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THE MEMORY FUNCTION FOR CYCLOTRON RESONANCE IN THE TWO DIMENSIONAL ELECTRON GAS

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A divergence-free and explicit way for evaluating the memory function in the cyclotron resonance for the two dimensional electron gas is presented. The usual divergence of the free electron polarization function in cyclotron resonance is removed automatically by including the fluctuation effects of the center of mass. The general behavior of the memory function and its dependence on the magnetic field, electron density, transport time and temperature are studied in the strong magnetic field and low temperature limit. Our theory gives an understanding of the peak splitting phenomenon, and gives a very good fit for many heretofore unexplained cyclotron resonance experiments in Si(001)-MOS systems when the electron density is between $2\times10^{11}cm^{-2}$ to $8\times10^{11}cm^{-2}$.

It is well known that the complex memory function $M(\omega) = M_1(\omega) + iM_2(\omega)$, which describes the complex frequency dependent scattering, is a key concept in the theory of high-frequency transport [1-13]. In a magnetic field, the conductivity may be written as [4]

$$\sigma_{\pm}(\omega) = \frac{in_s e^2/m}{\omega \mp \omega_c + M(\omega)} , \qquad (1)$$

where m, -e, and n_s are the electron band mass, charge, and surface density respectively, and $\omega_c = \frac{eB}{mc}$ is the cyclotron resonance (CR) frequency. In the simplest approximation, one considers the electron-electron interaction in the random phase approximation (RPA), the electron-impurity interaction as short ranged, and ignores other scattering mechanisms, to obtain for the two dimensional electron gas (2DEG) [4,12]

$$M(\omega) = \omega_c \frac{1}{4\pi\epsilon_f \tau} \frac{\omega_c}{(e^2/\kappa_{sc}\ell_0)} \int dx \frac{x^4}{y} \left\{\frac{1}{\epsilon(x,0)} - \frac{1}{\epsilon(x,y)}\right\} \qquad (2)$$

where $x = q\ell_0$, $y = \omega/\omega_c$, $\ell_0 = (\hbar/m\omega_c)^{1/2}$, $\tau = n_i m U^2$ is the transport time for short range impurity scattering with U denoting the Fourier transform of the impurity potential, κ_{sc} is the dielectric constant of the static lattice and $\epsilon(\vec{q},\omega)$ is the RPA dielectric function of wave vector $\vec{q}$.

Even though the above scheme has been widely used to calculate the a.c. conductivity [1-3], in particular to give a theoretical analysis of the CR spectrum of the 2DEG at low temperatures for comparison with the experimental results of the Si-MOS system [1,14-17], the evaluation of the memory function $M(\omega)$ of (2) remains a controversal question, arising from the proper handling of the well-known divergence of the free electron polarization function associated with the RPA dielectric function $\epsilon(\vec{q},\omega)$. In a recent paper [12], we have indicated that there is a physical effect which can eliminate the divergences viz fluctuations. Our theory has successfully explained the nature of the double peak CR spectrum [13] and we also obtained a good fit to the heretofore unexplained experimental data of the cyclotron effective mass and scattering time in certain electron density regions [14]. Here, we give a more detailed analysis of the key quantity, the memory function $M(\omega)$, and we compare our results to some other experiments [15,16].

Our approach is based on the use of a generalized quantum Langevin equation [11], where the system is described by the center of mass and relative electrons, and where the center of mass is a massive quantity with a negligible non-commutivity to its coordinate, and the divergences are removed automatically by inclusion of fluctuation effects. The a.c. conductivity and the corresponding memory function are formally the same as (1) and (2) but the electron polarization function associated with the dielectric function $\epsilon(\vec{q},\omega)$ is now [12]

$$\pi_{nn'}(q,\omega) = \frac{f(\varepsilon_n) - f(\varepsilon_{n'})}{\hbar\omega - (\varepsilon_{n'} - \varepsilon_n) + i\,Dq^2} , \qquad (3)$$

where $\varepsilon_n = (n + 1/2)\hbar\omega_c - \varepsilon_F$, ε_F is the Fermi energy of the electron gas, $f(x)$ is the Fermi distribution function, and D is the diffusion constant for the center-of-mass defined in terms of the ensemble average of the coordinate fluctuations $\delta R^2(t)$ of the center of mass

$$D = \lim_{t\to\infty} \frac{1}{2t} \overline{\delta R^2(t)} . \qquad (4)$$

As we have shown elsewhere [11], D is proportional to the inverse of the number of particles N and thus the fluctuation effects are most pronounced for small systems with low-electron concentrations.

Now, in general D is difficult to evaluate numerically but it is the only unknown quantity in the problem. On the other hand in the classical limit, one can easily derive analytically (in the limit $\omega_c\tau>>1$) that

$$D = T/(Nm\,\omega_c^2\tau). \qquad (5)$$

Guided by the above formula, we will use the property $D \sim T, N^{-1}, \tau^{-1}$ in our calculation. The advantage of this approach will be seen when we compare our theoretical results to the relevant experimental results. In the following we first study the basic properties of the memory function $M(\omega)$ of (2), and its dependence on n_s, B, τ and T, and then we compare it to the experimental results.

Taking the D term due to the fluctuations into account the memory function $M(\omega)$ of (2) due to lowest-order electron-impurity scattering and electron-electron interaction in the RPA approximation of the 2DEG in a magnetic field, has the following basic properties:

(i) In the low frequency region ($\omega/\omega_c << 1$), one has $M_1(\omega) = A^*\omega$, $M_2(\omega) = 1/\tau^*$, where A^* and τ^* are B, n_s τ, T dependent positive numbers. This $M_1(\omega) \sim \omega$, $M_2(\omega) \sim$ const. behavior is the same as in the absence of the magnetic field.

(ii) The real part $M_1(\omega)$ has a first maximum at $\omega_1<\omega_c$ and changes sign at a value of ω slightly larger than ω_c. The rich structure of $M_1(\omega)$ is in contrast to the classical Drude model where $M_1(\omega) \equiv 0$. As a result, double absorption peaks emerges in our model.

(iii) The imaginary part $M_2(\omega)$ has its maximum at $\omega_2 \gtrsim \omega_c$, and decreases dramatically near ω_2, having a Lorentz shape. This dramatic changing behavior of $M_2(\omega)$ at $\omega\sim\omega_c$ is also a strong deviation from the classical Drude model where one has $M_2(\omega) \equiv$ const.

The above common properties of $M(\omega)$ are illustrated in Figs. 1-3, where we display the B, n_s, τ, and T dependences of $M(\omega)$ separately. Fig. 1 shows the magnetic field B dependence of $M(\omega)$, where we have taken $n_s=3.2\times10^{11}cm^{-2}$, $\tau=1.4\times10^{-13}s$, and the dimensionless diffusion constant $D^*= mD/\hbar = 0.014$. In general when the magnetic field is decreased, one gets a decrease of the magnitude of the memory function, while the shape remains unchanged. As can be seen from the figure, at low field, the equation $M_1(y)/\omega_c = 1-y$, which is the main resonance condition determined by Eq. (1), has only one solution at $\omega<\omega_c$. When the magnitude of $M_1(\omega)$ increases following the increase of field, there will be another solution on the $\omega>\omega_c$ side. The magnetic field dependence of $M(\omega)$ calculated here and the resultant CR conductivity is in qualitative agreement with the results of Schlesinger et al. [13], in which they found a peak splitting phenomenon when one increases the magnetic field from the low field side (see Fig. 1 of Ref. 13). The corresponding $D^*=0$ result for $M(\omega)$ at B=5.16T is also shown as an insert in Fig. 1.

Fig. 2 shows the n_s dependence of $M(\omega)$, where B=5.16T, $\tau = 1.4\times10^{-13}s$ and $D^*=0.0045\times10^{12}cm^{-2}/n_s$ (see (5)) are used. One sees that

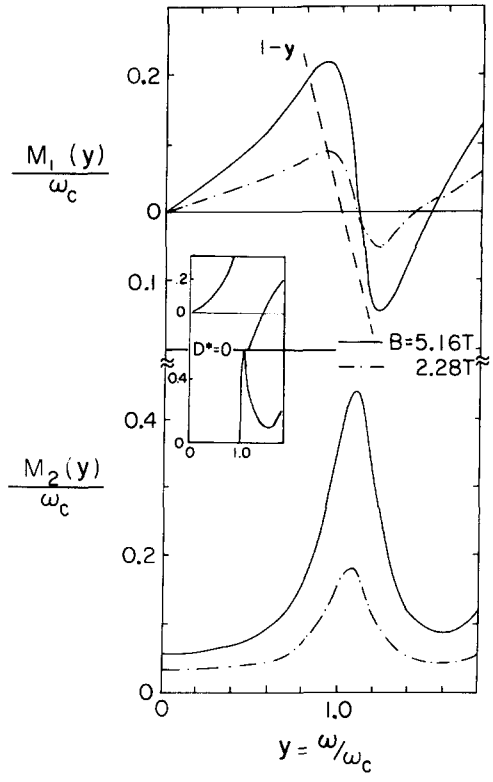

Fig. 1. Real and imaginary parts of the memory function as a function of frequency (all in units of ω_c) for different magnetic fields B and with $n_s = 3.2\times10^{11}cm^{-2}$, $\tau =1.3\times10^{-13}s$, $D^* = 0.014$. The insert illustrates the corresponding memory function at B = 5.16 in the $D^* = 0$ case. Also plotted is the line 1-y.

when the electron density n_s is increased, $M(\omega)$ will decrease. Our calculation of the large n_s limit is consistent with the small B limit as in Fig. 1, where in both cases one expects the classical limit (many occupied Landau levels) of one absorption peak at $\omega=\omega_c$. We note that the effective cyclotron mass m_c and the scattering time deduced from Fig. 2 is in very good agreement with the experimental observation of Kennedy et al. [14,15] who found that m_c will increase when n_s decreases below $8\times10^{11}cm^{-2}$ on the Si-MOS system. Also, we note that Cheng et al. [17] have observed a multiple-line structure in the CR spectrum in the low density region. Our results in Fig. 2 indicates that the $M_1/\omega \approx -y$ region at the $M_1<0$ side has a complicated structure. It will be very interesting to extend our theory to the extreme quantum limit region, which will be a subject of a future study.

Fig. 3 shows the τ dependence of $M(\omega)$, where B=6.15T, $n_s=2.3\times10^{11}cm^{-2}$ and $D^*=0.0224\times10^{-13}s/\tau$ are used. In general, when one uses samples with smaller τ (and hence smaller mobility) to study the CR, larger values of $M(\omega)$ will be obtained. As a result, the shift and broadening of the CR peak will become stronger when τ is decreased. Our numerical results are very consistent with the

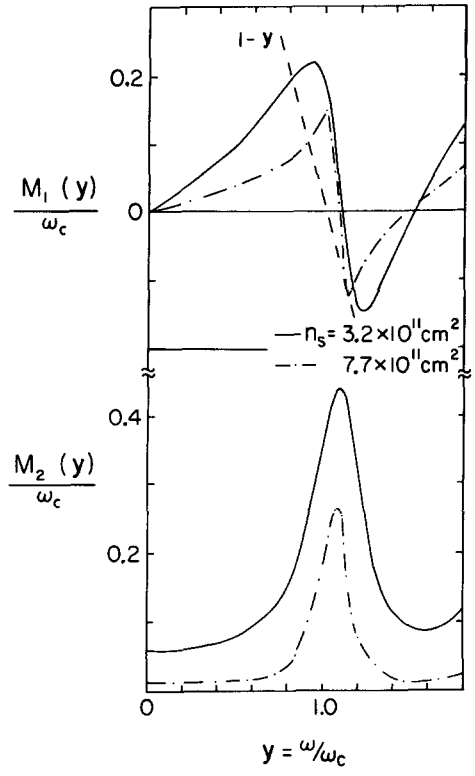

Fig. 2. Real and imaginary parts of the memory function as a function of frequency (all in units of ω_c) for different electron densities n_s and with B = 5.16T, τ = 1.3×10^{-13}s, D* = 0.0045 $\times10^{12}cm^{-2}/n_s$.

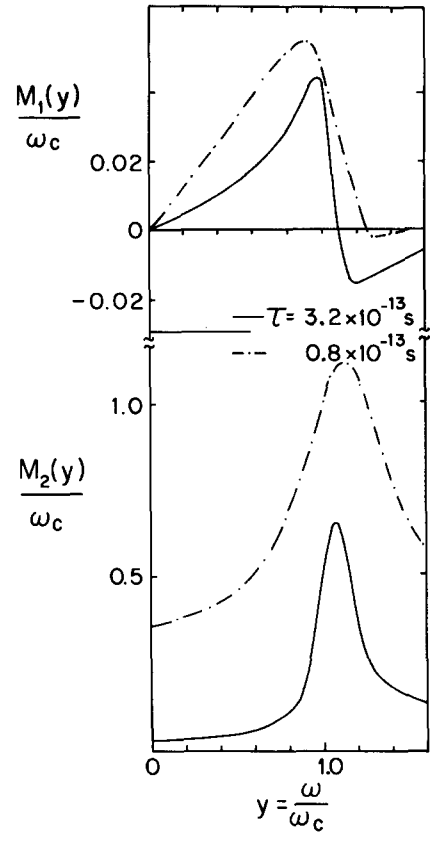

Fig. 3. Real and imaginary parts of the memory function as a function of frequency (all in units of ω_c) for different transport times τ and with B = 6.15T, n_s = $2.3\times10^{11}cm^{-2}$, D* = 0.0224 $\times10^{-13}s/\tau$.

experimental data. In Fig. 4a, we compare our theoretical result with the experimental data of Allen et al. [16], where they used two Si(001)-MOS samples (Valley degeneracy = 2) with different mobilities (one with mobility approximately 4 times larger than the other) to study the CR phenomenon. As illustrated by that figure, a very good fit of the peak position and comparable line shape are obtained. It is interesting to note that in the paper of Allen et al., they have also deduced values of $M(\omega)$ for the corresponding CR absorptions by means of Eq. (1). The values of their experimentally deduced memory functions fall close to our theretical values except in the vicinity of the $\omega=\omega_c$ where our $M(\omega)$ is considerably larger than the experimentally deduced value. This is shown as the insert of Fig. 4b.

We have also studied the temperature effect on the memory function and CR spectrum, using (1)-(3) and (5). Our model should be good when one can neglect the effect of electron-phonon interaction and the temperature dependence of the electron distribution, which is justified at low temperature and strong magnetic field. From (5) we know that in our model the T dependence of $M(\omega)$ is similar to the $1/\tau$ dependence. In general, the value of $M(\omega)$ will increase with increasing temperature. The insert in Fig. 5 shows the temperature dependence of the CR spectrum, where we have taken B=5.16T, n_s = $3.2\times10^{11}cm^2$, τ = 2.1×10^{-13}s, for three temperatures T, 2T and 5T. The value of D* at T is taken as D*=0.04, and the corresponding values of D* in the other two cases are determined by (5). One observes from the figure that when T is increased, the shift of the absortion peak will reduce while the half width will increase. This is very consistent with the experimental data on Si(001)-MOS system by Wagner et al. [15]. Actually, if we fix the lowest temperature (T) curve in Fig. 5 as T=3K°, then we obtain an excellent fit of the effective cyclotron mass m* and scattering time τ* to the relevant experiments [15], which is shown in Fig. 5.

In summary, we have presented a new way to evaluate the memory function for cyclotron resonance of the two-dimensional electron gas. The well known divergence problem of the polarization function is eliminated automatically in our approach when we include the fluctuation effect of the center of mass. The general behavior of the memory function and its dependence on the magnetic field B, the electron density n_s, the transport time τ and the temperature T are

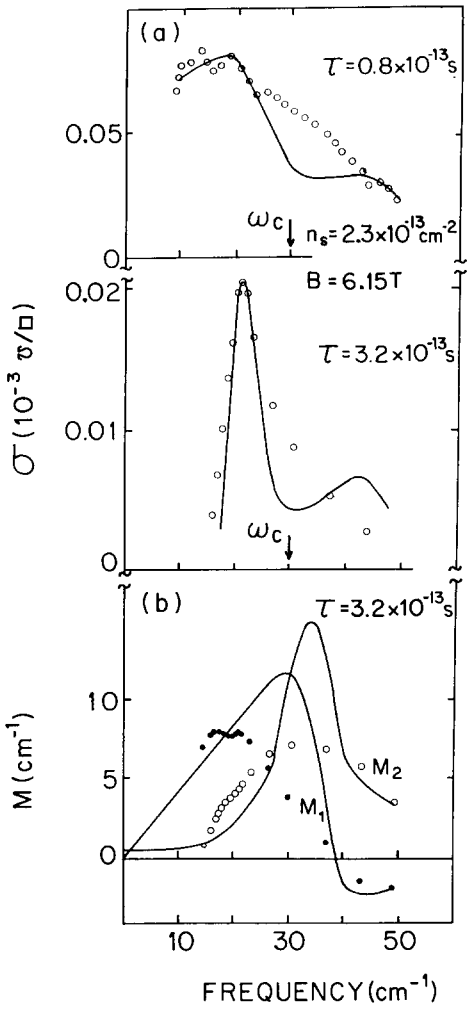

Fig. 4. A comparison between our theoretical results (denoted by solid lines) and the experimental data [16] (denoted by dots) in respect to (a) CR spectrum at different transport times, (b) the real (M_1) and imaginary (M_2) parts of the memory function as a function of frequency. $D^* = 0.0224 \times 10^{-13} s/\tau$ is used in fitting the experimental results.

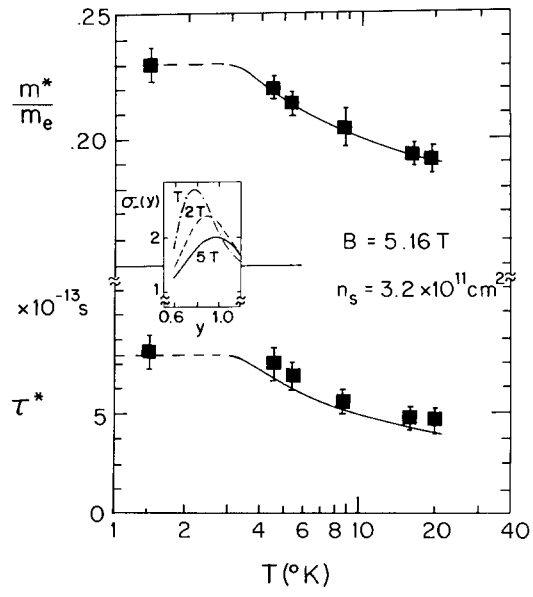

Fig. 5. The calculated effective cyclotron mass m* (in units of free electron mass m_e) and scattering time τ^* as a function of temperature at $\tau = 2.1 \times 10^{-13}$s and $D^* = 0.04$. Experimental data (denoted by dots) taken from ref. 15. The insert is the calculated CR spectrum at three different temperatures.

studied numerically. Our results show that these four physical quantities strongly influnce the memory function, which makes the cyclotron spectrum rather complicated. A direct comparison with experimental data shows that our theory is useful in understanding the CR peak splitting phenomenon, and is in quantative agreement with the CR experimental data for the effective mass, scattering time and their density, mobility, and temperature dependence for the 2DEG on the Si(001)-MOS system when $2 \times 10^{11} cm^{-2} < n_s < 8 \times 10^{11} cm^{-2}$.

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