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ELECTRIC FIELD EFFECT ON WEAK LOCALIZATION IN A SEMICONDUCTOR QUANTUM WIRE

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ABSTRACT

The influence of an electric field on weak localization in a semiconductor quantum wire is studied by a recently proposed generalized quantum Langevin equation approach to the conductivity problem. A new physical picture is presented. In our model the electronic motion is essentially one-dimensional, and the phase coherence length ℓ_ϕ is much larger than the elastic mean free path ℓ of electrons. We find that when the electric field E exceeds a critical value $E_c = \hbar V_F/e\ell_\phi^2$, where V_F is the Fermi velocity, it will introduce a new cut-off length $L_s = (E_c/E)^{1/2}\ell_\phi$ with implications for the experimental results on semiconductor quantum wires. Our theory is in good agreement with the experiments of Hiramoto and co-workers.

KEYWORDS

Weak localization; semiconductor quantum wire; high electric field effect; phase coherence length; coherent back scattering.

I. INTRODUCTION

The study of the influence of an electric field on weak localization has been quite controversial. Altshuler, Aronov and Khmel'nitsky (1981, 1982) predicted that a dc electric field does not break time-reversal invariance and, as long as the temperature is constant, the electric field has no influence on weak localization (Lee and Ramakrishnan, 1985). By contrast, Tsuzuki's study (1981) showed a reduction of the quantum correction to the conductivity in an electric field, and Mott and Kaveh (1981) found a critical length $L_D(\sim E^{-1/3})$, which diminishes weak localization as soon as $L_D < L_T$, the Thouless length ($L_T = (D\tau_\phi)^{1/2}$, where D and τ_ϕ denotes the diffusion constant and phase coherent time, respectively).

The purpose of this paper is to demonstrate that in a semiconductor quantum wire system, where the electronic motion is essentially one-dimensional and the phase coherence length ℓ_ϕ (in the low field limit) is much larger than the mean free path ℓ of the electrons, there is strong evidence supporting the viewpoint that the electric field does affect weak localization. We show that when the electric field E applied to the semiconductor quantum wire exceeds a critical value $E_c = \hbar V_F/e\ell_\phi^2$, it will introduce a new cut-off length (similar to the idea of Mott and Kaveh) $L_s = (E_c/E)^{1/2}\ell_\phi$ which is in good agreement with the experiments of Hiramoto and co-workers (1989).

In Sect. II, we present a new physical picture of weak localization in the semiconductor quantum wire. Based on the fact that the electron motion is essentially one-dimensional, we show that the coherent back scattering process is better described by a sudden reversal picture (as distinct from the diffusive picture). In Sect. III we implement the physical idea by a theoretical formulation based on the recently developed generalized quantum Langevin equation approach (Hu and O'Connell, 1987, 1988). In Sect. IV we show that our sudden reversal picture also leads to a cut-off length (different from that of Mott and Kaveh) and we compare our theoretical results with experiments. Our results are summarized in Sect. V.

II. PHYSICAL PICTURE

Weak localization is a quantum effect (Lee and Ramakrishnan, 1985) caused by coherent back scattering (CBS), where an electron with initial momentum $\vec{k}$ is finally scattered into the opposite state $-\vec{k}$ elastically. According to the diffusive picture, in real systems the CBS is realized through coherent scattering sequences (fan diagram), where an electron of Fermi momentum k_F moves in a diffusive way such that its momentum gradually changes to $-k_F+q$ (with $q/k_F << 1$). The average distance (the phase coherent length L_ϕ) over which the electron diffuses during these sequences, is estimated to be $\sqrt{D\tau_\phi}$, where D is the diffusion constant and τ_ϕ is the average time for a CBS process. This diffusive description of the electron motion serves as the basis of almost all the theoretical treatments of the quantum correction to the conductivity in the metallic regime.

While the diffusive picture for CBS is illuminating and correct for most of the weakly localized systems studied, it does not rule out other possible ways for electrons to achieve CBS in some unusual systems. The strong localization of the strictly one-dimensional (1D) system is one example, where the CBS process happens one dimensionally at a length scale of the mean free path $\ell = V_F \tau$, where V_F is the Fermi velocity and τ the momentum relaxation time. Here we propose another (which we shall call sudden reversal) picture for the CBS of electrons where the CBS process basically exhibits a 1D behavior but at a length scale much larger than ℓ. As in the diffusive picture, the CBS is realized by the coherent scattering sequence which has a total momentum transfer of $-2k_F+q$ ($q/k_F <<1$) for the electron. The difference is that instead of diffusing elastically through many different states gradually to achieve the CBS (as in the diffusive picture), the electrons are now assumed to be scattered by impurities only into two kinds of states. One is a small momentum transfer forward process which essentially does not change the velocity of electrons, the other is a large momentum transfer ($\sim 2k_F$) process which makes the electron move essentially in the reversed direction. In addition, the assumption that the system is weakly disordered makes the probability of the reversal scattering much less then the forward scattering. (The opposite case, i.e., when the reversal scattering dominates, corresponds to the strictly 1D case). In this way an electron will experience many forward scatterings with little change in its original speed. Eventually it will experience a reversal scattering. This is illustrated schematically in Fig. 1. Thus in our picture an electron will travel a distance $L_\phi \sim V_F \tau_\phi$ in a CBS process, as distinct from the result $L_\phi \sim \sqrt{D\tau_\phi}$ in the diffusive picture. Also, the L_ϕ in our picture is much larger than ℓ, in contrast to the 1D case (no lateral dimension of freedom) where $L_\phi \sim \ell$. These features of the sudden reversal picture will play an important role when we discuss later in Sect. IV the new cut-off length (due to the electric field).

From what we have described above, the sudden reversal picture of CBS proposed here should be applicable to many of the semiconductor quantum wires recently available through advances in microfabrication technology. The width of these thin wires is comparable to the Fermi wave length ($\sim 10^3$ Å), which makes the motion of electrons in them basically one-dimensional in a quantum mechanical way. On the other hand, the presence of a finite cross section also makes them totally different from the 1D system. Physically, due to the relatively large value of the Fermi wave length of the semiconductor, the dilute impurities in the quantum wire can not individually block the way of the moving electrons and hence ensures that the reversal scattering has a small probability (roughly proportional to the ratio of the size of the impurity to the width of the wire). At the same time, the lateral quantization of the sample restricts the motion of the electrons essentially in a 1D fashion and thus makes the other possible way of impurity scattering, the forward scattering, the dominant process.

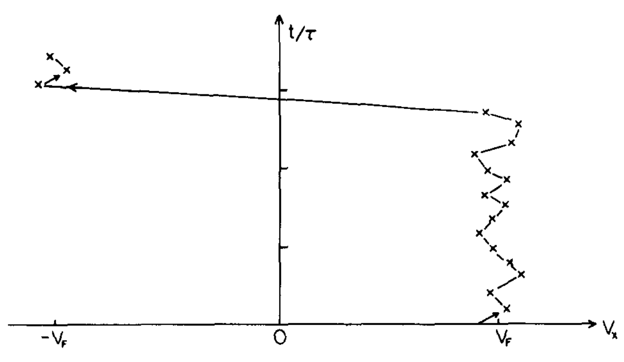

Fig. 1. Schematic picture of the velocity (V_x) evolution (time in units of the momentum relaxation time τ) of an electron in a quantum wire.

III. FORMULATION

Next, we implement the physical idea of the sudden reversal picture of CBS into a quantitative evaluation of the quantum corrections to the electric conductivity of the quantum wire (with a width of the order of the Fermi wave length and containing dilute impurities). For a simple discussion of the problem, the electron-electron interaction will be neglected in this paper.

Our calculation scheme is a generalized Langevin equation (GLE) approach, which we have developed in a series of papers (Hu and O'Connell, 1987, 1988, 1989). The main approximation involved is to assume that N>>1, where N is the total number of electrons in the system, which is certainly good for a realistic semiconductor quantum wire obtained from a two dimensional system. For instance, $N \sim 10^3$ for a system of $1\times0.1\ \mu m^2$ with surface density $n_s \sim 10^{12}\ cm^{-2}$. For our present purpose of obtaining the conductivity with high electric field and the CBS corrections for the semiconductor quantum wire, there are two main steps in the calculations. First, we recall that the static conductivity $\sigma(V_d)$ which includes the CBS contribution and high electric field, is (Hu and O'Connell, 1988, especially Eqs. (2.18) and (3.8))),

$$\sigma(V_d) = \frac{ne^2}{m}\frac{M}{\mu(V_d)}\ , \tag{1}$$

where n, e, m are the density, charge, and effective mass of electrons respectively, M is the center of mass, V_d is the drift velocity and $\mu(V_d)$ is the memory function in the GLE which contains all the information concerning the effect of the heat bath (the relative electrons and phonons) on the transport properties of the quantum particle (center of mass of the electrons). Secondly, a self-consistent expression for the memory function which includes the CBS contribution in the sudden reversal picture, is

$$\mu(V_d) = \mu^{(o)}(V_d)/\{1 + \frac{2m\gamma}{N\pi\hbar\tau}\sum_{q'}\frac{1}{q_v^2 - q'^2}\}\ , \tag{2}$$

where $q_v = mV_d/\hbar$, and the factor 2 in the last term takes account of the spin degeneracy of q'. In addition, γ is a factor which takes account of the lateral quantization into subbands. An explicit form for γ will be presented elsewhere since, as we shall see, it is not needed for our present purposes. Also, the approximations used in obtaining (2) are the use of a random distribution and a cumulant decoupling scheme for higher order scattering terms. These are standard in treating low concentration impurity systems. We note that the sum over q' in (2) is carried out by the standard continuum approximation and by introducing an upper and lower cut-off for q', $1/\ell$ and $1/L_\phi$ ($1/L$ if $L_\phi > L$, where L is the length of the system) respectively. Substituting (2) into (1), after some algebra, we obtain

$$\sigma(V_d) = \sigma^0(V_d)\ f(V_d, L_\phi)\ , \tag{3}$$

where $\sigma^0(V_d)$ is conductivity in the absence of the CBS contribution, and

$$f(V_d, L_\phi) = 1 - \frac{\alpha}{2\pi q_v \ell}\ \ell n\left|\frac{1-q_v\ell}{1+q_v\ell}\,\frac{1+q_v L_\phi}{1-q_v L_\phi}\right|\ . \tag{4}$$

where $\alpha = 2\gamma/k_F W$. In general, α has a value less than one representing the reduction of the probability of CBS events due to the finite width of the quantum wire. In the particular limit where γ=1 (which corresponds to a one-band case), we see that α is inversely proportional to the width W of the sample, which is consistent with the Thouless weak localization theory of effective 1D systems. Eqs. (3) and (4) tell us that there are two main electric field effects on the conductivity of the semiconductor quantum wire. One is the hot electron effect (Hu and O'Connell, 1989) contribution to the conductivity, $\sigma^0(V_d)$, in the absence of the CBS contribution which in general decreases with increase of V_d in the intermediate field region. The other one is the electric field effect on weak localization represented by $f(V_d, L_\phi)$. In this paper, we will concentrate on the effect of $f(V_d, L_\phi)$, while keeping in mind that the presence of $\sigma^0(V_d)$ will adjust the net influence of $f(V_d, L_\phi)$ on $\sigma(V_d)$ in a way to slighly decrease the value of $\sigma(V_d)$ when V_d increases. We note that the linear static conductivity is obtained from (3) and (4) by taking $V_d \to 0$, from which we obtain (using $\sigma^0 = ne^2\tau/m$, and $\mu^0(o) = M/\tau$),

$$\sigma(V_d \to 0) = \sigma^0[1 - \frac{\alpha}{\pi}(L_\phi - \ell)/\ell]\ , \quad (L_\phi < L)\ , \tag{5}$$

which is exactly the results of 1D perturbation theory (Lee and Ramakrishnan, 1985) if we take α=1 and recall that the 1D conductivity is $\sigma^0_{1D} = e^2\ \ell/\hbar\pi$.

Finally, we note that the experimental data is often published in terms of the phase coherence length. If we define the phase coherence length $\mathcal{L}_\phi(E)$ at finite electric field by keeping the form of (5), then a comparison of (3), (4) and (5) will give

$$\mathcal{L}_\phi(E) \equiv \ell + \frac{1}{2q_v} \ell n \left| \frac{1-q_v \ell}{1+q_v \ell} \frac{1+q_v L_\phi}{1-q_v L_\phi} \right| . \qquad (6)$$

The above expression will be used in evaluating the current dependence of $\mathcal{L}_\phi(E)$ in the intermediate field region of $q_v \ell$, $q_v L_\phi < 1$, when we make comparison with experimental results. For future reference, we note that, for very weak fields, L_ϕ is essentially a constant (ℓ_ϕ say) and also, from (6), $\mathcal{L}_\phi(E) \rightarrow \ell_\phi$.

IV. COMPARISON OF THE THEORY WITH EXPERIMENTS

In recent experiments, Hiramoto and co-workers (1989) have measured the conductivity of a n-GaAs quantum wire as a function of current and obtained some unexplained results for the phase coherence length $\mathcal{L}_\phi$ from a fitting to the existing weak localization theory. Their results (for a sample #16 ofn-GaAs quantum wire, effective electron mass $m^*/m_e = 0.066$, density $n_s = 0.5 \times 10^{12}$ cm^{-2}, length L=2.45 μm, width W=0.053 μm, phase coherence length at low field limit $\ell_\phi = 0.13$ μm, $\ell = 0.017$ μm) are shown in Fig. 2, where one sees a constant $\mathcal{L}_\phi = \ell_\phi$ at low current ($I \rightarrow 0$) and a fast drop of $\mathcal{L}_\phi$ after the current passes some critical value around $I_c \sim 10^{-7}$A. Here we show that their results can be qualitatively understood by adopting the idea of a cut-off length due to electric field and they are quantitatively in agreement with the theory if we incorporate Eq. (6) and the sudden reversal picture (discussed earlier) into the calculation.

According to Mott and Kaveh (1981) the electric field E effect on the conductivity is to introduce a new cut-off length

$$L_D = (\hbar D/eE)^{1/3} = (n_s e W \hbar \ell^2/m^* I)^{1/3} , \qquad (7)$$

(where we have used $D = \ell^2/\tau$ and $E = I/\sigma^0 W$) which will affect the experimental results when $L_D < \ell_\phi$ at large enough E. In other words, for small field (hence small current), L_D of (7) is larger than ℓ_ϕ and the conductivity is given by (3). For large enough field, $L_D < \ell_\phi$ and the L_ϕ in (3) should be replaced by L_D and show a field dependence. This is exactly the qualitative behavior of the field dependence of $\mathcal{L}_\phi$ found in Hiramoto and co-workers' experiments. Using the sample data for n_s, W, ℓ, ℓ_ϕ, one can estimate from (7), the critical value of current I_c beyond which the $\mathcal{L}_\phi$ shows a field dependence as $I_c \approx 1.0 \times 10^{-8}$A, which is considerably smaller than the experimental value of I_c (see Fig. 2).

We recall that the L_D of (7) is obtained from the diffusive picture, where one equates the energy gained (eEL_D) by the electron in diffusing a distance L_D to the broadening energy ($\hbar D/L_D^2$) caused by diffusion. On the other hand, according to the sudden reversal picture presented earlier, the broadening energy in a CBS process is $\hbar V_F/L_S$. Thus, the cut-off length (L_S) due to the electric field in this case is

$$L_S = (\hbar V_F/eE)^{1/2} = (n_s e W \hbar \ell/m^* I)^{1/2} \equiv (I_c/I)^{1/2} \ell_\phi \equiv \ell_\phi/\sqrt{z} \qquad (8)$$

From (8), we calculate a critical current $I_c = 7.5 \times 10^{-8}$A for the Hiramoto et al. sample in good agreement with the experiment. When $I < I_c$, the L_ϕ in (6) is not affected by the electric field and when $I > I_c$, it is replaced by the L_S of (8) and (6) reduces to (using $V_d = I/n_s eW$)

$$\mathcal{L}_\phi(z) = \ell + \frac{\ell_\phi}{2bz} \ell n \frac{1-az}{1+az} \frac{1+b\sqrt{z}}{1-b\sqrt{z}} , \qquad (I > I_c) \qquad (9)$$

where $b = m^* I_c \ell_\phi / n_s e W \hbar$ and $a = b(\ell/\ell_\phi)$. Using (9), $\mathcal{L}_\phi(z)$ can be evaluated without any fitting parameters. This is shown in Fig. 2 by the full line which is quite close to the experimental data. One observes that in the high current region our theory overestimates the delocalization effect of the electric field by giving a smaller value of $\mathcal{L}_\phi$ as compared to the experimental measurement (see Fig. 2). We think this deviation is understandable as the experiments measure the overall effect of the electric field on $\sigma(V_d)$ while in our calcultion we have neglected the hot electron effect on $\sigma^0(V_d)$ (see (3)) which, as discussed earlier, will reduce the $\sigma(V_d)$ (i.e., offset the delocalization due to the increasing of field) slighly. For comparison, we have also plotted in Fig.2 (dashed line), the theoretical curve of $\mathcal{L}_\phi$ in the Mott-Kaveh diffusive picture (using (6) and (7)).

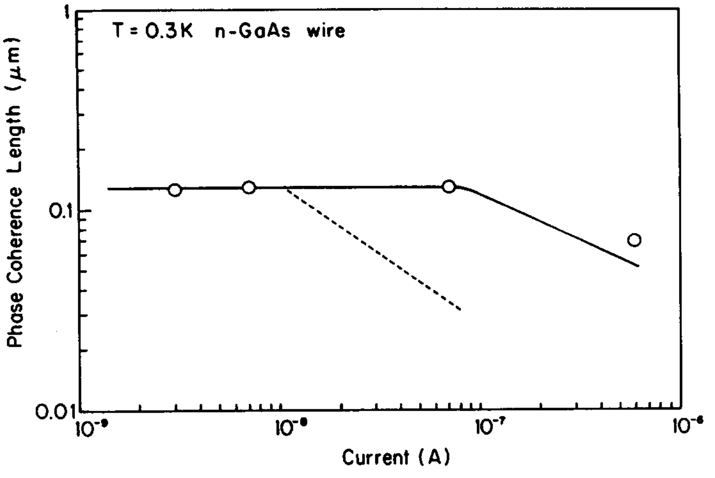

Fig. 2. A comparison between the theoretical and experimental results for the current dependence of the phase coherence length ℓ_ϕ for a n-GaAs wire. Open circles are experimental data from Hiramoto and co-workers (1989). The dashed line is the theoretical curve from Mott-Kaveh theory. The full line is our theoretical curve based on the sudden reversal picture.

V. SUMMARY

We have studied the influence of an electric field on weak localization in semiconductor quantum wires by the generalized quantum Langevin equation approach to the conductivity problem. For the semiconductor quantum wires, which have a width comparable to the Fermi wave length, we have presented a new (sudden reversal) picture of the weak localization theory. The electronic motion in our picture is essentially one dimensional and the phase coherence length of the system is much larger than the mean free path. In this way, an electron will experience many forward scatterings with little change in its original speed before eventually suffering a reversal scattering. Based on the sudden reversal picture, a general formula for the memory function (2) of the non-interacting electrons, in the presence of high order impurity scattering and an arbitrary electric field, is presented. In the low field limit, our formalism reduces to the well known scale dependent conductivity (5). In the high field case, we adopt a sudden reversal picture and find that when the electric field exceeds a critical value of $\hbar V_F/e\ell_\phi^2$, it will introduce a new cut-off length $L_s=(\hbar V_F/eE)^{1/2}$ which affects the experimental results. Our theory gives good agreement with the experiments of Hiramoto et al.

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