

1/f NOISE IN TWO-DIMENSIONAL MESOSCOPIC SYSTEMS FROM
A GENERALIZED QUANTUM LANGEVIN EQUATION APPROACH*

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In the $T \rightarrow 0$ limit for a two-dimensional weak localized electronic system, we derive an explicit form for the excess noise, which we show is inversely proportional to the total volume of the sample, the frequency, and the temperature. A model for mesoscopic electronic systems is proposed in which the system is described by many Brownian particles. The spectrum of the auto-correlation function of the velocity fluctuations is calculated by using the non-linear generalized Langevin equation, which has a random force which is a function of the coordinates of the Brownian particle.

I. INTRODUCTION

Recent advances in the study of quantum interference effects in mesoscopic systems [1] have stimulated much interest in 1/f noise [1-3], the origin of which is a long-standing unanswered problem. Two theoretical models which relate the quantum interference effect arising from defect motion to 1/f noise, the local interference (LI) model and the universal conductance fluctuation (UCF) model [2-3], have been proposed. On the other hand, the dynamics of the problem have not been fully explored and no systematic study has yet been performed. In this paper we study the fluctuation spectrum, from a dynamical point of view and basically from a first principles approach.

We approach the problem by adopting the UCF model, and by using the generalized quantum Langevin equation (GLE) method [4-6], which we recently developed to study a variety of transport phenomenon. In Sect. 2, we derive a formula for the excess noise based on the non-linear Langevin equation which considers the fluctuations of the coordinates of Brownian particles due to their environment. In Sect. 3, we treat the mesoscopic electronic system as a system of Brownian particles. We show how to obtain an explicit expression for the excess noise by adopting the UCF model without any assumption of defect motion and by using our previous results obtained in the study of weak localized systems by the GLE formulation [5]. Our results are summarized in Sect. 4.

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II. NON-LINEAR GENERALIZED LANGEVIN EQUATION AND EXCESS NOISE

The generalized Langevin equation method is now widely used in describing many-body systems in many different areas of physics [4-7]. The non-linear GLE is such that the memory function and random force are functions of the coordinates of the Brownian particle. Physically this dependence is due either to a strong external field or to the fluctuations of the coordinates of the Brownian particle due to its environment. In this paper we concentrate on the latter. The relevance of this model for mesoscopic systems will be discussed in Sect. 3.

We study the kinetics of a N Brownian particle system (all particles have the same mass m) by starting from the non-linear generalized Langevin equation (NGLE) of a Brownian particle in an external field $f(t)$

$$m\ddot{x}(t) + \int_{-\infty}^t \mu(x(t)-x(t'); t-t') \dot{x}(t') dt' = F(x,t) + f(t), \quad (1)$$

where the site index i ($i = 1, 2, \dots, N$) of the Brownian particle is suppressed as they all obey the same equation. Apart from the inclusion of the external field $f(t)$, (1) is a generalization of the linear GLE in the sense that the memory function and random force are treated as having a dependence on x (no such dependence in the linear GLE), the coordinate of the Brownian particle.

To study the fluctuation phenomenon for a system in a steady state in an external field $f(t)$, it is convenient to introduce the velocity fluctuation $v(t)$ of the Brownian particle, defined by the relation

$$\dot{x}(t) = v_d + v(t), \quad \langle v(t) \rangle = 0, \quad v^2 \equiv \langle v(t)^2 \rangle, \quad (2)$$

where v_d is the drift velocity of the steady state, and the x dependence of $v(t)$ is suppressed.

Now we study (1) by using (2) in the weak field limit ($v_d \rightarrow 0$) and in the presence of the lowest order fluctuation contribution v^2 . In other words, we expand the integrand in (1) to order v_d , $v(t)$, and $v(t)^2$. Thus terms like v_d^2 , $v(t)^3$ or higher orders will be neglected. Next, we decompose (1) into two equations, the steady state transport equation [4] and the velocity fluctuation equation. Those terms proportional to v_d and $v(t)^2$, or both, will not vanish after taking the ensemble average, and they form the steady state transport equation in the weak field limit. The velocity fluctuation equation, to order of $v(t)^2$ and in the weak field limit, is

$$m\dot{v}(t) + \int_{-\infty}^t \mu(t-t') v(t') dt' = F(x,t), \quad (3)$$

where $F(x,t)$ is understood to be expanded to order $v(t)^2$, and $\mu(t)$ is the $\mu(x,t)$ of (2) at $x=0$, i.e. it is independent of the coordinates. In obtaining (3) we have used the parity property for the isotropic system for the non-linear memory function $\mu(x,t)$, which implies that it does not contain terms odd in x . Also, we remark that the $v(t)^2$ term in the expansion of $\mu(x,t)$ does not appear in (3) as it contributes a term of higher order than $v(t)^2$.

From (3), it is now straightforward to obtain, to order v^2 , the spectrum of the auto correlation function of the velocity fluctuation:

$$S_V(v^2, \omega) = \frac{1}{|m\omega + i\mu(\omega)|^2} S_F(v^2, \omega), \quad (4)$$

where the Fourier transform of the auto correlation function of the non-linear random force is

$$S_F(v^2, \omega) = \int_{-\infty}^{\infty} dt \langle F(x, t) F(0, 0) \rangle e^{i\omega t} \quad (5)$$

and the v^2 in the bracket emphasizes the fact that (4) is correct to the order of v^2 .

Equation (4) is a key result of the present paper. In the linear GLE treatment, one neglects the x dependence of $F(x, t)$ and then (5) reduces to the linear result $S_F(\omega)$ and so (4) then simply gives the Nyquist noise ($2k_B T / m$, where τ is the relaxation time and T is the temperature). Now, the difference between the non-linear formula (4) and its linear correspondence is, after including the familiar factor of N^{-1} for a N particle system,

$$\Delta S_V(\omega) \equiv S_V(v^2, \omega) - S_V(\omega) = \frac{1}{N} \frac{S_F(v^2, \omega) - S_F(\omega)}{|m\omega + i\mu(\omega)|^2}, \quad (6)$$

which is a measure of the excess (relative to the Nyquist $S_V(\omega)$) noise. It is due to the non-linear effect caused by the movement of the Brownian particle and is valid to order v^2 . Obviously, in the linear GLE treatment $\Delta S_V(\omega) = 0$.

Since the linear functions $\mu(\omega)$ and $S_F(\omega)$ in (6) are known from the calculation of the linear GLE, it is desirable if we can evaluate the only unknown function $S_F(v^2, \omega)$ in (6) by means of these known functions. Actually, the difference between the non-linear random force $F(\vec{x}, t)$ and its linear correspondence $F(t) \equiv F(x=0, t)$ can be seen more clearly if we expand $F(\vec{x}, t)$ in a Fourier series:

$$F(\vec{x}, t) = \sum_{\vec{q}} F(\vec{q}, t) e^{i\vec{q} \cdot \vec{x}}, \quad F(t) = \sum_{\vec{q}} F(\vec{q}, t). \quad (7)$$

We find that there is a general relationship between the space Fourier transform of (5) and its linear counterpart. In the low frequency limit, we obtain

$$S_F(v^2, \omega) = S_F(\omega) + \sum_{\vec{q}, \alpha} \frac{q_\alpha^2 v^2}{\omega^2} [S_F(\vec{q}, \omega) - S_F(\vec{q}, 0)]. \quad (8)$$

The above equation is obtained by first deriving the second order time differential equation for the space Fourier transform of (5) and then solving the equation by Fourier transform techniques. Physically, this is similar to applying the non-Markovian law [7] to the diffusion process of the Brownian particles.

III. APPLICATION: 1/f NOISE IN MESOSCOPIC SYSTEMS

We can now evaluate the excess noise $\Delta S_V(\omega)$ of (6) supplemented by (8), if only the Fourier transform of both the linear random force autocorrelation function, $S_F(\vec{q}, \omega)$, and the linear memory function, $\mu(\omega) = \sum_{\vec{q}} \mu(\vec{q}, \omega)$, in the linear GLE is known.

Recently, we have studied the GLE of an electron system from a center of mass formulation. In our approach, we visualize the center of mass of the electrons as a Brownian "particle", while the phonons and relative electrons act as a heat bath, which is coupled to the center of mass through the electron-impurity and electron-phonon interactions. The only assumption used in obtaining the NGLE is that the number of electrons N_e composing this Brownian particle should be much larger than one. The application of our NGLE method to study the $1/f$ noise in the $T \rightarrow 0$ limit is motivated by a recent work [2] of Feng, Stone, and Lee (FSL), in which they presented compelling arguments that the weak localization mechanism is a source of $1/f$ noise in the quantum regime. Following FSL's work, we divide the electronic system into equivalent small boxes (each box being a "Brownian particle") and the length of each box is bounded by the dephasing length L_ϕ . Inside each box, there are strong interference effects between electrons. We use our quantum mechanical approach to derive a NGLE by the center of mass description. Between electrons in different boxes, the interference effects are expected to be very weak. Obviously, the total number N of Brownian particles representing the whole system in this description depends on $L_\phi(-T^2$ for electron-electron interactions). We emphasize that the mass m and the coordinates x refer to a "box" and that N is the total number of boxes with (N_e/N) electrons in each box.

Previously, we have derived the GLE of an electron-impurity system at $T=0$ in the weak-localization regime. For the $d=2$ case, the memory function we obtained in the low frequency region ($\omega\tau \ll 1$) is [5]

$$\mu(\omega) = \frac{1}{\tau} \left(1 + \frac{1}{k_F \lambda} \epsilon n \omega \tau \right)^{-1} \quad (9)$$

where τ is the relaxation time in the absence of the back scattering effect and k_F is the Fermi momentum. Also, the form of $\mu(q, \omega)$ defined by $\mu(\omega) = \int_{\vec{q}} \mu(\vec{q}, \omega)$ can be found in Ref. 5. It follows that we can obtain

the $S_F(\vec{q}, \omega)$ by using the fluctuation-dissipation theorem [6] in the $T=0$ limit

$$S_F(\vec{q}, \omega) = \hbar \omega \operatorname{Re} \mu(\vec{q}, \omega) \quad (10)$$

Substituting (9) and (10) into (8) and then using (6), and using the proportional property between the velocity and the current, we obtain the excess noise of the current fluctuation spectrum for a two-dimensional electron-impurity system

$$\mathcal{S}_I(\omega) = S_I(\omega)/I^2 = \frac{L_\phi^2}{\Omega \omega} \frac{\delta G^2}{G^2} \frac{3k_F^2}{\hbar} \frac{1}{1 + \frac{1}{k_F \lambda} \epsilon n(\omega\tau)} \quad (11)$$

where Ω is the total volume of the sample ($\Omega = N L_\phi^2$), G is the conductance ($G = e^2 n e^2 v / m$ for the $d=2$ case) of the box, the $1/N$ is a factor that comes from statistics for N boxes, the other terms are the one box contributions, and we have used the relation $v^2/v_0^2 = \delta G^2/G^2$.

Qualitatively, (11) is similar to the general Hooge formula in that $\mathcal{S}_I(\omega)$ is inversely proportional to ω and Ω , and $\mathcal{S}_I(\omega)$ increases when the density or k_F of the electrons decreases (this can be easily observed if we use the result for the universal conductance fluctuation [1,2] viz.

$\delta G^2/G^2 = 1/(k_B T)^2$ in (11)). Eq. (11) is a $T=0$ result. An extension of (11) to the finite temperature case is among our plans for the future. Nevertheless, it is clear that the factor N^{-1} in (11) will be kept for the finite temperature version, which indicates a decrease of $S_i(\omega)$ with increase of T (since $L^2 \sim T^{-1}$ i.e. the number of boxes increases with increasing temperature). This predicted decrease of $S_i(\omega)$ with increase of T is opposite to what one finds in the classical high-temperature case (where, around room temperature $S_i(\omega)$ increases with increase of T). Yet it is in conformity with the theoretical results of Feng, Lee and Stone [2], and confirmed by experiments [8,9]. We note that in Ref. 2 the basic idea is that the time variation of the positions of the impurities is the source of the $1/f$ noise. In our approach, the time variation of the random force exerted on the electrons due to the environment is the source of the $1/f$ noise. In addition, we have considered a dynamical model based on first principles.

IV. SUMMARY

We have analyzed the phenomenon of excess noise, based on the study of a non-linear Langevin equation which considers the fluctuations of the coordinates of Brownian particles due to the environment. Our basic results, given in (6) and (8), can be used to evaluate excess noise quite generally in the GLE formulation. For mesoscopic systems, we adopt the UCF model and divide the electronic system into equivalent small boxes (each box being a "Brownian particle"). The length of each box is bounded by the dephasing length, and the center of mass of the electrons in each box is designated as a Brownian particle. The memory function (9) of the Brownian particle, together with the fluctuation-dissipation theory (10), are then used to obtain for the 2d mesoscopic system the current fluctuation spectrum (11), which is inversely proportional to the total number of electrons, the frequency and the temperature.

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