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SCHIFF'S PROPOSED GYROSCOPE EXPERIMENT AS A TEST OF THE SCALAR-TENSOR THEORY OF GENERAL RELATIVITY

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We compare an explicit expression for the precession of a gyroscope in the Brans-Dicke scalar-tensor general relativity theory with the result derived by Schiff using Einstein's theory, and suggest that the gyroscope experiment offers the best possibility for testing the Brans-Dicke theory. Further, we conclude that the gyroscope in a satellite offers a more sensitive test than the earth-bound gyroscope.

Einstein's general theory of relativity is generally acclaimed as the correct theory of gravitation. Perhaps its only serious challenger is the scalar-tensor theory of Brans-Dicke (BD).[1] In the latter theory the gravitational constant is normalized to give the well-known red-shift result, and the dimensionless coupling constant ω is selected to be $\geqslant 6$ to ensure that the result for the precession of the perihelion of Mercury agrees, with an accuracy of 8% or less, with the computed value predicted by Einstein's theory. For $\omega = 6$, the BD theory gives a precession of 39.6″ arc/century which is about 3.43″ arc/century less than Einstein's value. The recent work of Dicke and Goldenberg[2] on the contribution of solar oblateness to the precession of the perihelion seems to favor the BD theory, but there is considerable controversy surrounding both the measurement itself[3] and the relation[4] between the surface oblateness and the interior oblateness (the latter being the source of the quadrupole moment). It has recently been shown[5] that the rate of gravitional radiation from a system of binary stars in BD theory is smaller than the value predicted by Einstein's theory by a factor of $(2\omega+3)/(2\omega+4)$; however, it seems that it will be a considerable time before this test is experimentally feasible. Cosmological tests[6] have likewise been unable to resolve the question. It is our purpose in this communication to suggest that perhaps the best test is the gyroscope experiment proposed by Schiff.[7,8] In particular, we write down an explicit expression for the precession of the gyroscope in BD theory for comparison with the Einstein value.

The angular velocity of precession in Einstein theory, $\vec{\Omega}_E$ say, may be written as[8]

$$\vec{\Omega}_E = \vec{\Omega}_T + \vec{\Omega}_{DS} + \vec{\Omega}_{LT}, \tag{1}$$

where $\vec{\Omega}_T$, $\vec{\Omega}_{DS}$ and $\vec{\Omega}_{LT}$ are the so-called Thomas, de Sitter, and Lense-Thirring contributions, respectively. Explicitly,[8]

$$\Omega_T = \tfrac{1}{2}(\vec{f}\times\vec{v}), \tag{2a}$$

$$\Omega_{DS} = (3m/2r^3)(\vec{r}\times\vec{v}), \tag{2b}$$

$$\Omega_{LT} = (I/r^3)[(3\vec{r}/r^2)(\vec{\omega}\cdot\vec{r})-\vec{\omega}], \tag{2c}$$

where $\vec{f}$ is the acceleration arising from any nongravitational constraint, m is the mass of the gyroscope ($c=G=1$), $\vec{r}$ its position vector with respect to the center of the earth, $\vec{v}$ is its velocity vector, and I and ω are the moment of inertia and rotational angular velocity of the earth, respectively.

Following Eddington[9] and Robertson[10], Schiff[11] has written the metric for the nonrotating earth in its most general isotropic form:

$$ds^2 = [1-2\alpha(m/r)+2\beta(m/r)^2+\cdots]dt^2 - [1+2\gamma(m/r)+\cdots](dx^2+dy^2+dz^2), \tag{3}$$

and deduces that the de Sitter term is modified by a factor $(\alpha+2\gamma)/3$. For the particular case of the BD theory it is easy to show that this factor is $(3\omega+4)/(3\omega+6)$. Being a special-relativistic effect only, the Thomas precession remains unchanged in the BD theory. However, there is a change in the Lense-Thirring effect which is deduced quite easily from an observation made by the present author and

Salmona[5] to the effect that, in the weak-field limit, the solutions of the transformed[12] BD equations (the so-called barred system) are exactly the same as the solutions to Einstein's equation except for a factor. Explicitly,[5]

$$(\bar{h}_{\mu\nu})_{\rm BD} = [(2\omega+3)/(2\omega+4)](h_{\mu\nu})_{\rm E} \quad (4)$$

in the weak-field limit. This immediately enables us to conclude that the Lense-Thirring precession is reduced by a similar factor.[13] Thus, the angular velocity of precession in BD theory, $\Omega_{\rm BD}$ say, may be written as

$$\vec{\Omega}_{\rm BD} = \vec{\Omega}_{\rm T} + [(4+3\omega)/(6+3\omega)]\vec{\Omega}_{\rm DS} + [(3+2\omega)/(4+2\omega)]\vec{\Omega}_{\rm LT}. \quad (5)$$

Note that the factor modifying the de Sitter term turns out to be identical to the factor appearing in the perihelion precession angle[1] in BD theory (this is not true in general, of course). For a value of $\omega = 6$ we obtain

$$\vec{\Omega}_{\rm BD} = \vec{\Omega}_{\rm T} + (11/12)\vec{\Omega}_{\rm DS} + \tfrac{15}{16}\vec{\Omega}_{\rm LT}. \quad (6)$$

It is clear that the most sensitive test of the BD theory occurs when $\Omega_{\rm DS}$ and $\Omega_{\rm LT}$ attain their maximum possible values. Now, for a gyroscope in a satellite at moderate altitude (orbiting the earth's equatorial plane and with the gyroscope spin axis normal to the earth's axis), $\Omega_{\rm DS}$ is about[11,14] 7″/yr, $\Omega_{\rm LT}$ is about 0.1″/yr, and $\Omega_{\rm T}$ is practically zero ($\Omega_{\rm T}$ can be made exactly zero if the slave satellite idea of Pugh[15] is adopted). For a gyroscope in an earth-bound laboratory[11,14] (with spin axis normal to the earth's axis), $\Omega_{\rm E}$ is roughly 0.4″/yr with $\Omega_{\rm T}$, $\Omega_{\rm DS}$, and $\Omega_{\rm LT}$ contributing to the same order of magnitude. It is thus clear that the gyroscope in a satellite offers the most sensitive test of the BD theory (particularly of the terms in the BD metric which contribute to the de Sitter effect); it is fortunate that this is also the most convenient experimental arrangement.[7,8,11,14] With regard to experimental accuracy, Schiff[11] states that the direction of the spin axis can be read out with an accuracy of 0.1″; an accuracy of 0.0″ now appears possible.[17] Thus we see the favorable possibilities that exist for distinguishing between $\Omega_{\rm E}$ and $\Omega_{\rm BD}$. As a further refinement, we note that it will also be possible to test separately the $(4+3\omega)/(6+3\omega)$ and $(3+2\omega)/(4+2\omega)$ terms appearing in Eq. (5). This arises because of the different angular dependences. For example,[8] at a laboratory latitude of 35°16′ the secular precession arising from the Lense-Thirring effect is zero.

To summarize, we consider that the Schiff gyroscope experiment offers the best possibility for testing the BD theory and that the gyroscope in a satellite offers a more sensitive test than the earth-bound gyroscope.

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K^+d STRUCTURE IN $I=0$ AT 1.2 GeV/c AS A RESULT OF S-STATE $K\to K^*(890)$ CHANNEL COUPLING

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A well-defined peak has been reported[1] in the K^+d total cross section at 1.2 GeV/c K^+ laboratory momentum. Subtraction of the K^+p cross section and making the Glauber correction for screening leaves an $I=0$ peak of approximately 6 mb above background and a width of about 150 MeV.[1]

We wish to report that a previous fit to the data[2] below 800 MeV/c contains a prediction of this experimental peak. The purpose of Ref. 2 was the explanation of the rise in the $I=0$, S-state phase shift between 400 and 800 MeV/c. The successful mechanism was the coupling of this K^+N state to the S-state $K^*(890)N$ channel whose threshold is at ~1000 MeV/c. A simple boundary condition model was used. The boundary radius r_0, in the theoretically indicated range, was taken from an earlier $I=1$ fit.[3] The three homogeneous boundary condition parameters were fitted to the scattering length and phase shifts below K^* threshold. The amount of K^* production predicted at higher energies was comparable with that observed in the $I=1$ channel and was quantitatively related assuming isovector exchange.

In Ref. 2 the K^* width was ignored, as all detailed comparisons were at energies more than 200 MeV/c below K^* threshold and thus insensitive to its width. We have now included the effect of the K^* width and recalculated results in the region 0-1.5 GeV/c. The nonvanishing width requires that Eq. (4) of Ref. 2 be replaced by[4-6]

$$f_{\text{eff}}^{\,0}=f^0-(f_c^{\,0})^2\int\frac{\rho(m)}{f_*^{\,0}-ir_0K(m)}dm, \qquad (1)$$

where $K(m)$ is the relativistic momentum of a K^* of mass m in the center-of-momentum system; f^0, f_c^0, and $f_*^{\,0}$ are constants; and $\rho(m)$ is the resonance shape in high-energy K^* production,

$$\rho(m)=N\frac{\gamma(q/q_*)^3}{(m^2-m_*^{\,2})^2+(m_*^{\,4}/m^2)\gamma^2(q/q^*)^6} \qquad (2)$$

for real pion momentum $q(m)$ in the K^* rest system, and vanishes for imaginary $q(m)$. The normalization is $\int\rho(m)dm=1$. $q(m)$ is given by

$$(q^2+m_K^{\,2})^{1/2}+(q^2+m_\pi^{\,2})^{1/2}=m. \qquad (3)$$

γ is the reduced width and m_* and q^* are the values of m and q at the K^* peak.

With the above $f_{\text{eff}}^{\,0}$ the complex amplitude is computed as in Ref. 2. Using $m_*=891$ MeV and $\gamma=50$ MeV the phase-shift fit of Ref. 2 was restored by small variations of the boundary conditions. The choice $f^0=4.1$, $(f_c^{\,0})^2=10.2$, and $f_*^{\,0}=1.3$ at $r_0=0.45m_\pi^{-1}$ fits the phase shift δ_{00} below threshold as shown in Fig. 1. The predicted K^+N scattering length is $a_0=0.036m_\pi^{-1}$. The same figure shows the calculated values of δ_{00} and η_{00} above threshold. The resulting $\sigma_{\text{tot}}^{\,0}(S_{\frac{1}{2}})$ is peaked as shown in Fig. 2. The experimental $\sigma_{\text{tot}}^{\,0}$ is obtained from $\sigma_{\text{tot}}^{K^+d}$ and $\sigma_{\text{tot}}^{K^+p}$ as in Ref. 1 and the error corridor obtained from

$$\Delta\sigma_{\text{tot}}^{\,0}=\{[2\Delta\sigma_{\text{tot}}(K^+d)]^2+[3\Delta\sigma_{\text{tot}}(K^+p)]\}^{1/2}, \qquad (4)$$

neglecting any error in the calculation of deuteron effects. On subtracting $\sigma_{\text{tot}}^{\,0}(S_{\frac{1}{2}})$ from the experimental cross section the peak is re-