale $d\tau = -\kappa_R dz$. With ϑ being the angle between the z-direction (pointing to the observer) and the line of sight, $\mu = \cos \vartheta$ can be defined. η_I, η_Q and η_V are given by (Unno, 1956)

$$\eta_I = \frac{1}{2}\eta_p \sin^2 \psi + \frac{1}{4}(\eta_l + \eta_r)(1 + \cos^2 \psi), \tag{5}$$

$$\eta_Q = \frac{1}{2}\eta_p - \frac{1}{4}(\eta_l + \eta_r)\sin^2 \psi, \tag{6}$$

$$\eta_I = \frac{1}{2}(\eta_r - \eta_l)\cos \psi, \tag{7}$$

where η_l, η_p and η_r are the monochromatic absorption coefficients for transitions with respectively $\Delta m = -1, 0$ and 1. ψ denotes the angle between the magnetic field direction and the line of sight. The meanings of the magneto-optical parameters ρ_R and ρ_W are described in the following section.

The direction of the x-axis, to which the angle of linear polarization refers, can be defined arbitrarily but for the integration of the local solutions over the stellar surface it is necessary to define a fixed coordinate system for the whole disk. When the local x-axis — which in our case is oriented with respect to the projection of the magnetic field on a plane perpendicular to the line of sight — differs from the global x-direction by an angle ϕ, the vector consisting of the Stokes parameters Q and U is transformed by the rotation

$$\begin{pmatrix} Q' \\ V' \end{pmatrix} = \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} Q \\ V \end{pmatrix}. \tag{8}$$

3. The magneto-optical parameters

3.1. The Faraday Effect

The Faraday effect has its origin in the fact that the refractive indices for left (n_-) and right (n_+) circularly polarized light are not equal in the presence of a magnetic field. For radiation with an angular frequency of ω, the difference between the phase velocities c/n_+ and c/n_- leads to a rotation angle of the electric field vector per unit path length l of (see e.g. O'Connell and Wallace, 1981)

$$\frac{d\theta}{dl} = \frac{\omega}{2c}(n_+ - n_-). \tag{9}$$

The magneto-optical parameter ρ_R that enters the equations of radiative transfer $(1) - (2)$ can be defined as

$$\rho_R \equiv -2\cos \psi \frac{d\theta}{\kappa_R dl}. \tag{10}$$

The quantities $n_\pm$ are connected to the dielectric constants $\epsilon_\pm$ as follows:

$$\epsilon_\pm \equiv \epsilon'_\pm + i\epsilon''_\pm = (n_\pm + i\kappa_\pm)^2. \tag{11}$$

$\kappa_\pm$ is the imaginary part of the refractive index. When $\epsilon_\pm$ is known, $n_\pm$ can be calculated from

$$n_\pm^2 = \sqrt{\frac{1}{2}\{[(\epsilon'_\pm)^2 + (\epsilon''_\pm)^2]^{1/2} + \epsilon'_\pm\}}. \tag{12}$$

The contribution of the spectral line components can be found with the help of the line dispersion function F (Wittmann, 1974; Martin and Wickramasinghe, 1981):

$$\rho_R = -\left(\sum_i \eta_{ri} F_{ri} - \sum_j \eta_{lj} F_{lj}\right)\cos \psi, \tag{13}$$

where the sum extends over all line components that contribute to the wavelength integrated absorption coefficients (divided by κ_R) η_l and η_r. Since we assume that the line components have Voigt profiles $H(a,v)$, the dispersion function can be expressed as following

$$F(a,v) = \frac{1}{a}\left(\frac{1}{2}H(a,v) + \frac{1}{4}\frac{\partial H(a,v)}{\partial v}\right). \tag{14}$$

The contribution of the continuum to ρ_R is obtained by solving the equation of motion for an electron in a static magnetic field B driven by an electromagnetic field $\mathbf{E}_\pm$ (see e.g. Smith, 1967):

$$m_e\ddot{\mathbf{r}} + m_e v_e\dot{\mathbf{r}} + m_e\omega_p\mathbf{r} = -e\left(\mathbf{E}_\pm - \frac{\dot{\mathbf{r}}}{c}\times \mathbf{B}\right). \tag{15}$$

(The magnetic component $H_\pm$ is much smaller than $v/c\mathbf{E}_\pm$ and can therefore be neglected.) This leads to the dielectric constants

$$\epsilon_\pm = 1 - \frac{\omega_p^2}{\omega(\omega \pm \omega_c + iv_e)}, \tag{16}$$

where ω_p denotes the plasma frequency $\sqrt{4\pi N_e e^2/m_e}$. N_e and v_e are the density and the collision frequency of the electrons. Now $n_\pm$, $d\theta/dl$ and ρ_R are easily calculated by inserting the real and imaginary parts of $\epsilon_\pm$

$$\epsilon'_\pm = 1 - \frac{\omega_p^2(\omega \pm \omega_c)}{\omega\left[(\omega \pm \omega_c)^2 + v_e^2\right]} \tag{17}$$

and

$$\epsilon''_\pm = \frac{\omega_p^2 v_e}{\omega\left[(\omega \pm \omega_c)^2 + v_e^2\right]} \tag{18}$$

into equation (12), (9) and (10). The asymptotic approximation of Pacholczyk (1976) that has been used by Martin and Wickramasinghe (1982) follows in the limit $\omega \gg \omega_c$, $\omega \gg \omega_p$ and $v_e = 0$:

$$\rho_R = -\frac{\omega_p^2 \omega_c}{c\kappa_R(\omega^2 - \omega_c^2)}\cos \psi. \tag{19}$$

We use

$$v_e = 7.751 \cdot 10^{15}\frac{P_e/(\text{g sec}^{-2}\text{cm}^{-1})}{(T/\text{K})^{5/2}}$$
$$\times \left(-8.835 + 2\ln \frac{T}{\text{K}} - \frac{1}{2}\frac{P_e}{(\text{g sec}^{-2}\text{cm}^{-1})}\right)\frac{1}{\text{sec}}. \tag{20}$$

for the electron collision frequency. T denotes the temperature and $P_e = N_e kT$ the electron pressure. The formula can be derived from Spitzer (1956) and is the same as the result of Ford and O'Connell (1980) except for a factor of $\pi/4$. Note that equation (20) is derived for free-space conditions and neglects the influence of the magnetic field on the motion of the
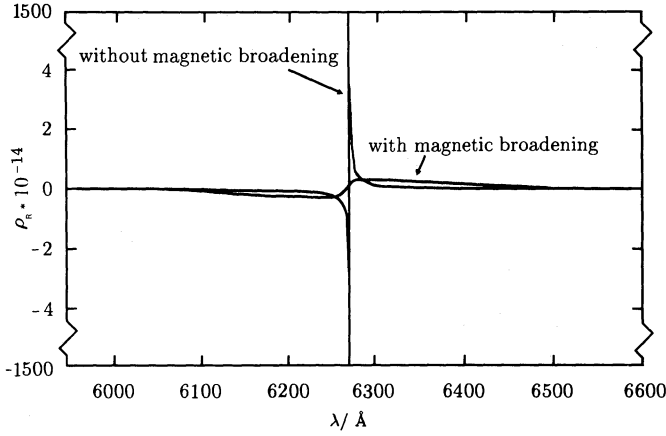
Fig. 1. The Faraday parameter ρ_R for a magnetic field of 170 MG at an optical depth $\tau = 2/3$ in a model atmosphere with $T_{\text{eff}} = 16000$ K and log $g = 8$. In the curve with the sharp resonance at the cyclotron frequency (corresponding to a wavelength of 6270 Å) ρ_R is only broadened by electron collisions while the other curve shows the effect of the additional broadening by the variation of the magnetic field over a surface element (extending over a latitude difference of 5° in the spherical coordinate system of the star. Please notice the scale change which is necessary in order to show the extreme height of the resonance.)

electrons (one-dimensional mobility only along the field lines). But since Fig. 1 shows that the magnetic broadening dominates the collisional broadening by orders of magnitude we did not take the modification due to the magnetic field into account.

Even with collision broadening included the resonance of ρ_R at the cyclotron frequency is extremely sharp. This is demonstrated in Fig. 1 for a field of $B = 170\text{MG}$ at an optical depth of $\tau_R = 2/3$ in a $T_{\text{eff}} = 16000\,\text{K}$ atmosphere. For a model calculation, in which the stellar surface is divided into a limited number of surface elements, the resulting profile is quite unrealistic, since the magnetic field and therefore the cyclotron frequency is not constant over a single area element. In a dipole field, where the strength of the magnetic field is a factor of two higher at the poles than at the equator, the field and cyclotron frequency variation is of the order of 10% on an element which extends over 5° in latitude.

Responsible for the sharp resonance are the terms

$$f(\omega) = \frac{1}{(\omega - \omega_c)^2 + v_e^2} \tag{21}$$

and

$$g(\omega) = \frac{\omega - \omega_c}{(\omega - \omega_c)^2 + v_e^2} \tag{22}$$

in equation (17) and (18). If ΔB is defined as the standard deviation of the magnetic field over a single surface element and $\Delta\omega_c = e\Delta B/2mc$, we can substitute the fast varying terms f and g by the means

$$\frac{1}{2 \cdot 5\Delta\omega_c} \int_{\omega_c - 5\Delta\omega_c}^{\omega_c + 5\Delta\omega_c} f(\omega) d\omega$$
$$\approx \frac{1}{10\Delta\omega_c} \int_{-\infty}^{+\infty} \frac{d(\omega - \omega_c)}{(\omega - \omega_c)^2 + v_e^2} = \frac{\pi}{10\Delta\omega_c v_e} \tag{23}$$

and

$$\frac{\omega - \omega_c}{5\Delta\omega_c|\omega - \omega_c|} \int_{\omega_c}^{\omega_c + 5\Delta\omega_c} g(\omega) d\omega$$
$$= \frac{(\omega - \omega_c)}{5\Delta\omega_c|\omega - \omega_c|} \int_0^{+5\Delta\omega_c} \frac{z dz}{z^2 + v_e^2} \tag{24}$$
$$= \frac{(\omega - \omega_c)}{10\Delta\omega_c|\omega - \omega_c|} \ln \frac{25\Delta\omega_c^2 + v_e^2}{v_e^2}$$

in the frequency interval $(\omega_c - 5\Delta\omega_c, \omega_c + 5\Delta\omega_c)$. The size of the interval is chosen in such a manner that a smooth behavior of ρ_R is reached. In equation (23) the limits can be replaced by $-\infty$ and $+\infty$ since practically only ω values in the vicinity of ω_c contributes to the integral. In Fig. 1 the resulting profile shows the situation with and without additional broadening by the magnetic field.

3.2. The Voigt Effect

The other magneto-optical effect that has to be taken into account is the Voigt effect. It arises from a phase shift between the linear polarized components of the electric field vector parallel and perpendicular to the magnetic field. The phase shift $d\varphi/d\tau$ and magneto-optical parameter ρ_W are connected to the refractive indices $n_\parallel$ and $n_\perp$ by:

$$\rho_W \equiv \frac{d\varphi}{d\tau} \sin^2 \psi = -\frac{\omega}{2c\kappa_R}(n_\parallel - n_\perp) \sin^2 \psi. \tag{25}$$

The contribution of the lines is obtained analogous to equation (13) (Wittmann, 1974)

$$\rho_W = -\left(\sum_k \eta_{pk} F_{pk} - \frac{1}{2} \sum_j \eta_{lj} F_{lj} \frac{1}{2} \sum_i \eta_{ri} F_{ri} \right) \tag{26}$$
$$\times \sin^2 \psi,$$

while the procedure to calculate ρ_W of the continuum is equivalent to that of ρ_R, if n_+ is replaced by $n_\parallel$ and n_- substituted by $n_\perp$. The real and imaginary parts of $\epsilon_{\parallel/\perp}$ are

$$\epsilon_\parallel' = 1 - \frac{\omega_p^2}{\omega^2 - v_e^2}, \tag{27}$$

$$\epsilon_\perp' = 1$$
$$- \frac{\omega_p^2 \left[(\omega_p^2 - \omega^2)(\omega_p^2 - \omega^2 + \omega_c^2) + \omega^2 v_e^2 \right]}{\omega^2(\omega_p^2 - \omega^2 + \omega_c^2 + v_e^2)^2 + v_e^2(\omega_p^2 - 2\omega^2)^2} \tag{28}$$

and

$$\epsilon_\parallel'' = \frac{\omega_p^2 v_e}{\omega(\omega^2 - v_e^2)}, \tag{29}$$

$$\epsilon_\perp'' = \frac{\omega_p^2 v_e \left[(\omega^2(\omega_c^2 + v_e^2) + (\omega_p^2 - \omega^2)^2 \right]}{\omega \left[\omega^2(\omega_p^2 - \omega^2 + \omega_c^2 + v_e^2)^2 + v_e^2(\omega_p^2 - 2\omega^2)^2 \right]}. \tag{30}$$

In the limit $\omega \gg \omega_c$, $\omega \gg \omega_p$ and $v_e = 0$ we obtain the second formulae of Pacholczyk (1976):

$$\rho_W = -\frac{\omega_p^2 \omega_c^2}{2c\omega\kappa_R(\omega^2 - \omega_c^2)} \sin^2 \psi. \tag{31}$$

Fig. 3. The linear polarization predicted by a model with and without magneto-optical effects taken into account. $B_p = 320$, $i = 30°$, $T_{eff} = 16000$ K and log $g = 8$.
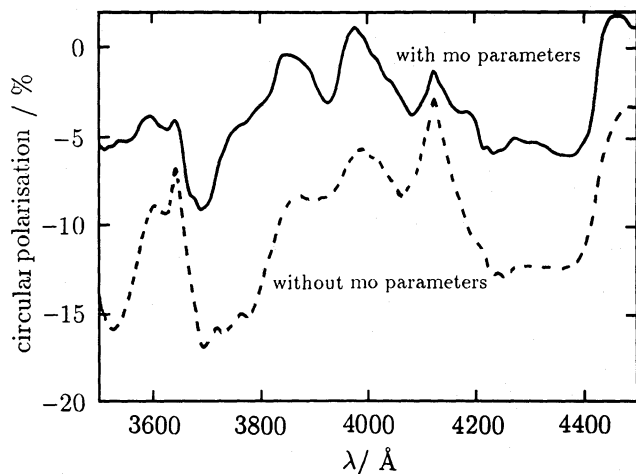
Fig. 4. The degree of circular polarization in percent predicted by a model with $B_p = 320$ MG, $i = 30°$, $T_{eff} = 16000$ K and log $g = 8$. The minimum of the polarization is reduced from -17 % to -8 % by the inclusion of magneto-optical effects.

of hydrogen will be discussed. Although the flux distribution can be reproduced much better now, our improvements cannot produce a better agreement between theory and observation for the polarization, until consistent calculations for the continuum opacities become available. This would be very important, since the exact configuration of the magnetic field — e.g. departures from a centered dipole — cannot be determined from the energy spectrum alone.

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