ON A GENERALISATION OF THE GELL-MANN ROSENFELD TRIANGLE FOR Σ-DECAYS

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Recent measurements of the asymmetry parameters in Σ-decay [1]) show that the well-known triangular relationship of Gell-Mann and Rosenfeld (hereafter referred to as G-M-R) [2]) is not satisfied. This relationship is a direct consequence of the $|\Delta t| = \frac{1}{2}$ rule for non-leptonic processes, which assumes that the interaction Hamiltonian for these processes is of the form

$$H_{\mathrm{int}} = H^{1/2}_{-1/2}\,, \qquad (1)$$

where the sub- and superscript refer to the third component of isospin and total isospin respectively. In view of the failure of this assumption for the above process (as well as for another non-leptonic process [3])) we wish to investigate here the effect of using a more general interaction Hamiltonian. As a first generalisaiton of eq. (1) we make the assumption that H_{int} is of the form

$$H = H^{1/2}_{-1/2} + H^{3/2}_{-1/2}\,. \qquad (2)$$

Let us now define

$$\begin{aligned} N_0^{1/2,3/2} &= \langle \mathrm{p}\pi^0 | H^{1/2,3/2}_{-1/2} \; |\Sigma^+\rangle\,, \\ N_+^{1/2,3/2} &= \langle \mathrm{n}\pi^+ | H^{1/2,3/2}_{-1/2} \; |\Sigma^+\rangle\,, \qquad (3) \\ N_-^{1/2,3/2} &= \langle \mathrm{n}\pi^- | H^{1/2,3/2}_{-1/2} \; |\Sigma^-\rangle\,, \end{aligned}$$

with

$$N_{\pm 0} = N^{1/2}_{\pm 0} + N^{3/2}_{\pm 0}\,. \qquad (4)$$

If we then expand the pion-nucleon states in terms of isospin states and apply the Wigner-Eckart theorem (which introduces a second set of Clebsch-Gordan coefficients) to the matrix elements obtained we find the following relations for the $N^{1/2}_{\pm 0}$ and $N^{3/2}_{\pm 0}$:

$$N_+^{1/2} + \sqrt{2}\, N_0^{1/2} - N_-^{1/2} = 0\,, \qquad (5)$$

$$N_+^{3/2} + \sqrt{2}\, N_0^{3/2} + 2\, N_-^{3/2} = 0\,. \qquad (6)$$

From eqs. (4), (5) and (6) we then have the relation

$$N_+ + \sqrt{2}\, N_0 - N_- + 3\, N_-^{3/2} = 0\,. \qquad (7)$$

This is the generalisation of the G-M-R triangle relation for non-zero $H^{3/2}_{-1/2}$. The surprising feature of the relation (7) is that of the three quantities $N^{3/2}_{\pm 0}$, *two* of which are linearly independent, only *one* $N_-^{3/2}$, appears. The purpose of this note is to discuss some ways in which this feature of eq. (7) may be exploited:

a. As a first consequence of eq. (7) we note that even if the G-M-R "triangle" did close, this would imply a $|\Delta t| = \frac{1}{2}$ rule for the Σ_- decay, but not necessarily for the Σ^+ decays.

b. In contrast, the fact that the G-M-R "triangle" does not close, implies that the $|\Delta t| = \frac{1}{2}$ rule does not hold for the Σ decay, and by eq. (6) this, in turn, implies that the rule does not hold for at least one of the two Σ^+ decay channels.

c. The factor 3 in eq. (7) shows that a relatively small admixture of $|\Delta t| = \frac{3}{2}$ to the Σ decay can lead to a significant departure from the G-M-R relation. Thus we can explain the rather large departure from that relation found by Tripp et al. [1]) in terms of a relatively small $|\Delta t| = \frac{3}{2}$ admixture.

d. We see immediately from eq. (7) that if $N_{\pm 0}$ are known experimentally, $N_-^{3/2}$, and hence $N_-^{1/2}$, can be obtained. As a result the triangle in the S-P plane defined by

$$N_- = N_-^{1/2} + N_-^{3/2}\,.$$

can be constructed.

e. In point of fact, however, the $N_{\pm 0}$, regarded as vectors in the S-P plane, are not yet known completely. For each of them the overall sign of the vector is not determined, and for each it is also not known which of its components is S and which P. This means that, in fact, eq. (7) leads to 64 possibilities for $N_-^{3/2}$, regarded as a vector in the S-P plane. If, however, we regard those $N_-^{3/2}$ as equivalent which can be obtained from each other by a reflexion in either of the axes, or by an interchange of the S and P axes these possibilities may be reduced to 16. Without making any other assumptions these possibilities cannot be further reduced. At this stage, therefore, *we make the assumption that $|N_-^{3/2}|$, though not zero, is small compared to the $|N_{\pm 0}|$*. Only three of the 16 possible $N^{3/2}$'s fulfill this condition. They are obtained as follows:

1. We take N_+ and N_- in the conventional directions [1]) as shown in fig. 1, and we take the lower

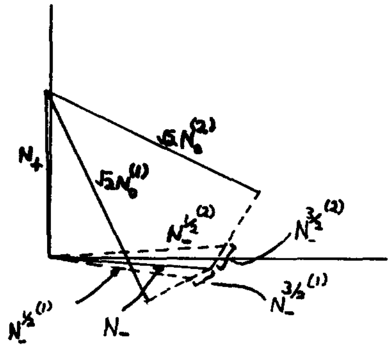

Fig. 1.

of the two possibilities of Tripp et al. for $\sqrt{2}\, N_0$. We call this $\sqrt{2}\, N_0(1)$. (The upper $\sqrt{2}\, N_0$ of Tripp et al. we call $\sqrt{2}\, N_0(2)$). In this case $N_-^{3/2}$ is the vector indicated by $N_-^{3/2}(1)$ in fig. 1. This turns out to be the smallest $|N_-^{3/2}|$ we can obtain from the data of Tripp.

2. We take N_+ and N_- in the conventional directions (fig. 1) and take $\sqrt{2}\, N_0(2)$ for $\sqrt{2}\, N_0$. In this case $N_-^{3/2}$ is the vector indicated by $N_-^{3/2}(2)$ (fig. 1). We see that $|N_-^{3/2}(2)|$ is slightly larger than $|N_-^{3/2}(1)|$.

3. We take N_+ in the conventional direction, and take N_- in the direction shown in fig. 2.

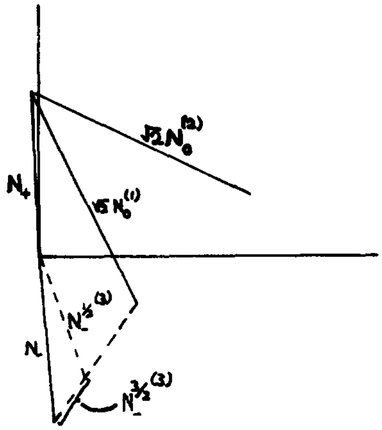

Fig. 2.

For $\sqrt{2}\, N_0$ we take $\sqrt{2}\, N_0(1)$. In this case $N_-^{3/2}$ is the vector indicated by $N_-^{3/2}(3)$ (fig. 2). We see that $|N_-^{3/2}(3)|$ is roughly twice as large as $|N_-^{3/2}(1)|$ and $|N_-^{3/2}(2)|$ but that it is still small compared with $|N_{\pm 0}|$ and so this possibility for N_- cannot be ruled out.

For each of these possibilities we have indicated the corresponding $N_-^{1/2}$ vector. In cases 1. and 3. it is seen that this vector lies farther from the axis than the vector N_- itself, and cannot lie along the axis even allowing for the experimental error. In these cases it would seem that the fact that N_- lies close to the axis is something of an accident. In case 2., however, the $N_-^{1/2}$ lies quite close to the axis, and could, in fact, lie along the axis within the experimental errors. Thus in case 2. it is possible that the $N_-^{1/2}$ is a pure S or P, the small deviation from pure S or P in N_- being due to the small $N_-^{3/2}$ correction. This together with the fact that the N_+ is almost exactly pure S or P suggests that $N_+^{1/2}$ might also be a pure S or P, with $N_+^{3/2}$ negligible. Incidentally, if $N_+^{3/2}$ is negligible then from eq. (6) we can determine $N_0^{3/2}$ (and hence construct the triangle with side N_0, $N_0^{1/2}$, $N_0^{3/2}$).

In conclusion it might not be out of place to investigate the effect of including an $H_{-1/2}^{5/2}$ term in eq. (2). For the $N_{0\pm}^{5/2}$, defined analogously to 3., the Wigner-Eckart theorem gives *two* relations, namely

$$N_0^{5/2} = \sqrt{2}\, N_+^{5/2} = \sqrt{2}\, N_-^{5/2} \tag{8}$$

and the generalisation of eq. (7) is

$$N_+ + \sqrt{2}\, N_0 - N_- + 3N_-^{3/2} - 2N_-^{5/2} = 0\ . \tag{9}$$

We note that eq. (9) is actually the most general expression we can obtain for the Σ-decays. This is because, even if the Hamiltonian contains terms of the form $H_{-1/2}^{t}$, $t \neq \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$, these terms will not contribute, since

$$N_{\pm 0}^{t} = 0\ ,\ t \neq \tfrac{1}{2}, \tfrac{3}{2}, \tfrac{5}{2}\ , \tag{10}$$

by the addition rule for isotopic spin.

References

1) R.D.Tripp, M.B.Watson and M.Ferro-Luzzi, Phys. Rev. Letters 9 (1962) 66.
2) M.Gell-Mann and A.H.Rosenfeld, Ann. Rev. Nuclear Sci. 7 (1957) 454.
3) G.Alexander, S.P.Almeida and F.S.Crawford Jr, Phys. Rev. Letters 9 (1962) 69.

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