Electron-electron interactions in quasi-one-dimensional electron systems

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We present an analytical calculation for the Coulomb matrix elements in a quasi-one-dimensional system using a harmonic-confinement-potential model. We show that the intrasubband Coulomb interaction keeps the typical logarithmic divergent behavior of the strictly one-dimensional system in the long-wavelength limit, while the intersubband Coulomb interaction approaches discrete values depending on the relevant subbands. In applying our analytical form of the Coulomb matrix to study the intersubband collective excitations with a single-subband separation in the long-wavelength limit, we obtain analytical expressions for the intersubband plasmon frequencies and find that there are M different modes composing the collective excitations of a system with M populated subbands. The physical origin of these M modes is due to the different Fermi momenta of the electrons in different subbands. The maximum value of the intersubband plasmon frequency estimated by our theory is in good agreement with the existing experimental data.

Through advances in high-resolution electron lithography, some recent experiments have been directed at adding lateral confinement to the two-dimensional (2D) electron gas in either the silicon inversion layer or the GaAs heterostructures.[1-3] Because of the confinement, the electronic states in these systems are quantized (usually with several subbands being occupied simultaneously) in the lateral direction (taken to be along the y axis here). For each subband in the y direction, there is an allowed continuous distribution of momenta in the x direction. Denoting the subband energies by ε_n it is clear that the maximum energy associated with the x direction is $\varepsilon_F-\varepsilon_n\equiv k_{Fn}^2/2m^*$, where $\varepsilon_F=k_F^2/2m^*$ is the Fermi energy, and m^* is the effective mass of the electron. Since, of course, ε_F is the same regardless of the subband energy, it is clear that there are fewer electrons in the higher subbands (larger n values), and that the larger the n, the smaller the subband Fermi momentum k_{Fn}. Such systems are termed as quasi-one-dimensional (Q1D) systems, where many novel effects have been detected with exciting possibilities for device applications. Many efforts have been made toward understanding the physics of these Q1D systems both experimentally[1-3] and theoretically.[4-7] Among them, the study of the collective excitations and the Coulomb interactions are subjects of increasing interest.

It is known that the Coulomb interaction of the electrons in a strictly 1D system ("strictly" referring to the fact that there exists only one energy band) has a logarithmic divergent behavior in the long-wavelength limit ($q\rightarrow 0$).[4] For a Q1D system, due to the lateral quantization, the occupation of many subbands is possible, and extra terms such as the intersubband Coulomb interaction emerge. The form of the Coulomb interaction is quite complicated and is generally evaluated numerically both for the intrasubband and intersubband cases[4-7] (with the ground-state intrasubband Coulomb interaction as the only analytical case appearing in the literature). The large amount of numerical work involved in evaluating the Coulomb interaction not only constitutes an obstacle to setting up a practical transport theory for the Q1D system, but also gives no hint of the answer to some basic physical questions, such as the difference between the intrasubband and intersubband interactions. Therefore it is very desirable to find a simple form for the Q1D Coulomb potential matrix elements.

In this paper we present an analytical calculation for the Coulomb interaction in Q1D electron systems using the simplest possible, but practical, model. We show that the intrasubband Coulomb interaction keeps the typical logarithmic divergent behavior of the 1D system in the long-wavelength limit ($q\rightarrow 0$), while the *intersubband* Coulomb interaction approaches *discrete values* depending on the band index of the relevant subbands. This latter intriguing behavior of the intersubband Coulomb interaction was unnoticed in numerical studies of the same problem appearing in the literature. We also find that the general rule for the intersubband Coulomb interaction in the $q\rightarrow 0$ limit makes it possible to study plasmon excitations for a Q1D system with an unlimited number of subbands.

The model we use in our calculation is actually a widely used one in the literature,[1-7] where one assumes that the electrons are in a zero thickness xy plane with a harmonic confinement potential in the y direction (so that $\varepsilon_n=\hbar\omega_0(n+\frac{1}{2})$ where $n=0,1,2,\ldots$, and ω_0 is the characteristic frequency). Our main contribution here is an analytical evaluation of the known form of the Coulomb interaction matrix elements.

In the harmonic-confinement model, the one-electron wave function is

$$\psi(x,y,z)=\frac{e^{iqx}}{\sqrt{L}}\xi_n(y)\delta(z)\ , \tag{1}$$

where the δ function indicates that we neglect the z-direction motion of the electrons, and

$$\xi_n(y)=\left[\frac{1}{2^n n!\sqrt{\pi}b}\right]^{1/2} e^{-y^2/2b^2}H_n(y/b)\ . \tag{2}$$

Here b is the characteristic length of the harmonic potential defined by $b=\sqrt{\hbar/m^*\omega_0}$, and $H_n(x)$ is the Hermite polynomial. In the strictly 1D system only $n=0$ is considered but our interest extends to arbitrary n (Q1D systems).

Using (1) and (2), the matrix element of the Coulomb interaction takes the familiar form[7]

$$V_{\{N\}}(q)=\frac{2e^2}{\kappa}\int dy_1\int dy_2\,\xi_{n_1}(y_1)\xi^*_{n_2}(y_1)\times\xi^*_{n_3}(y_2)\xi_{n_4}(y_2)K_0(qy)\ , \tag{3}$$

where $y=|y_1-y_2|$, $\{N\}$ is a collective index representing $\{n_1,n_2,n_3,n_4\}$ and n_1,n_2,n_3,n_4 are the subband indices, $K_0(x)$ is the modified Bessel function of the second kind, and κ is the dielectric constant for the averaged background lattice. We find that (3) can be evaluated analytically. Details are given in Appendix A and the result is

$$V_{\{N\}}(q)=\begin{cases}\dfrac{e^2}{\kappa}E_{\{N\}}(qb), & N \text{ even} \quad (4a)\\ 0, & N \text{ odd} \quad (4b)\end{cases}$$

where $N=n_1+n_2+n_3+n_4$, and

$$E_{\{N\}}(qb)=\sum_{s=0}^{N/2}B_s^{\{N\}}\frac{\sqrt{2\pi}}{qb}e^{q^2b^2/4}W_{-s,0}(\tfrac{1}{2}q^2b^2)$$
$$=\sqrt{\pi}\sum_{s}^{N/2}B_s^{\{N\}}U(\tfrac{1}{2}+s;1;\tfrac{1}{2}q^2b^2)\ . \tag{4c}$$

Here $W_{\lambda\mu}(x)$ is the Whittaker function and $U(x,y,z)$ is Kummer's confluent hypergeometric function.[8] Also, $B_s^{\{N\}}$ is defined by (A9), and some values of a related coefficient $C_s^{\{N\}}$ [see (5b) below] are listed in Table I.

Equations (4a)–(4c) are general expressions for the Coulomb matrix elements of the Q1D electron system in the harmonic-confinement model. They are the key results of this communication. The vanishing of the $V_{\{N\}}(q)$ at odd N is actually a general property for any symmetric confinement potential, a fact first pointed out by Jain and Das Sarma,[5] it is actually a reflection of the fact that $\xi_n(y)$ has parity $(-1)^n$. For even N, a Coulomb matrix element is composed of the sum of $(N/2)+1$ terms.

In the following we give detailed calculations for the two most important special cases of (4) viz. the diagonal part of the Coulomb matrix elements for both the intrasubband ($n=n_1=n_2=n_3=n_4$) and the intersubband

TABLE I. Coefficients $C_s^{\{N\}}$ of the Coulomb matrix elements defined by Eqs. (4a), (4c), and (5), in the integral representation of the Whittaker function. Only part of the off-diagonal elements and only values of $n_i\le 3$ are listed.

		s						
	$C_s^{\{N\}}$	0	1	2	3	4	5	6
$\{N\}$	(00,00)	1						
	(11,11)	$\frac{3}{4}$	$-\frac{1}{2}$	$\frac{3}{4}$				
	(22,22)	$\frac{41}{64}$	$-\frac{13}{16}$	$2\frac{11}{32}$	$-2\frac{13}{16}$	$1\frac{41}{64}$		
	(33,33)	$\frac{147}{256}$	$-1\frac{7}{128}$	$4\frac{125}{256}$	$-10\frac{25}{64}$	$15\frac{45}{256}$	$-12\frac{39}{128}$	$4\frac{131}{256}$
	(00,11)	$\frac{1}{2}$	$\frac{1}{2}$					
	(00,22)	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{3}{8}$				
	(00,33)	$\frac{5}{16}$	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{5}{16}$			
	(11,22)	$\frac{7}{16}$	$\frac{9}{16}$	$-\frac{15}{16}$	$\frac{15}{16}$			
	(11,33)	$\frac{11}{32}$	$\frac{1}{4}$	$\frac{9}{16}$	$-1\frac{1}{4}$	$1\frac{3}{32}$		
	(22,33)	$\frac{51}{128}$	$\frac{79}{128}$	$-2\frac{13}{64}$	$4\frac{59}{64}$	$-5\frac{25}{128}$	$2\frac{59}{128}$	
	(01,01)	$\frac{1}{2}$	$-\frac{1}{2}$					
	(02,02)	$\frac{3}{8}$	$-\frac{3}{4}$	$\frac{3}{8}$				
	(03,03)	$\frac{5}{16}$	$-\frac{15}{16}$	$\frac{15}{16}$	$-\frac{5}{16}$			
	(12,12)	$\frac{7}{16}$	$-\frac{13}{16}$	$\frac{21}{16}$	$-\frac{15}{16}$			
	(13,13)	$\frac{11}{32}$	-1	$2\frac{1}{16}$	$-2\frac{1}{2}$	$1\frac{3}{32}$		
	(23,23)	$\frac{51}{128}$	$-1\frac{7}{128}$	$3\frac{15}{64}$	$-5\frac{55}{64}$	$5\frac{95}{128}$	$-2\frac{59}{128}$	
	(00,02)	$-\frac{1}{2\sqrt{2}}$	$\frac{1}{2\sqrt{2}}$					
	(00,13)	$-\frac{3}{4\sqrt{6}}$	0	$\frac{3}{4\sqrt{6}}$				
	(11,13)	$-\frac{3}{8\sqrt{6}}$	$\frac{9}{8\sqrt{6}}$	$-\frac{21}{8\sqrt{6}}$	$\frac{15}{8\sqrt{6}}$			
	(01,12)	$\frac{1}{4\sqrt{2}}$	$\frac{1}{2\sqrt{2}}$	$-\frac{3}{4\sqrt{2}}$				
	(01,23)	$\frac{\sqrt{3}}{16}$	$\frac{\sqrt{3}}{16}$	$\frac{3\sqrt{3}}{16}$	$-\frac{5\sqrt{3}}{16}$			
	(12,23)	$\frac{15}{32\sqrt{6}}$	$\frac{9}{8\sqrt{6}}$	$-\frac{63}{16\sqrt{6}}$	$\frac{45}{8\sqrt{6}}$	$-\frac{105}{32\sqrt{6}}$		

($n=n_1=n_3$, $n'=n_2=n_4$) interaction. Then, we analyze the plasmon spectrum based on our analytic expression (4), where we also discuss the off-diagonal terms.

First, the evaluation of (4c) is facilitated by using the integral representation of the Whittaker function, from which we have

$$E_{\{N\}}(qb)=\int_0^\infty dy \frac{e^{-y/2}}{[y(y+b^2q^2)]^{1/2}} \times \sum_{s=0}^{N/2} C_s^{\{N\}} \left[\frac{y}{y+b^2q^2}\right]^s , \tag{5a}$$

where

$$C_s^{\{N\}}=B_s^{\{N\}}\frac{2^{2s-1}(s-1)!}{(2s-1)!} . \tag{5b}$$

The coefficients $B_s^{\{N\}}$ and $C_s^{\{N\}}$ in (4c) and (5a) can be directly evaluated by using the complete expressions for $B_s^{\{N\}}$ of (A9) and (5b), for any value of the subband index. In Table I, we have listed their values for $n_1,n_2,n_3,n_4\le 3$. From that table, one observes that there are some strict rules governing these coefficients $B_s^{\{N\}}$ and $C_s^{\{N\}}$. For example, one has

$$\sum_{s=0}^{2n} C_s^{nnnn}=1, \quad \sum_{s=0}^{n+n'} C_s^{nnn'n'}=1 , \tag{6a}$$

and

$$\sum_{s=0}^{n+n'} C_s^{nn'nn'}=0 . \tag{6b}$$

The above relations also follow from the definitions of $B_s^{\{N\}}$ and $C_s^{\{N\}}$. An immediate application of these relations is an analysis of the $q\to 0$ behavior for $E_n(bq)\equiv E_{nnnn}(bq)$ using (5a). We recall that for the ground state ($n=0$), $C_0^{\{0\}}=1$, and (5a) reduces to the well-known result[4]

$$E_0(qb)=e^{b^2q^2/4}K_0(q^2b^2/4) ,$$

and

$$E_0(bq\to 0)\to -\ln(bq) . \tag{7a}$$

Now, it follows from (5a), (6a), and (7a) that for the nth *intrasubband* Coulomb matrix $E_n(bq)$, as $q\to 0$ one has the same logarithmic divergent behavior as for the $n=0$ case:

$$E_n(bq\to 0)\to -\ln(bq) . \tag{7b}$$

On the other hand, (6b) implies that the *intersubband* Coulomb matrix should have different long-wavelength behavior as compared to the intrasubband case. This is because as $q\to 0$, $[y/(y+b^2q^2)]^s\to 1$, and the sum over s in (5a) will contribute a vanishing factor which makes the Coulomb matrix no longer divergent, and consequently

$$E_{nn'}(bq\to 0)\to \beta_{nn'} , \tag{8}$$

where $\beta_{nn'}$ is a constant depending on the values of n and n', and $E_{nn'}$ stands for $E_{nn'nn'}$.

The finite value of the intersubband Coulomb matrix in the long-wavelength limit is actually the source of the well-known depolarization effect of the collective excitation, a point which we will fully explore later in our discussion. In the experiments which study the collective excitation of the Q1D system, usually many subbands are involved, so it is important to know the structure of the Coulomb matrix. It turns out that (8) is also crucial in determining the off-diagonal elements. Here we present the full expressions of $E_{01}(bq)$ and $E_{02}(bq)$ as two examples. From Table I and (5a), after some algebra we have

$$E_{01}(bq)=\tfrac{1}{4}b^2q^2e^{b^2q^2/4}[K_1(\tfrac{1}{4}b^2q^2)-K_0(\tfrac{1}{4}b^2q^2)] , \tag{9a}$$

$$E_{02}(bq)=\tfrac{1}{8}b^4q^4e^{b^2q^2/4}\left[\left[\frac{1}{b^2q^2}-1\right]K_1(\tfrac{1}{4}b^2q^2)+\tfrac{1}{2}K_0(\tfrac{1}{4}b^2q^2)\right] . \tag{9b}$$

We note that (9a) and (9b) can also be obtained from (3) by directly using the definitions of $H_0(x)$, $H_1(x)$, and $H_2(x)$. From (9a) and (9b), it is easy to obtain

$$E_{01}(bq\to 0)=1 , \tag{10a}$$

$$E_{02}(bq\to 0)=\tfrac{1}{2} . \tag{10b}$$

Equations (10a) and (10b) show that the diagonal part of the intersubband Coulomb matrix consists of discrete values. We end our discussion of the behavior of $E_n(bq)$ and $E_{nn'}(bq)$ by mentioning that in the short-wavelength limit ($q\to\infty$), they both keep the $1/q$ behavior, as can be observed easily from (5a). The $1/q$ behavior is known to be a property of the Coulomb interaction of 2D systems.[9]

The overall properties of $E_n(bq)$ and $E_{nn'}(bq)$ can be studied numerically by using (5a) and Table I. In Fig. 1 we have plotted $E_n(bq)$ and $E_{nn'}(bq)$ for $n,n'\le 3$, as a function of bq. The $bq\to 0$ and $bq\to\infty$ behavior of these functions discussed above are clearly demonstrated in the figure. Also one observes that the magnitude of $E_{nn'}(bq)$ is smaller than that of $E_n(bq)$.

Next, we analyze the collective excitation of the Q1D system as an application of our analytic expression (4). The emphasis here will be a rigorous discussion of the depolarization effect of the collective excitation in the $q\to 0$ limit for a Q1D system with an unlimited number of bands.

In a Q1D system, the dielectric function takes the form of a matrix. The generalized dielectric function is given by[7,9]

$$\varepsilon_{ij,lm}(q,\omega)=\delta_{il}\delta_{jm}-\chi_{ij}(q\omega)V_{ij,lm}(q) , \tag{11}$$

where the Coulomb matrix $V_{ij,lm}(q)$ is defined by (4), and $\chi_{ij}(q\omega)$ is the generalized polarizability function. In the Bohm-Pines random-phase approximation (RPA), the $\chi_{ij}(q\omega)$ in (11) is replaced by the noninteracting polarizability $\chi_{ij}^0(q\omega)$. In addition, we restrict our discussion to $T=0$, for which[10]

$$\chi_{ij}^0(q,\omega)=\frac{2k_{Fi}q^2}{m^*\pi\omega^2}+O(q^4), \quad i=j \tag{12}$$

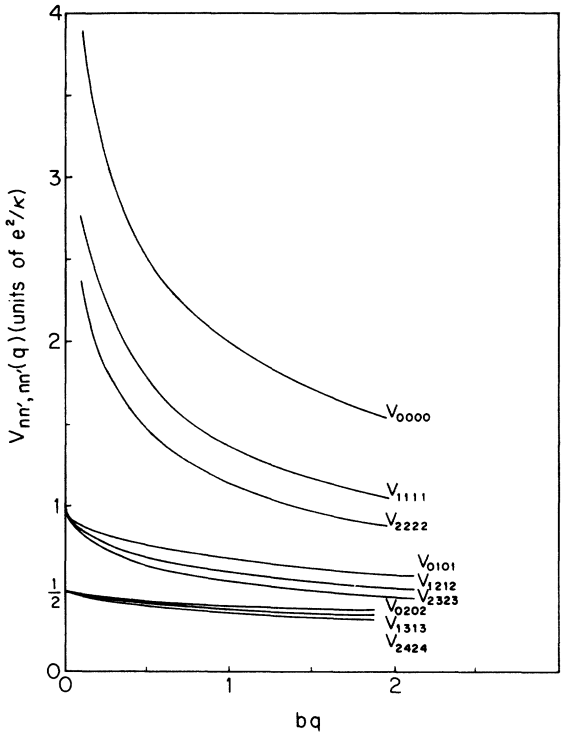

FIG. 1. Diagonal part of the Coulomb matrix elements $V_{nn',nn'}(q)$ (in units of e^2/κ) as a function of q (in units of b^{-1}) for a Q1D system in the harmonic confinement potential model, where $b=\sqrt{\hbar/m^*\omega_0}$ is the characteristic length of the harmonic potential.

$$\chi^0_{ij}(q,\omega)=\frac{(k_{F,i}-k_{F,j})/\pi}{\omega-\omega_{ji}}+O(q^2),\quad i\neq j \tag{13}$$

where $\omega_{ji}=(j-i)\hbar\omega_0$ is the separation between the jth and ith subbands and $k_{F,i}$ is the Fermi momentum of the ith subband.

The plasmon spectrum is determined by the condition (following the convention used for the subband indices)

$$\det(\varepsilon_{ij,lm})=0,\quad i,j,l,m=0,1,2,\ldots,M-1 \tag{14}$$

which has a dimension of $M^2\times M^2$. In the literature, (14) is solved numerically by using a few-bands approximation,[6,7] since the matrix elements of (14) appearing in those studies are quite complicated. The analytical derivation of the Coulomb matrix elements (4) makes it possible to solve (14) without the few-bands restriction. This is the subject of the following discussion.

Applying our general expression (4) to (14), one observes directly that the matrix elements in (14) can be regrouped into two decoupled (which we shall call the even and odd) blocks. The even block contains all those matrix elements with $i+j$ and $l+m$ being even numbers. In other words, the even block has all the M intrasubband terms as its diagonal part plus those intersubband terms with even number separations for the related subbands, and the odd block has all the intersubband terms with odd number separations for the related subbands. Also, the dimensionality of the even and odd blocks are the same, except that when M is odd, the even block is one dimension larger than that of the odd block. Because of this decoupling, the evaluation of (14) is dramatically simplified. We note that in the numerical study of (14) in the few-band approximation,[7] the above decoupling rule emerged.

The other implication of (4) to (14) is that it is now possible to solve (14) rigorously for the many-subbands situation, as all the matrix elements are in a relatively simple form. As an example, we solve (14) in the $bq\to 0$ limit in the following. Our main concern here is to take a close look at the depolarization effect on the one-level collective excitation, i.e., electron transitions from the ith to the $(i-1)$th or to the $(i+1)$th subbands. The diagonal elements in (14) directly involved in the one-level collective excitations are obviously contained in the odd block discussed before. Thus our interest here is to solve the one-level subblock contained in the odd block of (14).

Using (4) and (11)–(13), in the $T=0$ K and $bq\to 0$ limit, all the matrix elements in the one-level excitation subblock of (14) can be worked out after a considerable amount of algebra. The details are given in Appendix B, and the one-level excitation subblock of (14) for a system with M occupied subbands in the $T=0$ K and $bq\to 0$ limit can be explicitly written as

$$\det\begin{bmatrix} D_{12}(\gamma) & O_{12}^{23} & \cdots & O_{12}^{M,M+1} \\ O_{23}^{12} & D_{23}(\gamma) & \cdots & O_{23}^{M,M+1} \\ \vdots & \vdots & & \vdots \\ O_{M,M+1}^{12} & O_{M,M+1}^{23} & \cdots & D_{M,M+1}(\gamma) \end{bmatrix}=0\ , \tag{15}$$

where $\gamma=\omega/\omega_0$, and we have introduced symbols for the 2×2 matrices

$$D_{i,i+1}(\gamma)=\begin{bmatrix} \gamma^2-1-(\gamma+1)C_{i-1} & -(\gamma+1)C_{i-1} \\ (\gamma-1)C_{i-1} & \gamma^2-1+(\gamma-1)C_{i-1} \end{bmatrix}, \tag{16}$$

$$O_{i,i+1}^{l,l+1}=\frac{C_{i-1}}{(l-1)!(i-1)!\sqrt{li}}\begin{bmatrix} -1-\gamma & -1-\gamma \\ -1+\gamma & -1+\gamma \end{bmatrix}, \tag{17}$$

with

$$C_i=2b^2(k_{Fi}-k_{Fi+1})/\pi a_B^*\ , \tag{18}$$

where $a_B^*=\kappa\hbar/m^*e^2$.

A few comments are in order. First, when all the C's contained in (15) are taken to be zero, (15) is diagonalized and one has the single-particle transition solution $\gamma=1$ ($\omega=\omega_0$). Physically, this corresponds to taking $b\to 0$ ($\omega_0\to\infty$), the extreme large-subband separation case. Secondly, one notices that the most effective off-diagonal elements in (15) are those C_i and C_{i+1} contained in the $D_{i,i+1}(\gamma)$ of (16), as the others in the $O_{i,i+1}^{l,l+1}$ of (17) are a factor of $[(l-1)!(i-1)!\sqrt{li}\,]$ smaller than those C_i and C_{i+1}. Therefore, in the first-order approximation, one can neglect all the $O_{i,i+1}^{l,l+1}$, and (15) reduces to

$$\det D_{12}(\gamma)\det D_{23}(\gamma)\cdots\det D_{M,M+1}(\gamma)=0\ . \tag{19}$$

Substituting (16) and (18) into (19), we obtain solutions

for the collective excitations

$$\gamma_i^2=1+\frac{4b^2}{\pi a_B^*}(k_{F,i}-k_{F,i+1}),\quad i=0,1,2,\ldots,M-1\ . \tag{20}$$

Equation (20) tells us that there are M different modes for the collective excitations of a single subband separation of a Q1D system with M occupied subbands. Each of these modes has a plasmon frequency shifted from the single-particle value $\gamma=1$ by a factor depending on the related Fermi momentum, the characteristic length b of the harmonic potential model, and the intrinsic properties a_B^* of the system. This is the so-called *depolarization effect,* known from the study of the subband effect in the two-dimensional systems.[9] The difference here is that numerous populated subbands are involved, and we find that many modes exist. Equation (20) shows that each mode contributes a different amount to the depolarization effect, and the largest contribution comes from the $i=M-1$ case, assuming that the $M-1$ band is nearly full, where the $k_{F,i+1}$ in (20) vanishes as the Mth subband is empty. The general rule for the other $(M-1)$ modes is that the higher the index of the subband, the larger the frequency shift it has. The largest value of the plasmon frequency can be obtained from (20) at $i=M-1$ as

$$\omega_p^M=\omega_0\left[1+\frac{4}{\pi}\frac{b^2k_{F,M-1}}{a_B^*}\right]^{1/2}\ . \tag{21}$$

Using the data from the experiments of Damel *et al.*,[2] one deduces that $a_B^*=104$ Å, $\omega_0=1$ meV, $b=329$ Å, $k_Fb=5.9$, $k_{F,M-1}/k_F=0.2$, so that (21) gives $\omega_p^M=2.4$ meV. This number is in reasonable agreement with the experimental results of Demel *et al.*, who found a plasmon peak around $\omega=3.4$ meV. A better agreement with the experiment can be reached for the following reasons: (i) When the O matrix is included, which represents the plasmon-plasmon coupling in (15), one can qualitatively argue that the estimate of ω_p^M will increase as compared to the values given by (21), while the basic picture is unchanged. (ii) The above-deduced values of ω_0 and k_F, etc. from the experiments have some uncertainties. If the ω_0 has a larger value than the 1 meV used above, then the ω_p^M in (21) will also increase. Finally, the theoretical picture presented here also implies that the one-level collective excitation should have a fine structure, as it contains M different modes for a system with M occupied subbands. We hope that our theory stimulates more experimental interest concerning the subband structure.

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APPENDIX A: THE DERIVATION OF THE COULOMB MATRIX ELEMENTS (4a)–(4c)

Combining (2) and (3), we have

$$V_{\{N\}}(q)=\frac{2e^2}{\pi\kappa}\left[\frac{2^{-N}}{n_1!n_2!n_3!n_4!}\right]^{1/2}\int dx_1\int dx_2\,e^{-(x_1^2+x_2^2)}K_0(qb\bar{x})H_{n_1}(x_1)H_{n_2}(x_1)H_{n_3}(x_2)H_{n_4}(x_2)\ , \tag{A1}$$

where $x_i=y_i/b$, $\bar{x}=|x_1-x_2|$. Also $\{N\}$ is a collective index representing $\{n_1,n_2,n_3,n_4\}$, and $N=n_1+n_2+n_3+n_4$.

To evaluate (A1), we first transform it into the center-of-mass coordinates $x=(x_1-x_2)/\sqrt{2}$, $x'=(x_1+x_2)/\sqrt{2}$, so that

$$V_{\{N\}}(q)=\begin{cases}\dfrac{e^2}{\kappa}\displaystyle\int_0^\infty dx\,e^{-x^2}K_0(\sqrt{2}qbx)I_{\{N\}}(x) & \text{when } I_{\{N\}}(x)=I_{\{N\}}(-x) \quad \text{(A2)}\\ 0 \quad \text{when } I_{\{N\}}(x)=-I_{\{N\}}(-x)\ , & \text{(A3)}\end{cases}$$

where

$$I_{\{N\}}(x)=\frac{4}{\pi}\frac{2^{-N}}{n_1!n_1!n_3!n_4!}\int dx'\,e^{-(x')^2}H_{n_1}\left[\frac{x'+x}{\sqrt{2}}\right]H_{n_2}\left[\frac{x'+x}{\sqrt{2}}\right]H_{n_3}\left[\frac{x'-x}{\sqrt{2}}\right]H_{n_4}\left[\frac{x'-x}{\sqrt{2}}\right]\ . \tag{A4}$$

We note that, in writing (A2) and (A3), we have used the fact that $I_{\{N\}}(x)$ is either an even or odd function of x, a fact easily observed from (A4).

Our main task now consists in the evaluation of (A4) and (A2). After some lengthy algebra, we found (A4) can be evaluated analytically, and the result is

$$I_{\{N\}}(x)=\sum_{l=0}^{N_1}\sum_{l'=0}^{N_2}\sum_{k=0}^{n_1+n_2-l}\sum_{m=0}^{M}\sum_{j=0}^{M-m}(-1)^{j-k}\frac{(2M-2m)!}{j!(2M-2m-2j)!}A_{ll'km}^{\{N\}}(2x)^{2M-2m-2j}\ ,\quad N \text{ even} \tag{A5}$$

$$I_{\{N\}}(x)=-I_{\{N\}}(-x),\quad N \text{ odd}\ , \tag{A6}$$

where

$$N_1=\left\lfloor\frac{n_1+n_2}{2}\right\rfloor,\quad N_2=\left\lfloor\frac{n_3+n_4}{2}\right\rfloor,\quad M=\frac{N}{2}-l'-l-k\ ,$$
$$A_{ll'km}^{\{N\}}=\frac{2^{2l+2l'+k+m-N}\,l!\,l'!\,k!\,m!}{(n_1!\,n_2!\,n_3!\,n_4!)^{1/2}}C_{n_1}^{l}C_{n_2}^{l}C_{n_3}^{l'}C_{n_4}^{l'}C_{n_1+n_2-2l}^{k}C_{n_3+n_4-2l'}^{m}C_{n_1+n_2-2l-k}^{m}C_{n_3+n_4-2l'-k}^{k}\ , \tag{A7}$$

and the floor function $\lfloor\ \rfloor$ denotes the largest integer contained in the relevant expression. Substituting (A5) into (A2), the integral of (A2) can be carried out using the formula[8]

$$\int_0^\infty dx\,e^{-x^2}K_0(cx)x^{2s}=\frac{1}{2c}\Gamma^2(s+\tfrac{1}{2})e^{c^2/8}W_{-s,0}(\tfrac{1}{4}c^2)\ , \tag{A8}$$

where K_0 and $W_{\lambda\mu}$ are the modified Bessel and Whittaker functions, respectively. The end result for $V_{\{N\}}(q)$ as given in (A1) is (4a)–(4c) in the text, with

$$B_s^{\{N\}}=\sum_{l,l',k,m}\sum_{j=0}^{M-m}(-1)^{n_1+n_2+j-k}\frac{(2M-2m)!\,[(2s-1)!!]^2}{j!\,2s!}A_{ll'km}^{\{N\}}\ , \tag{A9}$$

where the ranges of the sum over l,l',k,m are the same as that of (A5), and $s=(N/2)-l-l'-k-m-j$.

APPENDIX B: THE DERIVATION OF THE DIELECTRIC MATRIX (15) FOR THE ONE-LEVEL COLLECTIVE EXCITATION

First, the dielectric matrix elements which consists of (15) are given by those terms of (11) with $j=i\pm1$ and $m=l\pm1$. Explicitly, they are

$$\varepsilon_{i,i\pm l;l,l+\eta}(q,\omega)=\delta_{il}\delta_{i\pm1,l+\eta}-\chi^0_{i,i\pm1}(q,\omega)V_{i,i\pm1;l,l+\eta}(q)\ , \tag{B1}$$

where $\eta=1$ or -1. Also, we have used RPA approximation and $\chi^0_{ij}(q\to0,\omega)$ is defined by (13). The central task is to evaluate the $q\to0$ Coulomb matrix $V(q)$ appeared in (B1). When $n_1,n_2,n_3,n_4\le3$, by using Table I, (4a), (5a), and (5b), one can show that the $q\to0$ two-subband Coulomb matrix element

$$V_{ij,ij}(q\to0)\equiv V_{ij,ji}(q\to0)=\frac{e^2}{\kappa}\frac{1}{|i-j|}\ , \tag{B2}$$

and the multisubband Coulomb matrix element ($ijlm$ takes at least three different indices) is

$$V_{i,i+1;l,l+1}(q\to0)\equiv V_{i,i+1;l+1,l}(q\to0)=\frac{e^2}{\kappa}\frac{1}{l!\,i!\sqrt{(l+1)(i+1)}}\ . \tag{B3}$$

We note that (10a) and (10b) are two examples of (B2). One can also numerically check that (B2) and (B3) is generally true for those Coulomb matrix elements having high subband indices.

Next, substituting (B2), (B3), and (13) into (B1), one obtains the two-subband $q\to0$ dielectric matrix function

$$\varepsilon_{i,i+1;i,i+1}(q\to0,\omega)=1-\frac{C_i}{\gamma-1}+O(q^2)\ , \tag{B4}$$
$$\varepsilon_{i+1,i;i+1,i}(q\to0,\omega)=1+\frac{C_i}{\gamma+1}+O(q^2)\ , \tag{B5}$$
$$\varepsilon_{i,i+1;i+1,i}(q\to0,\omega)=-\frac{C_i}{\gamma-1}+O(q^2)\ , \tag{B6}$$
$$\varepsilon_{i+1,i;i,i+1}(q\to0,\omega)=\frac{C_i}{\gamma+1}+O(q^2)\ , \tag{B7}$$

where $\gamma=\omega/\omega_0$ and C_i is defined by (18). Similarly, the $q\to0$ multisubband dielectric function is obtained as

$$\varepsilon_{i,i+1;l,l+1}(q\to0,\omega)=\frac{-C_i}{l!\,i!\sqrt{(l+1)(i+1)}}\frac{1}{\gamma-1}+O(q^2)\ , \tag{B8}$$
$$\varepsilon_{i+1,i;l,l+1}(q\to0,\omega)=\frac{C_i}{l!\,i!\sqrt{(l+1)(i+1)}}\frac{1}{\gamma+1}+O(q^2)\ . \tag{B9}$$

Finally, combining (14), (B1), and (B4)–(B9), and after multiplying by a factor γ^2-1, we obtain (15), where the submatrix D originates from (B4)–(B7), while the submatrix O originates from (B8) and (B9).

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[10]Similar expressions can be found in Ref. 7, where k_{Fl} in our Eqs. (12) and (13) is replaced by k_F, which corresponds to $l=1$ in our case.