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STRONG ELECTRIC FIELD EFFECT ON WEAK LOCALIZATION

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The influence of an electric field on weak localization is studied by a recently proposed generalized quantum Langevin equation approach to the conductivity problem. A general formula for the memory function of the non-interacting electron gas, in the presence of high order impurity scattering and an arbitrary electric field, is derived. In the low field case, a scale and frequency dependent conductivity is obtained, which reduces to the well known scale dependent conductivity in the static limit. In the high field case, the conductivity is field dependent through the drift velocity. It is shown that the presence of strong electric fields tends to delocalize the one and two dimensional electron systems if one adopts the electron heating model. However, if the electron gas does not heat up, the conductivity will be field independent.

1. Introduction

Since the proposal of a scaling theory of the Anderson transition for weak localization systems by Abrahams et al. [1, 2], there have been considerable theoretical [1–15] and experimental [3, 16–20] developments in this field. By treating the dimensionless conductance g as a function of sample length L as the only relevant scaling parameter, the behavior of the scaling function $\beta(g)$ in the limit of large g was found to be $d-2-\alpha/g$ (d denotes the number of dimensions and α is a constant of order unity) [1–3]. Furthermore, the perturbation theory of the conductivity of weak localized systems has given results consistent with the scaling theory. A key goal of these calculations is to include the quantum interference effect by evaluating the back scattering contributions of the electrons [3]. Up to now the methods used have been the Kubo formula [1–2, 6–8], the Boltzmann equation [9], the self-consistent theory [10, 11], and the study of the diffusion pole equation [12–15]. In this paper we study the weak localization, in particular the influence of an electric field, by a generalized quantum Langevin equation (GLE) approach recently

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proposed by us for the study of the quantum transport of a system of electrons, phonons and impurities [21, 22].

The study of the influence of an electric field on weak localization has been quite controversial. The theoretical papers of Altshuler et al. [7, 8] predicted that a dc electric field does not break time-reversal invariance and, as long as the temperature is constant, the electric field has no influence on weak localization [3]. By contrast, Tsuzuki's study [12] showed a reduction of the quantum correction to the conductivity in an electric field E, and Kaveh et al. [13–15] found a critical length L_E ($\sim E^{-1/3}$), which destroys weak localization as soon as $L_E < L_T$, the Thouless length ($L_T = (D\tau_{in})^{1/2}$, D and τ_{in} denote the diffusion constant and the inelastic scattering time, respectively). This controversy however is not unexpected as the influence of an electric field on weak localization is a non-linear (in electron–impurity scattering) and non-equilibrium phenomenon, which goes beyond the scope of the Kubo formula. Therefore, a comprehensive study of the subject is needed. Recently, Hershfield and Ambegaokar [9] have developed a weak localization theory which can include the non-linear effect of the electric field, based upon the Boltzmann equation with the right side generalized to include the coherent backscattering. On the other hand, we have previously developed a GLE approach to the electron quantum transport, which is capable of dealing both with the non-linearity of impurity scattering [21] and the non-equilibrium situation due to the high electric field [22], basically from a first principles approach. Our GLE approach is developed by applying the philosophy of Ford et al. [23, 24] (who have demonstrated the usefulness of the GLE approach to heat bath problems) to the study of electronic quantum transport, using the center-of-mass system [25–27]. The basic idea of our theory is the following: we visualize the center of mass of the electrons as a quantum particle, while the relative electrons act as a heat bath, which is coupled to the center of mass through electron–impurity and electron–phonon interactions [21]. In the course of solving the second order Heisenberg equations [23] for the fluctuating density of the relative electrons, the GLE of the center of mass of electrons is obtained directly. At the same time, the memory function, which contains all the information concerning the effect of the heat bath on the transport properties of the quantum particle, is also obtained without any assumed approximations for the electron–impurity, electron–phonon and electron–electron interactions. The advantages of our approach stem from the directness by which the GLE is obtained and the convenient way in which higher order approximations [21], as well as the high electric field effects [22] are included. Here we attempt to develop a systematic rigorous theory of weak localization under the influence of a high field, based on the GLE approach.

In section 2 we derive the high field conductivity formula in the presence of

high order impurity scattering, originating from a nonlinear generalized quantum Langevin equation. A general formula for the memory function of the non-interacting electron gas in the presence of high order impurity scattering and an arbitrary electric field is presented. In section 3, we show the usefulness of our formalism by rederiving the usual scale dependent conductivity in a very simple way. Furthermore, we show that the scale and low frequency dependence of the conductivity can be dealt with in one formula (see (2.18) below supplemented by (3.7) and (4.3)) in our approach. In section 4, we study the influence of the electric field on the weak localization. In section 5, the main results are summarized.

2. High field conductivity with high order impurity scattering

Recently we have obtained a general formula for the low field conductivity of an interacting electron system with high order impurity scattering based on a new method [21], where the central idea is to set up the GLE of the center of mass of the electronic momentum. The method has also been developed to treat the high field electronic transport with the lowest order impurity and phonon scattering, where an extra GLE for the center-of-mass energy has been studied [22]. Here we go a step further and derive a general formula for the electrical conductivity for high field transport with high order impurity scattering. The system under investigation will be assumed to be at *low temperature*, so we can generally use the lowest order electron–phonon scattering approximation. Also, for simplicity, the electron–electron interaction will be neglected here but, hopefully, will be the subject of a future study.

The GLE for the motion of the center of mass is obtained in the procedure of solving the second order Heisenberg equations for the fluctuating density of the relative electrons [21]. Following the same procedure as in ref. [21] (linear GLE with high order impurity scattering) and in ref. [22] (high field GLE with the lowest order impurity scattering), the high field GLE in presence of high order scattering is obtained as

$$M\ddot{\boldsymbol{R}}(t) = \boldsymbol{G}(\dot{\boldsymbol{R}}; t) + \boldsymbol{F}(t) + Ne\boldsymbol{E} , \tag{2.1}$$

where $\boldsymbol{R}$ and $M = Nm$ are, respectively, the center-of-mass coordinate and the mass of N non-interacting electrons, $\boldsymbol{E}$ is the external field, and $\boldsymbol{F}(t)$, $\boldsymbol{G}(t)$ denote the random force and frictional force, respectively. The general form of $\boldsymbol{F}(t)$ is listed in eq. (11) of ref. [21], and it will vanish after averaging over the

system. The frictional force $\boldsymbol{G}(t)$ in (2.1) is defined as

$$G_\alpha(\dot{\boldsymbol{R}};t) = -\int_{-\infty}^{t} \mu^{(0)}_{\alpha\beta}(\dot{\boldsymbol{R}};t,t')\dot{R}_\beta(t')\,\mathrm{d}t' + G^{(1)}_\alpha(\dot{\boldsymbol{R}};t)$$
$$\equiv G^{(0)}_\alpha(\dot{\boldsymbol{R}};t) + G^{(1)}_\alpha(\dot{\boldsymbol{R}};t)\,, \tag{2.2}$$

where $G^{(0)}_\alpha(\dot{\boldsymbol{R}};t)$ is the frictional force due to the lowest order impurity and phonon scatterings, which contains a non-Markovian memory function,

$$\mu^{(0)}_{\alpha\beta}(\dot{\boldsymbol{R}};t,t') \equiv \sum_{\substack{kq\\ s}} \mu^{s}_{kq,\alpha\beta}(\dot{\boldsymbol{R}};t,t')$$
$$\equiv \sum_{\substack{kq\\ s}} d^{s}_{kq,\alpha\beta} \exp\left\{-\mathrm{i}\omega^{s}_{kq}(t-t') + \mathrm{i}\boldsymbol{q}\cdot\oint_{t'}^{t} \dot{\boldsymbol{R}}(t_1)\,\mathrm{d}t_1\right\}, \tag{2.3}$$

where s denotes different scattering mechanisms, i.e. s = i, A, E, for impurity, phonon–absorption and phonon–emission respectively, and

$$\omega^{\mathrm{i}}_{kq} = \omega_{kq}\,, \qquad \omega^{\mathrm{A}}_{kq} = \omega_{kq} + \Omega_q\,, \qquad \omega^{\mathrm{E}}_{kq} = \omega_{kq} - \Omega_q\,, \tag{2.4}$$

with $\omega_{kq} = \boldsymbol{k}\cdot\boldsymbol{q}/m$, Ω_q is the energy of a $\boldsymbol{q}$ mode phonon and Planck's constant $\hbar = 1$ is used. Furthermore

$$d^{\mathrm{im}}_{kq,\alpha\beta} = \frac{n_{\mathrm{i}} q_\alpha q_\beta |U(\boldsymbol{q})|^2}{\omega_{kq}}\,\phi_{kq}\,, \tag{2.5}$$

$$d^{\mathrm{A}}_{kq,\alpha\beta} = \frac{q_\alpha q_\beta |M(\boldsymbol{q})|^2}{\omega^{\mathrm{A}}_{kq}}\,\phi_{kq} b_q b^{\dagger}_q\,, \tag{2.6}$$

$$d^{\mathrm{E}}_{kq,\alpha\beta} = \frac{q_\alpha q_\beta |M(\boldsymbol{q})|^2}{\omega^{\mathrm{E}}_{kq}}\,\phi_{kq} b^{\dagger}_{-q} b_{-q}\,, \tag{2.7}$$

where n_{i} is the impurity density, $U(\boldsymbol{q})$, $M(\boldsymbol{q})$ are the electron–impurity and electron–phonon interacting strength respectively, and

$$\phi_{kq} = [\rho_{kq}, \rho^{\dagger}_q] = C^{\dagger}_{k-q/2} C_{k-q/2} - C^{\dagger}_{k+q/2} C_{k+q/2}\,. \tag{2.8}$$

The $G^{(1)}_\alpha(\dot{\boldsymbol{R}};t)$ in (2.2) represents the frictional force due to higher order impurity scattering, which is

$$G^{(1)}_\alpha(\dot{\boldsymbol{R}};t) \equiv \sum_{kq} G^{(1)}_{kq,\alpha}(\dot{\boldsymbol{R}};t) = -\mathrm{i}\sum_{kqq'} \frac{q_\alpha}{\omega_{kq}}\, U^{\mathrm{i}}_{q}(\boldsymbol{R}) U^{\mathrm{i}}_{-q'}(\boldsymbol{R})\phi_{kqq'}\,, \tag{2.9}$$

where $U_q^i(\boldsymbol{R}) = U(q)\,\Sigma_a\, e^{iq\cdot(\boldsymbol{R}-\boldsymbol{R}_a)}$, $\boldsymbol{R}_a$ is the position of impurity a, and

$$\dot{\phi}_{kqq'} = -i[[\rho_{kq}, \rho_{q'}^{\dagger}], H]\,. \tag{2.10}$$

We note that in ref. [21] we were interested in the low field case, where the $\dot{\boldsymbol{R}}$ term in $\mu_{\alpha\beta}^{(0)}$ of (2.3) was dropped, making $\mu_{\alpha\beta}^{(0)}$ independent of the field; hence $G_\alpha^{(0)}$ and $G_\alpha^{(1)}$ of (2.2) became linear in $\dot{\boldsymbol{R}}$, and a self-consistent formula for G_α was obtained. Also, in ref. [22], we have shown that in the high field steady state case, if we neglect the velocity fluctuation and introduce the drift velocity $V_d = \langle \dot{R}_x \rangle$, then after taking the ensemble average over the center-of-mass coordinates, and taking the field $\boldsymbol{E} = E\hat{\boldsymbol{x}}$, the memory function will have the same form as its low field correspondence except the ω_{kq}^s in (2.3) is replaced by $\omega_{kq}^s - q_x V_d$. It follows then that a self-consistent formula for the G_α of (2.2) can be derived with the same procedure as in ref. [21]. The result is expressed in terms of the Fourier transform of G_α,

$$G_\alpha(V_d;\omega) = -\mu_{\alpha\beta}(V_d;\omega)\dot{R}_\beta(\omega) \equiv G_\alpha^{(0)}(V_d;\omega) + G_\alpha^{(1)}(V_d;\omega)\,, \tag{2.11}$$

where the Fourier transform is defined by

$$\mu_{\alpha\beta}(V_d;\omega) = \int_0^\infty \mu_{\alpha\beta}(V_d;t)\, e^{i\omega t}\, dt\,, \qquad \mathrm{Im}\,\omega > 0\,, \tag{2.12}$$

and $G_\alpha(V_d;\omega)$ in the same way. If we write

$$\mu_{\alpha\beta}(V_d;\omega) = \sum_q \mu_q(V_d;\omega) = \sum_{kq} \mu_{kq,\alpha\beta}(V_d;\omega)\,,$$

then we find that the self-consistent equation for the memory function is

$$\mu_q(V_d;\omega) = \mu_q^{(0)}(V_d;\omega) + \sum_k P_{kq}(V_d;\omega)\mu_{kq}(V_d;\omega) + \sum_{kq'} Q_{kq}(V_d;\omega)\mu_{k,-q'}(V_d;\omega)\,, \tag{2.13}$$

where

$$P_{kq}(V_d;\omega) = n_i \sum_{q'} |U(q')|^2 \left\{ \frac{2}{(\omega+\omega_0)^2 - \omega_{kq}^2} - \frac{1}{(\omega+\omega_0)^2 - \omega_{k+q',q}^2} - \frac{1}{(\omega+\omega_0)^2 - \omega_{k-q',q}^2} \right\}, \tag{2.14}$$

$$Q_{kq}(V_d;\omega) = n_i \frac{q}{q'} |U_q(q)|^2 \sum_{i=1,4} (-1)^i \frac{1}{(\omega+\omega_0)^2 - \omega_{k_iq}^2}, \tag{2.15}$$

and $k_{1,3} = k \pm (q'+q)/2$, $k_{2,4} = k \pm (q'-q)/2$, $\omega_0 = q_x V_d$. We note that if we take $\omega_0 = 0$ in eq. (2.13), then we recover the low field expression for μ as given in ref. [21], eqs. (28)–(30). The real part of the first term on the RHS of (2.13) is [22, 26]

$$\text{Re}\,\mu_q^{(0)}(V_d;\omega) = \frac{n_i|U(q)|^2 q_x^2}{\omega+\omega_0}\chi_2^0(q,\omega+\omega_0) + \frac{2|M(q)|^2 q_x^2}{\omega+\omega_0} \times [n(\Omega_q/T) - n((\omega+\omega_0+\Omega_q)/T_e)]\chi_2^0(q,\omega+\omega_0+\Omega_q), \tag{2.16}$$

where $n(x) = 1/(e^x - 1)$, $\chi^0 = \chi_1^0 + i\chi_2^0$, and

$$\chi^0(q,\omega) = \sum_k \frac{f_{k-q/2} - f_{k+q/2}}{\omega - \omega_{kq} + i\delta}, \tag{2.17}$$

the Fermi distribution function $f_k = \{\exp(\varepsilon_k - \varepsilon_f)/T_e + 1\}^{-1}$, and T_e is the electron temperature. The imaginary part of $\mu^{(0)}(V_d,\omega)$ has the property that $\text{Im}\,\mu^{(0)}(V_d,0) = 0$.

Finally, the form of the conductivity in the present case is found to be the same as in the low field case (ref. [21], eq. (18)):

$$\sigma(V_d;\omega) = \frac{ine^2}{m}\left\{\omega + \frac{i\mu(V_d;\omega)}{M}\right\}^{-1}, \tag{2.18}$$

where n is the electron density.

In summary, we have obtained a general expression (2.13) for the memory function which includes the high field and high order impurity scattering effects. The ac conductivity can be calculated by substituting the solution of (2.13) into (2.18). Strictly speaking, one has yet to determine the electron temperature T_e appearing in (2.16) and (2.17), which is a function of the high electric field and is usually calculated by studying the energy transport equation [22, 26]. However, we are interested in this paper in the weak localization problem, which is represented by the last two terms in (2.13) not directly related to T_e. Therefore, we leave the discussion of the energy transport to appendix A, and continue to discuss, in the following sections, the application of the present formalism to the weak localization problem.

3. Weak localization in the low field limit

The theory of weak localization in the case of a low electric field is well established now [3–5]. It is known that the quantum interference effect due to the back-scattering of electrons will reduce the conductivity. Including only this process, the static conductivity σ is [3]

$$\sigma = \frac{ne^2\tau}{m} - \frac{2e^2}{\pi\hbar}\frac{1}{L^d}\sum_Q Q^{-2}\,, \tag{3.1}$$

where τ is the momentum relaxation time, L and d refer to the length and dimension of the system, respectively, and Q is the magnitude of the sum of the momenta of the initial and final states in the electron back-scattering. The lower cut-off for Q for a system of length L is $1/L$, the upper cut-off being $Q_c \sim l^{-1}$, where $l = V_f\tau$ is the elastic scattering length. The scale dependent *weak* electric field conductivity is then obtained for different dimensional cases as

$$\sigma_{1D}(L) = \sigma_0 - \frac{e^2}{\hbar\pi}(L - l)\,, \tag{3.2a}$$

$$\sigma_{2D}(L) = \sigma_0 - \frac{e^2}{\hbar\pi^2}\ln\frac{L}{l}\,, \tag{3.2b}$$

$$\sigma_{3D}(L) = \sigma_0 - \frac{e^2}{\hbar\pi^3}\left[\frac{1}{l} - \frac{1}{L}\right], \tag{3.2c}$$

where $\sigma_0 = ne^2\tau/m$. These results refer to the *zero temperature* situation. At finite temperatures L is to be taken as the inelastic diffusion length [3]. As we mentioned in the introduction, the existing theoretical methods for obtaining (3.2) are often complicated and not easily extended to the high field case. We now apply our conductivity formalism, (2.13) and (2.18), to study the weak localization problem, concentrating on the low field case in this section and on the high field case in the next section.

In the low field limit, the conductivity (2.18) as well as the memory function (2.13) are field independent. At $T = 0$, we *neglect* the *phonon contribution*, and the $\mu_q^{(0)}$ of (2.16) will contain only the first term on the RHS due to the impurity scattering.

Then the memory function, given by (2.13)–(2.15), is reduced to

$$\mu_q(\omega) = \mu_q^{(0)}(\omega) + \sum_k P_{kq}(\omega)\mu_{kq}(\omega) + \sum_{kq} Q_{kq'}(\omega)\mu_{k,-q'}(\omega)\,, \tag{3.3}$$

$$P_{kq}(\omega) = n_i \sum_{q'} |U(q')|^2 \left\{ \frac{2}{\omega^2 - \omega_{kq}^2} - \frac{1}{\omega^2 - \omega_{k+q',q}^2} - \frac{1}{\omega^2 - \omega_{k-q',q}^2} \right\}, \tag{3.4}$$

$$Q_{kq}(\omega) = n_i \frac{q}{q'} |U(q)|^2 \sum_{i=1,4} (-1)^i \frac{1}{\omega^2 - \omega_{k_i q}^2}, \tag{3.5}$$

where k_i is the same as in (2.15). The last two terms on the RHS of (3.3) are the higher order impurity scattering contributions. In the static limit, it is straightforward to check that the last two terms contained in $P_{kq}(\omega)$ of (3.4) diverge and make the dominant contributions when $k \pm q' \to 0$, $q = q_f$. Thus, in the $\omega \to 0$ limit, (3.3) can be rewritten as

$$\mu_q(\omega) = \mu_q^0(\omega) - 2n_i U^2 \mu_q(\omega) \sum_{k'} \frac{1}{\omega^2 - \omega_{k'q}^2}, \tag{3.6}$$

where we have shifted variables from $k \pm q'$ to k', and used the relation $\mu_q(\omega) = \Sigma_k \mu_{kq}(\omega)$, and for short range impurities $|U(q)|^2 = U^2$. It so happens that the higher order impurity scattering contribution in (3.6) just corresponds to the "back-scattering" electrons with original state near q_f, scattered to the final state near $-q_f$, with the sum of the wave vectors of the two states $k' \to 0$, and energy change $\omega_{k'q} = k' q_f / m$. It is clear now that the dominant contribution to the memory function $\mu(\omega) = \Sigma_q \mu_q(\omega)$ comes from q values near q_f. Summing both sides of (3.6) over q and using the approximation $\mu(\omega) \approx \mu_{q_f}(\omega)$ for the second term we obtain, after reinserting Planck's constant $\hbar$,

$$\mu(\omega) = \mu^0(\omega) - \mu(\omega) \frac{2m}{N\pi\hbar\tau} \sum_k \frac{1}{k_\omega^2 - k'^2}, \tag{3.7}$$

where $k_\omega = m\omega/\hbar q_f$, $\hbar/\tau \equiv 2\pi n_i U^2 N(0)$, $N(0) = N/2\varepsilon_f$ is the density of states per spin, and the factor 2 in the last term takes account of the spin degeneracy of k'. Substituting the $\mu(\omega)$ obtained from (3.7) into (2.18) we obtain the *low frequency limit* conductivity including the back-scattering effects:

$$\sigma(\omega) = \frac{ine^2/m}{\omega + \dfrac{i\mu^0(\omega)}{M} \left\{ 1 + \dfrac{2m}{N\pi\hbar\tau} \sum_{k'} \dfrac{1}{k_\omega^2 - k'^2} \right\}^{-1}}. \tag{3.8}$$

The above conductivity formula is the main result of this section. In the static limit ($\omega = 0$), we have $k_\omega = 0$ and $\mu^0(0)/M = 1/\tau$, the latter result following

from (2.16) at $T=0$ and $T_e=0$. Thus (3.8) is reduced to

$$\sigma(0)=\frac{ne^2\tau}{m}\left\{1-\frac{2m}{N\pi\hbar\tau}\sum_{k'}\frac{1}{k'^2}\right\} \tag{3.9}$$

which is the same as the well known result of (3.1). Thus, our theory gives the same results as the scaling theory and other perturbation theories in the low field limit.

The low frequency conductivity near the weak localization limit can also be obtained directly from (3.8). If we use the lowest order expansion for the memory function $\mu^0(\omega\to 0)/M=1/\tau+i\omega A$, with $A=\{\mathrm{Im}\,\mu^0(\omega)/M\omega\}_{\omega=0}$ a finite number, then it follows immediately from (3.8) that

$$\sigma_d(\omega\to 0;L)=\sigma_0\frac{f_d(\omega,L)}{1+\omega^2\tau^2[f_d(\omega,L)-A]^2}\,, \tag{3.10a}$$

with

$$f_d=1+\frac{2m}{N\pi\hbar\tau}\sum_{k'}\frac{1}{k_\omega^2-k'^2}\,, \tag{3.10b}$$

where $\sigma_0=ne^2\tau/m$ and again d denotes dimensions. For $d=3$ it turns out that $\sigma_3(\omega)>\sigma_0$ and thus no weak localization occurs. For $d=1$ and 2, there exists weak localization. $f_d(\omega,L)$ can be obtained by carrying out the sum contained in the denominator of (3.8) with the results

$$f_1(\omega,L)=1-\frac{m}{n\tau\hbar\pi^2}\frac{1}{2k_\omega}\ln\left|\frac{1-k_\omega l}{1+k_\omega l}\frac{1+k_\omega L}{1-k_\omega L}\right|\,, \tag{3.11}$$

$$f_2(\omega,L)=1-\frac{m}{n\tau\hbar\pi^2}\frac{1}{2}\ln\left|\frac{l^{-2}-k_\omega^2}{L^{-2}-k_\omega^2}\right|\,, \tag{3.12}$$

where $k_\omega=m\omega/\hbar q_f$, $k_\omega l=\omega\tau$, $k_\omega L=\omega\tau_{in}$, and τ_{in} is the inelastic scattering time. When $L\to\infty$, we denote $\sigma_d(\omega\to 0,L\to\infty)$ by $\sigma_d(\omega)$, and obtain the low frequency conductivity (3.10) by using (3.11) or (3.12) as follows:

$$\sigma_{1D}(\omega)=\frac{e^2V_f\tau}{3\hbar\pi^2}\omega^2\tau^2\,, \tag{3.13}$$

$$\sigma_{2D}(\omega)=\sigma_0+\frac{e^2}{\hbar\pi^2}\ln\omega\tau\,, \tag{3.14}$$

where in (3.13) we have used the fact that $\sigma_{1D}(0)=0$. The above results show that the $\omega\to 0$ behavior of $\sigma_d(\omega)$ depends on dimensionality. While $\sigma_{1D}(\omega)$

rises with ω^2, the $\sigma_{2D}(\omega)$ falls logarithmically as frequency decreases, so that $\sigma_{2D}(\omega \to 0) \to 0$. We note that, unlike the static case, where our results (3.9) are in total agreement with (3.1) of the other treatments, the results of the low frequency conductivity seen in (3.13) and (3.14) are only in qualitative agreement with other approaches. In the 2D case, the logarithmic term in our expression (3.14) is two times larger than the Gorkov et al. results [6] obtained by a diagram calculation. Also, the $\sigma_{1D} \sim \omega^2$ in (3.13) is qualitatively the same behavior as compared with the famous Mott formula $\sigma_{1D}(\omega \to 0) \sim \omega^2$, although the latter formula is obtained from a totally different approach, the resonant absorption theory [28].

4. The electric field dependence of the weak localization

In this section we study the static conductivity $\sigma(V_d)$ in the presence of high order impurity scattering and high electric field. Using our formalism, the static conductivity in this case can be written by using (2.13) and (2.18) as

$$\sigma(V_d) = \frac{Ne^2}{m} \frac{M}{\mu(V_d)} , \tag{4.1}$$

$$\mu(V_d) = \mu^{(0)}(V_d) + \sum_{kq} P_{kq}(V_d)\mu_{kq}(V_d) + \sum_{kqq'} Q_{kq}(V_d)\mu_{k,-q'}(V_d) , \tag{4.2}$$

where $\mu^{(0)}(V_d)$ is defined by (2.16) and $P_{kq}(V_d)$, $Q_{kq'}(V_d)$ are the $\omega = 0$ expressions of (2.14) and (2.15) respectively.

First we study the static conductivity (4.1) for the case of intermediate field strength, and restrict ourselves to the calculation of the first order electric field contributions to the conductivity, i.e. we study the electric field corrections to the conductivity (3.8) given in the last section.

The high field static conductivity, including the back-scattering contribution, can be put, using (4.2), into a form analogous to (3.7) as

$$\mu(V_d) = \mu^0(V_d) - \mu(V_d) \frac{2m}{N\pi\hbar\tau} \sum_{k'} \frac{1}{k_V^2 - k'^2} , \tag{4.3}$$

where $k_V = mV_d/\hbar$ and τ is the same as that of (3.7). The first term on the RHS of (4.3) is the high field memory function (in Fourier space and $\omega = 0$) due to the lowest order electron–impurity and electron–phonon scattering, the expression for which is listed in (2.16). The second term on the RHS of (4.3) is the electron back-scattering contribution due to the high order impurity scattering, where the k'^{-1} is understood to have cut-off length l and L.

It is clear from (4.3) that the electron field effect on the memory function (and on the conductivity through (2.18)) is to revise both the $\mu^{(0)}$, and the high order impurity scattering contribution. The calculation of the $\mu^{(0)}(V_d)$ is an interesting subject in the study of the hot electron transport and is not the main concern to the present study. So we will just introduce $1/\tau(V_d) = \mu^0(V_d)/M$ to represent the electric field effect on the transport time [29]. The evaluation of the back scattering contribution in (4.3) is the core of the present investigation. For the case of an intermediate electric field, we identify two different cases for the drift velocity $V_d \ll \hbar/mL$ and $\hbar/mL \ll V_d \ll \hbar/ml$.

In the first case, when $V_d \ll \hbar/mL$ ($k_V \ll 1/L$), we observe immediately that the presence of the k_V in (4.3) is to enhance slightly the back-scattering contribution. The system will keep the basic properties of the static conductivity of (3.1) discussed in the last section. An example of this situation is the experiment of Roukes et al. [30] where $L \sim 1$ cm at a lattice temperature of 25 mK and the corresponding electric field is 10^{-6} V cm^{-1}. Thus we are in the hot electron region but, because of the relatively low drift velocity, the conductivity is essentially independent of the electric field.

In the second case, when $\hbar/mL \ll V_d \ll \hbar/ml$, the situation is much more interesting and will be taken as the definition of the intermediate region of the electric field E. The experiments of Dolan and Osheroff [16] (which were the first to obtain observations of the ln T dependence of the resistance of thin metal films) is an example where this condition is fulfilled. These authors are dealing with lattice temperatures as low as 10 mK (so that L is larger) and electric fields of the order of 10^{-3} V cm^{-1} (see fig. 1 of ref. 16), with the result that drift velocities are much larger so that the condition $(\hbar/mL) \ll V_d$ can be fulfilled. In this case, the static conductivity is, by using (2.18) and (4.3),

$$\sigma_d(V_d, L) = \sigma_d^0(V_d) - \frac{e^2}{\hbar\pi^2} f_d(V_d, L), \tag{4.4}$$

where

$$f_{1D}(V_d, L) = \frac{1}{m^2 V_d^2 L^2} \{L - l\}, \tag{4.5}$$

$$f_{2D}(V_d, L) = \ln \frac{\hbar}{mV_d l}, \tag{4.6}$$

$$f_{3D}(V_d, L) = \frac{1}{\pi} \left\{ \frac{1}{l} - \frac{1}{L} \right\}. \tag{4.7}$$

The above results show that the electric field correction to the back-scattering contribution of the conductivity is strongly dimensionally dependent while in

the 3D case the $f_{3D}(V_d, L)$ of (4.7) is essentially the same as its low field correspondence in (3.2), the $f_{2D}(V_d, L)$ and $f_{1D}(V_d, L)$ of (4.5) and (4.6) are quite different compared with that of (3.2). In both cases, the localization effect is reduced when we increase the electric field, represented by the fact that the L of the low field limit conductivity (3.2) is now replaced by a smaller one; for the 2D case it is $\hbar/mV_d$, and for the 1D case it is $\hbar^2/m^2V_d^2L$. Thus, we conclude that the electric field, even with intermediate range strength, will delocalize the 1D and 2D electron systems.

We note that *in our formalism* (4.4)–(4.7), *the temperature effect does not enter explicitly*. Actually, the drift velocity V_d in these equations is a function of electron temperatures T_e, if we adopt the heating concept of high field transport. To lowest order of V_d, it is straightforward to show that $V_d^2 \sim T_e - T$ [31]. In other words, if we allow the electrons to be heated up, then the localization effect will have a field dependence represented by (4.5) and (4.6) for the 1D and 2D cases respectively. This is consistent with the experimental result [16, 17] and the theoretical heating model [2]. On the other hand, if we assume that the electrons will always be kept at the same temperature as the lattice, then we will get $V_d \to 0$ in our treatment, and no field influence will show in the localization effect. This latter conclusion is in agreement with Bergmann's experiment [19] and Altshuler's theoretical results [7]. Thus, our theory is capable of explaining both sides of the controversy concerning the electric field influence on the localization effect [5].

Finally, we note that the high electric field case should be expected qualitatively to have even stronger delocalization behavior compared to that of the intermediate electric field case. This is because when the drift velocity V_d is comparable to V_f, the divergent nature of the second term on the RHS of (4.3) does not exist any longer.

5. Conclusion

In this paper we have presented a generalized quantum Langevin equation analysis of the weak localization phenomenon in the presence of an electric field of *arbitrary strength*. In our formalism, the conductivity (see (2.18)) is solely determined by the memory function, which appears in the GLE as the kernel of the frictional force and can be determined under different levels of approximation. A general memory function formula (2.13) for the non-interacting electron gas, in the presence of high order impurity scattering and arbitrary electric field, is derived. Our approach is basically from first principles and rigorous. By using our conductivity and memory function formulas, the well known dimension dependent results (3.2) for the conductivity in the low

field and weak localization limits are easily rederived. At the same time the low frequency conductivity $\sigma(\omega)$ of (3.9) is also obtained. Our results for $\sigma_{2D}(\omega)$ show a similar logarithmic dependence for $\omega\tau$ as $\omega\tau \to 0$, in agreement with other approaches. In addition, the $\sigma_{1D}(\omega + 0)$ is found to rise as ω^2, consistent with other theories. For the purpose of studying the influence of a strong electric field on the weak localization phenomenon, we have deduced the $\omega = 0$ memory function (4.3) from the general formula (2.16). By using this formula, we have shown that under the electron heating up assumption, a strong electric field will delocalize the 1D and 2D electron systems (which are localized in the low field limit). Also, we have shown that the electric field will have no influence on the weak localization if one always keeps the electron and lattice at the same temperature.

Concerning the nature of our approach to the weak localization problems, it is totally different from the other approaches in the literature [1–15], where the standard diagrammatic analysis and its combination with the mode coupling theory are the methods commonly used. Whereas all these methods are basically extensions of the Kubo formalism to the non-linear problem for the current–current correlation function [3], our approach dispenses with the need of intermediate steps involving the evaluation of correlation functions. The infinite set of electron back-scattering contributions (appearing in other theories as maximally crossed diagrams) is taken care of in our theory by deriving the self-consistent memory function formula (2.13) which makes our theory much simpler than the other ones.

Apart from the simplicity of our approach discussed above, the main advantages of the present theory for the weak localization problem is the capability to incorporate strong electric field effects quite conveniently and the ability to explain the controversial results of the electric field dependence of weak localization under different electron temperature assumptions.

In the future, we hope to extend our analysis to include the presence of a magnetic field. Speaking qualitatively, it is clear that the influence of a magnetic field will be to introduce an extra term ω_c^2 (where ω_c is the cyclotron frequency) into the denominators of (3.4) and (3.5). This in turn will suppress the divergence of these terms in the $\omega \to 0$ limit leading to the expected suppression of the weak localization due to the presence of a magnetic field [3].

Finally, what about the higher-order impurity terms discussed by Huberman and Chester [32]? These involve an expansion in powers of λ^2/ε and are evaluated [32] by first taking the steady-state limit ($\varepsilon \to 0$) and then $\lambda \to 0$ (weak scattering limit) or else by using the technique of Van Hove [33]. However, for $T = 0$ (which is the main thrust of the present paper and also ref. [21]) these terms make no difference [32]. For $T \neq 0$ and high frequencies ($\omega\tau \gg 1$), ε is replaced by $\omega + i\varepsilon$ and again these terms do not contribute [34].

But even for low frequencies at $T \neq 0$, ε must be replaced by a finite quantity (because phonon effects imply broadening [22]) and this case requires further study (neglecting the phonon interaction while keeping temperature effects in the statistics [32] is inconsistent and misses the key point that dissipation and broadening cannot be provided by impurities alone).

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Appendix A

The high field energy transport equation in the high order impurity scattering

The energy evolution of the center-of-mass electrons can also be studied via its energy GLE. The non-linear GLE energy equations for the electronic high field transport in the lowest order electron–impurity interaction was derived by us in ref. 21. In the presence of high order impurity scattering, from the momentum GLE (2.11), (2.12), using similar techniques, we obtain the energy GLE as

$$\frac{\mathrm{d}}{\mathrm{d}t}\,\varepsilon_{\dot{R}}(t) + \dot{H}_{\mathrm{e}} + \dot{H}_{\mathrm{ph}} = NeE \cdot V\,, \tag{A.1}$$

where

$$\dot{H}_{\mathrm{e}} = \dot{H}_{e}^{(0)} + \dot{R}_{\alpha} G_{\alpha}^{(1)}(\dot{R}; t)\,, \qquad \dot{H}_{\mathrm{ph}} = \dot{H}_{\mathrm{ph}}^{(0)}\,, \tag{A.2}$$

$\dot{H}_{\mathrm{e}}^{(0)}$, $\dot{H}_{\mathrm{ph}}^{(0)}$ are the energy changing rate of relative electrons and phonons in the lowest order electron–impurity and electron–phonon scattering, and are functions of non-Markovian memory functions [22], respectively. The energy GLE (A.1) is quite general and is in its microscopic form. One possible way to solve it is through the balance equation method. We have shown that in the steady state, when the velocity fluctuation is neglected, the energy GLE (A.1) will be reduced to the following energy balance equations [22, 26]:

$$\dot{H}_{\mathrm{e}} = 0\,, \tag{A.3}$$

$$NeEV + \dot{H}_{\mathrm{ph}} \equiv 2 \sum_{q} \Omega_{q} |M(q)|^{2} [n(\Omega_{q}/T) - n((\omega_{0} + \Omega_{q})/T_{\mathrm{e}})] \times \chi_{2}^{0}(\omega_{0} + \Omega_{q})/T_{\mathrm{e}}\,, \tag{A.4}$$

where $\dot{H}_e$ is defined by (A.2). From the above equations, the electron temperature T_e can be obtained, and then the drift velocity V_d, the memory function (2.24) as well as the conductivity (2.21) can be calculated.

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