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REGRESSION OF THE NODE OF THE ORBIT OF MERCURY DUE TO A SOLAR QUADRUPOLE MOMENT

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ABSTRACT

Assuming that the solar quadrupole moment is given by the value deduced by Dicke and Goldenberg from their solar oblateness measurements, Audretsch, Dehnen, and Hönl recently concluded that the node of the orbit of Mercury regresses at a rate of 3″.4 per century. However, this rate is with respect to the Sun's equatorial system. Now, the observations of all planetary orbits are described with respect to the equinox and ecliptic of a given epoch, and we show that, with respect to this system, the rate of regression is only 0″.27 per century, which is actually smaller than the experimental error of 1″ per century.

From their recent measurements of solar oblateness, Dicke and Goldenberg (1967) concluded that the Sun has a quadrupole moment which contributes 3″.4 per century to the precession of the perihelion of Mercury. However, the considerable controversy surrounding the question of whether or not the Sun actually has a quadrupole moment led Audretsch, Dehnen, and Hönl (1967) to suggest a measurement of the regression of the node of the orbit of Mercury as an independent test of this hypothesis. They calculated that the node regresses 3″.4 per century. However, this rate is with respect to the Sun's equatorial system. Now, *the observations of all planetary orbits are described with respect to the equinox and ecliptic of a given epoch* (Clemence 1943, 1947, 1949; Shapiro 1965), and thus it is important to calculate the rate of regression with respect to this system.

Let ω, Ω, and i denote the argument of perihelion, the longitude of the ascending node, and the inclination of the orbit of Mercury, respectively, in relation to the equinox and ecliptic of a given epoch, and let ω', Ω', and i' denote the corresponding quantities with respect to the Sun's equatorial system. Now, making use of results well known from artificial-satellite theory (King-Hele 1958; Sterne 1960; Massey 1961; Danby 1962; Shapiro 1963, 1965), we obtain the following secular rates of change

$$\frac{d\omega'}{dt} \simeq 6''.8 \text{ per century}, \tag{1a}$$

$$\frac{d\Omega'}{dt} \simeq -3''.4 \text{ per century}, \tag{1b}$$

$$\frac{di'}{dt} = 0 . \tag{1c}$$

To obtain the secular rates of change with respect to the equinox and ecliptic of a given epoch, we make use of the transformation equations given by Shapiro (1965) and obtain

$$\frac{d\omega}{dt} \simeq 3''.66 \text{ per century}, \tag{2a}$$

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$$\frac{d\Omega}{dt} \simeq -0''.27 \text{ per century}, \tag{2b}$$

$$\frac{di}{dt} \simeq -0''.20 \text{ per century}. \tag{2c}$$

It is thus clear from equations (1b) and (2b) that the 3''.4 per century regression of the node of the orbit of Mercury quoted by Audretsch *et al.* (1967) is with respect to the Sun's equator but that the regression with respect to the frame of reference to which measurements are referred is only 0''.27 per century, which is less than the experimental error of 1'' per century (Clemence 1943; Duncombe 1958). Thus a measurement of $d\Omega/dt$ will *not* provide a crucial test for the existence of a solar quadrupole moment. It is clear that the relative smallness of $d\Omega/dt$ and di/dt, compared to the magnitude of $d\omega/dt$, was also apparent to Dicke (1964) and Gilvarry and Sturrock (1967). It should also be noted that the value of $-0''.20$ per century for di/dt is consistent with the discrepancy of $-0''.12 \pm 0''.16$ per century (Clemence 1943; Duncombe 1958) between the calculated and observed values of di/dt.

As a final remark, we draw attention to a point well known from earth-satellite theory (King-Hele 1958), viz., that when the orbit is nearly in the plane of the ecliptic (and for Mercury i is only 7°), the forward precession of the perihelion tends to become enmeshed with the regression of the orbital plane. Thus the net effect is a precession of the perihelion at a rate ($d\omega_{\text{eff}}/dt$) given by

$$\frac{d\omega_{\text{eff}}}{dt} \simeq \frac{d\omega}{dt} + \frac{d\Omega}{dt}, \tag{3}$$

which again results in a rate of 3''.4 per century for the precession of the perihelion of Mercury with respect to the equinox and ecliptic of a given epoch.

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