

# Magneto-optical effects from free electrons in magnetic white dwarfs

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**Abstract.** When calculating synthetic spectra and theoretical values for the polarization of magnetic white dwarfs, it is important to take the magneto-optical effects arising from free electrons into account. The hitherto used formalism of Pacholczyk breaks down at fields higher than 100 MG where the cyclotron resonance occurs in the optical region of the electromagnetic spectrum. In a new program for the analysis of magnetic white dwarfs a consistent approach is used that allows for damping by electron collisions. Inaccuracies originating from the discrete division of the stellar disc into a limited number of surface elements are reduced by taking into account the variation of the magnetic field over the individual elements.

**Key words:** stars: white dwarfs – stars: magnetic field – synthetic spectra – polarization

## 1. Introduction

The magnetic field structure of magnetic white dwarfs can be determined by comparing the observed flux distribution and wavelength dependent polarization with detailed model calculations. For this purpose the radiative transfer equations (Unno, 1956; Beckers, 1969) must be solved for the Stokes parameters  $I$ ,  $Q$ ,  $U$  and  $V$  in a realistic model atmosphere at different parts of the stellar surface.

With the help of such models Martin and Wickramasinghe have analyzed five magnetic white dwarfs with polar field strengths between 5 and 36 MG (Wickramasinghe and Martin, 1979; Martin and Wickramasinghe, 1979). In the high field region Wickramasinghe and Ferrario (1988) obtained a good fit for the famous star Grw + 70°8247 between 3400 Å and 6000 Å by assuming a dipole model with a polar field strength of  $320 \pm 20$  MG. They found serious discrepancies between theory and observation for the polarization at all wavelengths and for the flux at  $\lambda > 6000$  Å. This is also true for comparisons of polarization measurements of highly magnetic white dwarfs between 0.35 and  $1.65 \mu\text{m}$  and models published by West (1989). The probable reason is the lack of accurate continuum opacities, but this assumption can only be checked by ruling out other sources of uncertainty, e.g. the treatment of magnetic birefringence.

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Beside absorption, Faraday and Voigt effect have influence on the radiative transport in a magnetized plasma. Both, the change of the refractive index in the neighborhood of spectral lines and the anomalous dispersion arising from free electrons in the continuum, affect the emerging spectrum and polarization. This was demonstrated by Martin and Wickramasinghe (1981 and 1982) for fields lower than  $10^8$  G. Their main result was that — compared to the situation without magneto-optical effects taken into consideration — the lines are deepened (especially for transitions with a change of the magnetic quantum number by  $\Delta m = 0$ ) and the linear polarization  $\sqrt{Q^2 + U^2}/I$  is greatly reduced. The wavelength dependent profile of the circular polarization was slightly altered, but no large modification of its magnitude was found.

For the magneto-optical parameters of the continuum — their effect is larger than the contribution from the lines — Martin and Wickramasinghe (1982) used two formulae of Pacholczyk (1976) (equation (19) and (31) in this paper) that can be applied, provided the frequency of the radiation is much larger than the cyclotron frequency  $\omega_c = eB/m_e c$  ( $e$  is the electron charge,  $m_e$  the electron mass and  $c$  the velocity of light). This assumption is not fulfilled for stars like Grw + 70°8247, where the magnetic field  $B$  exceeds 100 MG and the cyclotron resonance occurs in the optical region. With the consistent approach of this paper the magnetic birefringence can be calculated at arbitrary fields and wavelengths.

## 2. The radiative transfer equations

When magneto-optical effects are neglected the radiative transfer in a magnetized plasma can be described by the three Stokes parameters  $I$ ,  $Q$  and  $V$  (Unno, 1956). This simplification is no longer possible if the Faraday rotation leads to different azimuths of the plane of polarization at various optical depths. Then the fourth Stokes parameter  $U$  is necessary and four partial differential equations have to be solved (Becker, 1969; Hardorp et al., 1976):

$$\mu \frac{dI}{d\tau} = \eta_I(I - B) + \eta_Q Q + \eta_V V, \quad (1)$$

$$\mu \frac{dQ}{d\tau} = \eta_Q(I - B) + \eta_I Q + \rho_R U, \quad (2)$$

$$\mu \frac{dU}{d\tau} = \rho_R Q + \eta_I U - \rho_W V, \quad (3)$$

$$\mu \frac{dV}{d\tau} = \eta_V(I - B) + \rho_W U + \eta_V V. \quad (4)$$

We used the Rosseland absorption coefficient  $\kappa_R$  to define the optical depth scale  $d\tau = -\kappa_R dz$ . With  $\vartheta$  being the angle between the  $z$ -direction (pointing to the observer) and the line of sight,  $\mu = \cos \vartheta$  can be defined.  $\eta_l$ ,  $\eta_Q$  and  $\eta_V$  are given by (Unno, 1956)

$$\eta_l = \frac{1}{2} \eta_p \sin^2 \psi + \frac{1}{4} (\eta_l + \eta_r) (1 + \cos^2 \psi), \quad (5)$$

$$\eta_Q = \frac{1}{2} \eta_p - \frac{1}{4} (\eta_l + \eta_r) \sin^2 \psi, \quad (6)$$

$$\eta_V = \frac{1}{2} (\eta_r - \eta_l) \cos \psi, \quad (7)$$

where  $\eta_l$ ,  $\eta_p$  and  $\eta_r$  are the monochromatic absorption coefficients for transitions with respectively  $\Delta m = -1, 0$  and  $1$ .  $\psi$  denotes the angle between the magnetic field direction and the line of sight. The meanings of the magneto-optical parameters  $\rho_R$  and  $\rho_W$  are described in the following section.

The direction of the  $x$ -axis, to which the angle of linear polarization refers, can be defined arbitrarily but for the integration of the local solutions over the stellar surface it is necessary to define a fixed coordinate system for the whole disk. When the local  $x$ -axis — which in our case is oriented with respect to the projection of the magnetic field on a plane perpendicular to the line of sight — differs from the global  $x$ -direction by an angle  $\phi$ , the vector consisting of the Stokes parameters  $Q$  and  $U$  is transformed by the rotation

$$\begin{pmatrix} Q' \\ U' \end{pmatrix} = \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} Q \\ U \end{pmatrix}. \quad (8)$$

### 3. The magneto-optical parameters

#### 3.1. The Faraday Effect

The Faraday effect has its origin in the fact that the refractive indices for left ( $n_-$ ) and right ( $n_+$ ) circularly polarized light are not equal in the presence of a magnetic field. For radiation with an angular frequency of  $\omega$ , the difference between the phase velocities  $c/n_+$  and  $c/n_-$  leads to a rotation angle of the electric field vector per unit path length  $l$  of (see e.g. O'Connell and Wallace, 1981)

$$\frac{d\theta}{dl} = \frac{\omega}{2c} (n_+ - n_-). \quad (9)$$

The magneto-optical parameter  $\rho_R$  that enters the equations of radiative transfer (1) – (2) can be defined as

$$\rho_R \equiv -2 \cos \psi \frac{d\theta}{\kappa_R dl}. \quad (10)$$

The quantities  $n_{\pm}$  are connected to the dielectric constants  $\epsilon_{\pm}$  as follows:

$$\epsilon_{\pm} \equiv \epsilon'_{\pm} + i\epsilon''_{\pm} = (n_{\pm} + ik_{\pm})^2. \quad (11)$$

$\kappa_{\pm}$  is the imaginary part of the refractive index. When  $\epsilon_{\pm}$  is known,  $n_{\pm}$  can be calculated from

$$n_{\pm}^2 = \sqrt{\frac{1}{2} \{ (\epsilon'_{\pm})^2 + (\epsilon''_{\pm})^2 \}^{1/2} + \epsilon'_{\pm}}. \quad (12)$$

The contribution of the spectral line components can be found with the help of the line dispersion function  $F$  (Wittmann, 1974; Martin and Wickramasinghe, 1981):

$$\rho_R = - \left( \sum_i \eta_{ri} F_{ri} - \sum_j \eta_{lj} F_{lj} \right) \cos \psi, \quad (13)$$

where the sum extends over all line components that contribute to the wavelength integrated absorption coefficients (divided by  $\kappa_R$ )  $\eta_l$  and  $\eta_r$ . Since we assume that the line components have Voigt profiles  $H(a, v)$ , the dispersion function can be expressed as following

$$F(a, v) = \frac{1}{a} \left( \frac{1}{2} H(a, v) + \frac{1}{4} \frac{\partial H(a, v)}{\partial v} \right). \quad (14)$$

The contribution of the continuum to  $\rho_R$  is obtained by solving the equation of motion for an electron in a static magnetic field  $B$  driven by an electromagnetic field  $\mathbf{E}_{\pm}$  (see e.g. Smith, 1967):

$$m_e \ddot{\mathbf{r}} + m_e \nu_e \dot{\mathbf{r}} + m_e \omega_p \mathbf{r} = -e \left( \mathbf{E}_{\pm} - \frac{\dot{\mathbf{r}}}{c} \times \mathbf{B} \right). \quad (15)$$

(The magnetic component  $H_{\pm}$  is much smaller than  $v/cE_{\pm}$  and can therefore be neglected.) This leads to the dielectric constants

$$\epsilon_{\pm} = 1 - \frac{\omega_p^2}{\omega(\omega \pm \omega_c + i\nu_e)}, \quad (16)$$

where  $\omega_p$  denotes the plasma frequency  $\sqrt{4\pi N_e e^2 / m_e}$ .  $N_e$  and  $\nu_e$  are the density and the collision frequency of the electrons. Now  $n_{\pm}$ ,  $d\theta/dl$  and  $\rho_R$  are easily calculated by inserting the real and imaginary parts of  $\epsilon_{\pm}$

$$\epsilon'_{\pm} = 1 - \frac{\omega_p^2 (\omega \pm \omega_c)}{\omega [(\omega \pm \omega_c)^2 + \nu_e^2]} \quad (17)$$

and

$$\epsilon''_{\pm} = \frac{\omega_p^2 \nu_e}{\omega [(\omega \pm \omega_c)^2 + \nu_e^2]} \quad (18)$$

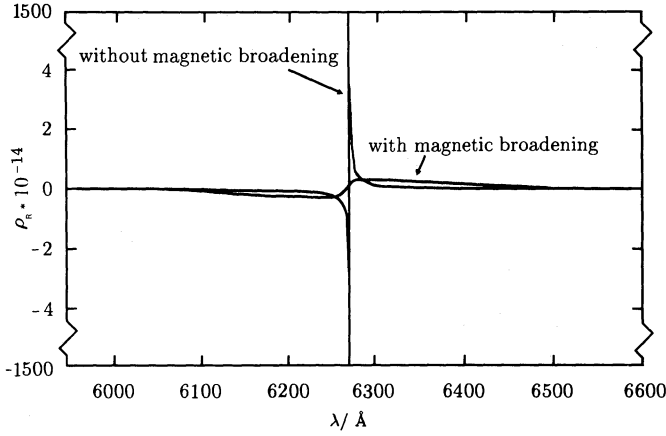
into equation (12), (9) and (10). The asymptotic approximation of Pacholczyk (1976) that has been used by Martin and Wickramasinghe (1982) follows in the limit  $\omega \gg \omega_c$ ,  $\omega \gg \omega_p$  and  $\nu_e = 0$ :

$$\rho_R = - \frac{\omega_p^2 \omega_c}{c \kappa_R (\omega^2 - \omega_c^2)} \cos \psi. \quad (19)$$

We use

$$\begin{aligned} \nu_e &= 7.751 \cdot 10^{15} \frac{P_e / (\text{g sec}^{-2} \text{cm}^{-1})}{(T/\text{K})^{5/2}} \\ &\times \left( -8.835 + 2 \ln \frac{T}{\text{K}} - \frac{1}{2} \frac{P_e}{(\text{g sec}^{-2} \text{cm}^{-1})} \right) \frac{1}{\text{sec}}. \end{aligned} \quad (20)$$

for the electron collision frequency.  $T$  denotes the temperature and  $P_e = N_e k T$  the electron pressure. The formula can be derived from Spitzer (1956) and is the same as the result of Ford and O'Connell (1980) except for a factor of  $\pi/4$ . Note that equation (20) is derived for free-space conditions and neglects the influence of the magnetic field on the motion of the



**Fig. 1.** The Faraday parameter  $\rho_R$  for a magnetic field of 170 MG at an optical depth  $\tau = 2/3$  in a model atmosphere with  $T_{\text{eff}} = 16000$  K and  $\log g = 8$ . In the curve with the sharp resonance at the cyclotron frequency (corresponding to a wavelength of 6270 Å)  $\rho_R$  is only broadened by electron collisions while the other curve shows the effect of the additional broadening by the variation of the magnetic field over a surface element (extending over a latitude difference of  $5^\circ$  in the spherical coordinate system of the star. Please notice the scale change which is necessary in order to show the extreme height of the resonance.)

electrons (one-dimensional mobility only along the field lines). But since Fig. 1 shows that the magnetic broadening dominates the collisional broadening by orders of magnitude we did not take the modification due to the magnetic field into account.

Even with collision broadening included the resonance of  $\rho_R$  at the cyclotron frequency is extremely sharp. This is demonstrated in Fig. 1 for a field of  $B = 170$  MG at an optical depth of  $\tau_R = 2/3$  in a  $T_{\text{eff}} = 16000$  K atmosphere. For a model calculation, in which the stellar surface is divided into a limited number of surface elements, the resulting profile is quite unrealistic, since the magnetic field and therefore the cyclotron frequency is not constant over a single area element. In a dipole field, where the strength of the magnetic field is a factor of two higher at the poles than at the equator, the field and cyclotron frequency variation is of the order of 10% on an element which extends over  $5^\circ$  in latitude.

Responsible for the sharp resonance are the terms

$$f(\omega) = \frac{1}{(\omega - \omega_c)^2 + v_e^2} \quad (21)$$

and

$$g(\omega) = \frac{\omega - \omega_c}{(\omega - \omega_c)^2 + v_e^2} \quad (22)$$

in equation (17) and (18). If  $\Delta B$  is defined as the standard deviation of the magnetic field over a single surface element and  $\Delta\omega_c = e\Delta B/2mc$ , we can substitute the fast varying terms  $f$  and  $g$  by the means

$$\begin{aligned} & \frac{1}{2 \cdot 5\Delta\omega_c} \int_{\omega_c - 5\Delta\omega_c}^{\omega_c + 5\Delta\omega_c} f(\omega) d\omega \\ & \approx \frac{1}{10\Delta\omega_c} \int_{-\infty}^{+\infty} \frac{d(\omega - \omega_c)}{(\omega - \omega_c)^2 + v_e^2} = \frac{\pi}{10\Delta\omega_c v_e} \end{aligned} \quad (23)$$

and

$$\begin{aligned} & \frac{\omega - \omega_c}{5\Delta\omega_c |\omega - \omega_c|} \int_{\omega_c}^{\omega_c + 5\Delta\omega_c} g(\omega) d\omega \\ & = \frac{(\omega - \omega_c)}{5\Delta\omega_c |\omega - \omega_c|} \int_0^{+5\Delta\omega_c} \frac{z dz}{z^2 + v_e^2} \\ & = \frac{(\omega - \omega_c)}{10\Delta\omega_c |\omega - \omega_c|} \ln \frac{25\Delta\omega_c^2 + v_e^2}{v_e^2} \end{aligned} \quad (24)$$

in the frequency interval  $(\omega_c - 5\Delta\omega_c, \omega_c + 5\Delta\omega_c)$ . The size of the interval is chosen in such a manner that a smooth behavior of  $\rho_R$  is reached. In equation (23) the limits can be replaced by  $-\infty$  and  $+\infty$  since practically only  $\omega$  values in the vicinity of  $\omega_c$  contributes to the integral. In Fig. 1 the resulting profile shows the situation with and without additional broadening by the magnetic field.

### 3.2. The Voigt Effect

The other magneto-optical effect that has to be taken into account is the Voigt effect. It arises from a phase shift between the linear polarized components of the electric field vector parallel and perpendicular to the magnetic field. The phase shift  $d\phi/d\tau$  and magneto-optical parameter  $\rho_W$  are connected to the refractive indices  $n_{\parallel}$  and  $n_{\perp}$  by:

$$\rho_W \equiv \frac{d\phi}{d\tau} \sin^2 \psi = -\frac{\omega}{2c\kappa_R} (n_{\parallel} - n_{\perp}) \sin^2 \psi. \quad (25)$$

The contribution of the lines is obtained analogous to equation (13) (Wittmann, 1974)

$$\begin{aligned} \rho_W = - & \left( \sum_k \eta_{pk} F_{pk} - \frac{1}{2} \sum_j \eta_{lj} F_{lj} \frac{1}{2} \sum_i \eta_{ri} F_{ri} \right) \\ & \times \sin^2 \psi, \end{aligned} \quad (26)$$

while the procedure to calculate  $\rho_W$  of the continuum is equivalent to that of  $\rho_R$ , if  $n_+$  is replaced by  $n_{\parallel}$  and  $n_-$  substituted by  $n_{\perp}$ . The real and imaginary parts of  $\epsilon_{\parallel/\perp}$  are

$$\epsilon'_{\parallel} = 1 - \frac{\omega_p^2}{\omega^2 - v_e^2}, \quad (27)$$

$$\begin{aligned} \epsilon'_{\perp} = & 1 \\ & - \frac{\omega_p^2 [(\omega_p^2 - \omega^2)(\omega_p^2 - \omega^2 + \omega_c^2) + \omega^2 v_e^2]}{\omega^2 (\omega_p^2 - \omega^2 + \omega_c^2 + v_e^2)^2 + v_e^2 (\omega_p^2 - 2\omega^2)^2} \end{aligned} \quad (28)$$

and

$$\epsilon''_{\parallel} = \frac{\omega_p^2 v_e}{\omega(\omega^2 - v_e^2)}, \quad (29)$$

$$\epsilon''_{\perp} = \frac{\omega_p^2 v_e [(\omega^2 (\omega_c^2 + v_e^2) + (\omega_p^2 - \omega^2)^2)]}{\omega [(\omega^2 (\omega_p^2 - \omega^2 + \omega_c^2 + v_e^2)^2 + v_e^2 (\omega_p^2 - 2\omega^2)^2)]}. \quad (30)$$

In the limit  $\omega \gg \omega_c$ ,  $\omega \gg \omega_p$  and  $v_e = 0$  we obtain the second formulae of Pacholczyk (1976):

$$\rho_W = -\frac{\omega_p^2 \omega_c^2}{2c\omega\kappa_R (\omega^2 - \omega_c^2)} \sin^2 \psi. \quad (31)$$

#### 4. The Model

In a new computer program for the calculation of synthetic spectra (Jordan, 1988) special attention is focused on magnetic fields  $\gtrsim 100$  MG. A brief description is given in Jordan (1989). Since the complete model will be presented in a forthcoming paper, only the basic assumptions and the improvements to the calculations of Martin and Wickramasinghe should be mentioned here.

The temperature and pressure structure is determined by zero field model atmospheres (see Koester et al., 1979). The radiative transfer equations are solved on a large number (20-100) of surface elements with an algorithm that is based on the one described in Martin and Wickramasinghe (1979). Subsequently the resulting Stokes parameters are integrated over all elements in order to get the emerging flux and polarization from the whole star.

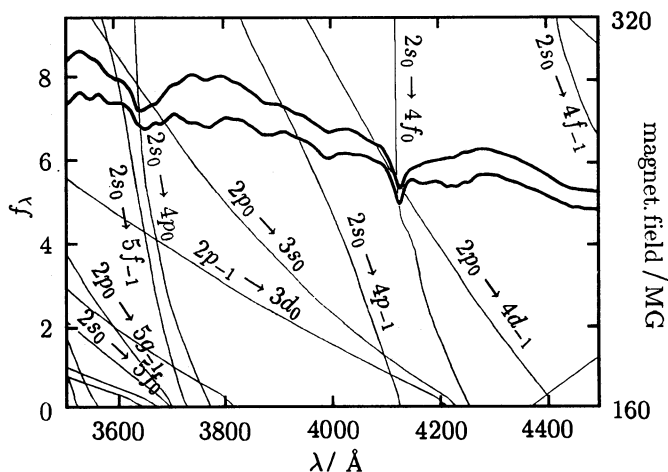
New tables of the Tübingen group (Wunner, 1986; Wunner et al., 1987) were applied for the line opacities. They represent a narrower data grid than all previously published tables (Rösner et al., 1984; Henry and O'Connell, 1984; Wunner et al., 1985) and can account for quick oscillations of the line strengths with the magnetic field.

The main improvement compared to the models of Martin and Wickramasinghe consists — beside the different treatment of the magneto-optical parameters — in the inclusion of the cyclotron absorption and a somewhat more realistic treatment of the bound-free opacities: the difference between the energies of the bound states and the Landau levels are used to get the correct position of absorption edges in the magnetic case. West (1989) uses a similar energy-level scheme for the photoionization, while previous models apply an algorithm by Lamb and Sutherland (1974), which cannot be extended to very high fields.

#### 5. Results

To demonstrate the effect of the magneto-optical parameters on the emerging spectrum and polarization, models were calculated with and without  $\rho_R$  and  $\rho_W$ . A detailed description of the computer program and the underlying physical assumptions will be given in a forthcoming paper. It should be mentioned here that — although correct wavelengths for the absorption edges are used in the new program — the main uncertainty for calculations in the high field region still consists in the treatment of the bound-free opacities (Jordan, 1989; Wickramasinghe and Ferrario, 1988). Up to now only very crude approximations exist and particularly the polarization reacts very sensitively to departures from the correct values of the continuum absorption. While comparisons of the synthetic spectra with observations of the high field object Grw + 70°8247 (ca. 320 MG) show that the flux distribution can be well reproduced (Jordan, 1989), the results for the polarization show serious discrepancies between the calculations and the observations.

Fig. 2 shows the theoretical flux distribution between 3400 Å and 4500 Å for a  $T_{\text{eff}} = 16000$  K hydrogen model atmosphere. The magnetic pole of the dipole field has a strength of 320 MG and is observed under a viewing angle of  $i = 30^\circ$ . The flux reduction by the lines, especially the  $\pi$ -components (corresponding to  $\Delta m = 0$  transitions), is significantly stronger under the influence of the Faraday and Voigt effects. For lower fields Martin and Wickramasinghe (1981 and 1982) have found a similar result.



**Fig. 2.** Comparison between a synthetic spectrum without (upper curve) and with (lower curve) magneto-optical effects taken into account. The polar field strength  $B_p$  of the dipole amounts to 320 MG and the viewing angle is  $i = 30^\circ$ .  $T_{\text{eff}}$  is 16000 K and  $\log g = 8.0$ . The zero point is the same for both spectra, which means that the reduction of the flux by the spectral lines is stronger with  $\rho_R$  and  $\rho_W$  taken into account. The position of the line components as a function of the magnetic field indicate which transitions contribute at a certain wavelength.

Here the superposition of the line components leads to a lower flux over the whole wavelength interval shown in Fig. 2.

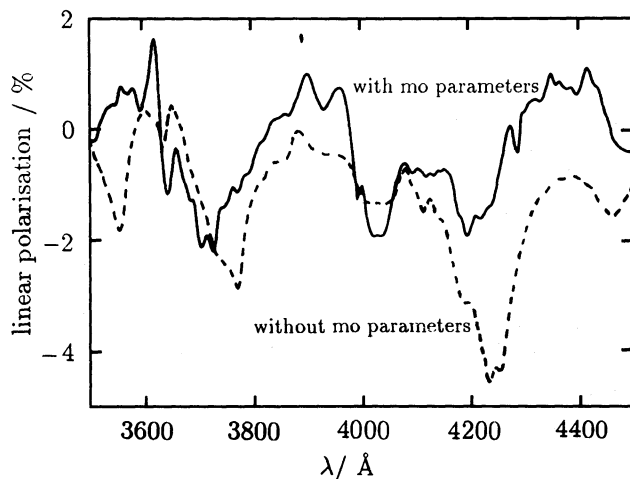
Martin and Wickramasinghe tried to give some qualitative arguments to explain the influence of  $\rho_R$  and  $\rho_W$  on the Stokes parameters by looking at the different terms in the radiative transfer equations. This may help to get a crude insight into the mechanisms, but the interaction between the Stokes parameters is complicated and can only be comprehended in detail by numerical calculations.

At most wavelengths the linear polarization is reduced if magnetic birefringence is taken into account (Fig. 3). This is not surprising since the Faraday effect rotates the plane of linear polarization. But, as Landstreet (1980) has pointed out, the Faraday rotation does not lead to a simple depolarization. For  $\cos \psi = 1$  (no linear polarization anyway) and  $\cos \psi = 0$  the magneto-optical parameters do not alter the Stokes parameters  $Q$  and  $U$  at all. Even repolarization is possible (Martin and Wickramasinghe, 1982) and the degree of polarization can be larger or smaller, depending on the wavelength, opacity, magnetic birefringence and orientation of the magnetic field.

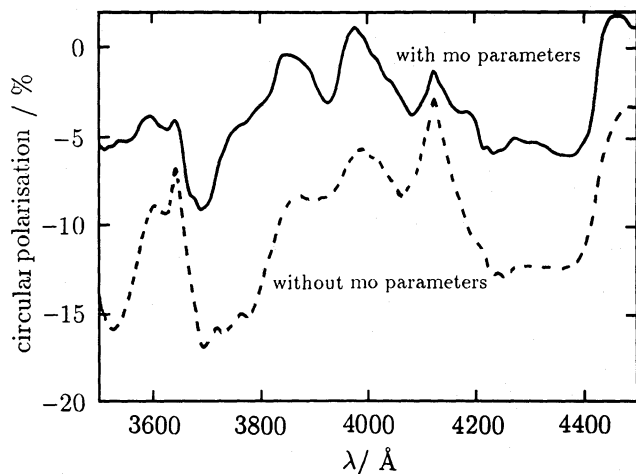
The linear polarization is very sensitive to variations of the angles  $\psi$ ,  $\theta$  and  $\phi$ . So the integration over a limited number of surface elements can lead to numerical inaccuracies and the results should be interpreted with caution. As our test calculations show, the circular polarization depends to a much lower degree on the number of surface elements.

In the neighborhood of the cyclotron frequency  $\rho_R$  and  $\rho_W$  are orders of magnitude larger compared to the situation when  $|\omega - \omega_c| \gg \omega_c$ . As a consequence the effect of the magnetic birefringence is more pronounced than in the low field region that was investigated by Martin and Wickramasinghe. Fig. 4 compares the result for the circular polarization with and without magneto-optical parameters.

In a forthcoming paper applications of the new computer program to the well known object Grw + 70°8247 will be shown and some improvements concerning the bound-free absorption



**Fig. 3.** The linear polarization predicted by a model with and without magneto-optical effects taken into account.  $B_p = 320$ ,  $i = 30^\circ$ ,  $T_{\text{eff}} = 16000$  K and  $\log g = 8$ .



**Fig. 4.** The degree of circular polarization in percent predicted by a model with  $B_p = 320$  MG,  $i = 30^\circ$ ,  $T_{\text{eff}} = 16000$  K and  $\log g = 8$ . The minimum of the polarization is reduced from -17 % to -8 % by the inclusion of magneto-optical effects.

of hydrogen will be discussed. Although the flux distribution can be reproduced much better now, our improvements cannot produce a better agreement between theory and observation for the polarization, until consistent calculations for the continuum opacities become available. This would be very important, since the exact configuration of the magnetic field — e.g. departures from a centered dipole — cannot be determined from the energy spectrum alone.

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