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Fluctuation effects on the cyclotron resonance spectrum for a two-dimensional electron gas

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Existing theories of the frequency-dependent magnetoconductivity of a two-dimensional electron gas are plagued by the appearance of divergences in the associated memory function. Here we show that such divergences may be removed by the inclusion of fluctuation effects in the calculation of the conductivity. Our calculations are based on a new approach (viz., by the use of a generalized quantum Langevin equation), to quantum transport, which we have recently expounded. Our results display splitting of the cyclotron absorption line, for a range of magnetic field values at low temperature, which was found experimentally by Schlesinger, Allen, Hwang, and Platzman. In addition, our theory gives a very good fit to the heretofore unexplained experimental results of Kennedy, Wagner, McCombe, and Tsui, concerning the increase of the cyclotron resonance mass with decreasing n_s in certain regions. The fluctuation effects are most pronounced for small systems with low electron concentrations.

Existing theories of the frequency-dependent magnetoconductivity of a two-dimensional gas are plagued by the appearance of divergences in the polarization function and, concomitantly, in the associated density-density correlation function, the memory function, and magnetoconductivity.[1-3] It is generally thought that the elimination of such divergences requires a calculation beyond lowest order in the impurity concentration—a nontrivial task.[2] Here we point out that there is another physical effect which can eliminate the divergencies, viz. fluctuations. Before getting into the details of our calculation, we refer to a recent paper by Ford, Lewis, and O'Connell[4] which enables us to make our basic point in a lucid fashion. This paper treats a simple model of noninteracting harmonically bound electrons and neglects scattering from phonons and impurities yet there is a damping term whose origin is scattering associated with radiation reaction forces (which, in turn, arise from fluctuations).

We have recently developed a new approach to quantum transport based on the use of a generalized quantum Langevin equation.[5] Generalizing these results to include the presence of a dc magnetic field, B, we obtain the frequency-dependent magnetoconductivity of a two-dimensional electron gas in the random-phase approximation (RPA) and at low temperature. We work to lowest order in the impurity concentration and, for the convenience of the reader, we will present our result using a notation similar to that used in Ref. 1. In fact, the difference between our results and those of Ref. 1 can be succintly stated as follows. In Ref. 1, Eq. (47), the divergence associated with the sharpness of the Landau energy levels is removed in an "...intuitive way..." by essentially incorporating higher-order impurity contributions in a phenomenological manner, whereas in our calculation the divergences are removed automatically by inclusion of fluctuation effects. Our result for the polarization function is

$$\Pi_{nn'}(\mathbf{q},\omega)=\frac{f(\epsilon_n)-f(\epsilon_{n'})}{\hbar\omega-(\epsilon_{n'}-\epsilon_n)+iDq^2}\ , \tag{1}$$

where

$$\epsilon_n=(n+\tfrac{1}{2})\hbar\omega_c-\varepsilon_F\ ,$$

and where ε_F is the Fermi energy of the electron gas, ω_c is the cyclotron frequency, $f(x)$ is the Fermi distribution function, D is the diffusion constant for the center of mass, and q is the wave vector associated with the electron density operator.

To underline the origin of the fluctuation, we return to our original paper, Ref. 5, where a crucial role was played by $\mathbf{R}(t)$, the coordinate of the center of mass of the system. There, we worked to linear order in the velocity $\dot{\mathbf{R}}(t)$ which, as we pointed out, is equivalent to linear-response theory after averaging over impurities and phonons [see Eq. (16) of Ref. 5]. However, if one includes the fluctuation of the center-of-mass coordinates, then more care is needed. In general, at any time t, we can denote the velocity of the center of mass $\dot{\mathbf{R}}(t)$ as the sum of the drift velocity $\langle\dot{\mathbf{R}}(t)\rangle_{\rm av}$ and its fluctuation $\delta\dot{\mathbf{R}}(t)=\dot{\mathbf{R}}(t)-\langle\dot{\mathbf{R}}(t)\rangle_{\rm av}$, where we use angular brackets to denote the ensemble average over the center-of-mass coordinates. In carrying out the more careful calculation,[6] one must carry out a cumulant average over the quantity $\exp(i\mathbf{q}\cdot\delta\dot{\mathbf{R}})$, which appears in the expression for the memory function, to arrive at the lowest-order contribution from $\langle\delta R^2(t)\rangle_{\rm av}$. The end result is the Dq^2 term in (1), where

$$D=\lim_{t\to\infty}\frac{1}{2t}\langle\delta R^2(t)\rangle_{\rm av}\ . \tag{2}$$

As we have shown elsewhere,[6] D is proportional to the inverse of N, the total number of particles and, thus, the fluctuation effects are most pronounced for small systems

with low-electron concentrations. Also, as a direct result of this feature of $D \sim 1/N$, our expression (1) is valid for $ql_0 < N$. From the polarization function one gets the dielectric function in the RPA,[1]

$$\epsilon(\mathbf{q},\omega) = 1 - V(q)\chi_0(\mathbf{q},\omega) \ ,$$
$$\chi_0(\mathbf{q},\omega) = \sum_{nn'} C_{nn'}(q)\Pi_{nn'}(\mathbf{q},\omega) \ , \tag{3}$$

where $C_{nn'}(q)$ is given by Eq. (43) of Ref. 1, and $V(q) = 2\pi e^2/\kappa_{sc} q$ is the Fourier transform of the Coulomb potential of the two-dimensional electron gas (2D EG), with κ_{sc} denoting the static dielectric constant of the lattice. In the case of the absorption of right-circularly-polarized radiation, the magnetoconductivity is given by

$$\sigma_+(\omega) = \frac{in_s e^2}{m}[(\omega - \omega_c) + M(\omega)]^{-1} \ , \tag{4}$$

where n_s is the number of electrons per unit area, m is the effective band mass, and $M(\omega) \equiv M_1(\omega) + iM_2(\omega)$ is the memory function. As we pointed out in Ref. 5, $M(\omega) = (i/M)\mu(\omega)$ where $M = Nm$, N is the total number of electrons and $\mu(\omega)$, the memory function which appears in the quantum Langevin equation, is given explicitly in terms of ϵ as

$$\mu(\omega) = -iM\omega_c \frac{1}{4\pi\epsilon_F \tau} \frac{\omega_c}{e^2/\kappa_{sc} l_0} \times \int dx \frac{x^4}{y}\left[\frac{1}{\epsilon(x,0)} - \frac{1}{\epsilon(x,y)}\right] \ , \tag{5}$$

where $x = ql_0$, $y = \omega/\omega_c$, $l_0 = (\hbar/m\omega_c)^{1/2}$, and $1/\tau = n_i m U^2$ is the transport time for short-range impurity scattering [the Fourier transform of the impurity potential is $U(\mathbf{q}) \equiv U$]. We note that (5) is obtained directly from Eq. (19) of Ref. 5 with $U(\mathbf{q}) \equiv U$, and the $\epsilon(x,y)$ is now defined through (3). For $x > N$ the contribution to the integral in (5) is negligible.

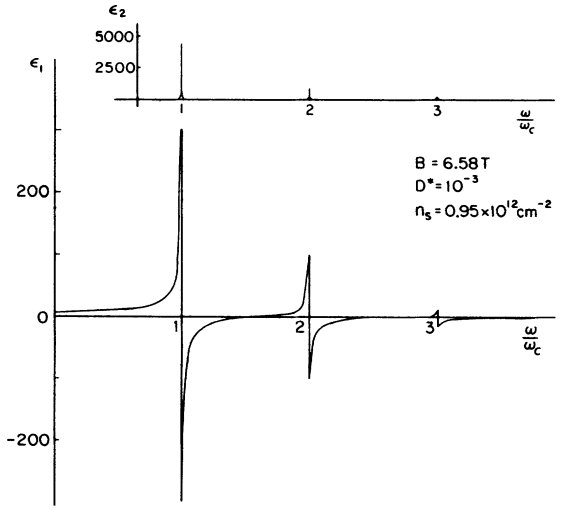

FIG. 1. Plot of the dielectric function $\epsilon \equiv \epsilon_1 + i\epsilon_2$ vs ω/ω_c (frequency/cyclotron frequency), in the case $D^* = 10^{-3}$, $B = 6.58$ T, $n_s = 0.95 \times 10^{12}$ cm^{-2}, and $ql_0 = 0.6$.

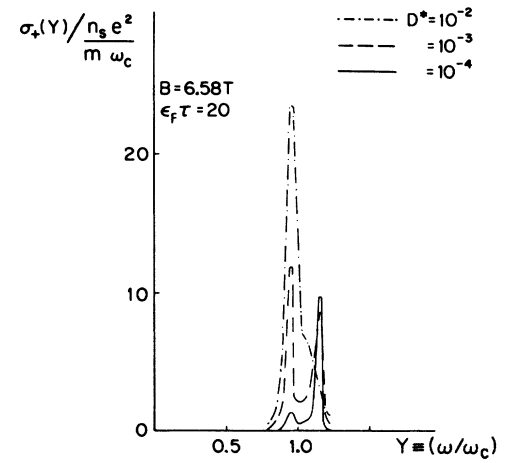

FIG. 2. The influence of the diffusion constant D^* on the cyclotron resonance spectrum.

We now turn to a numerical evaluation of our results, Eqs. (3)–(5). In Figs. 1–3 we plot representative curves corresponding to what we regard as typical experimental scenarios and in Fig. 4 we compare our theory to actual experimental results. In Fig. 1, we plot $\epsilon_1(\mathbf{q},\omega)$ and $\epsilon_2(\mathbf{q},\omega)$ versus frequency for fixed q. We choose $ql_0 = 0.6$ to show the general behavior in the region $ql_0 \sim 1$, where we have taken $B = 6.58$ T, $n_s = 0.95 \times 10^{12}$ cm^{-2}, $D^* \equiv mD/\hbar = 0.001$, and the corresponding number of filled Landau levels $\nu = 3$. The general features are as follows.

(i) $\epsilon_1(\mathbf{q},0) \approx 1$ for all values of $\mathbf{q}$. This indicates that the low-frequency charge disturbance is not strongly screened for the 2D EG in a uniform magnetic field, as distinct from the static dielectric constant of the 2D EG in the absence of the magnetic field, which is known to have the property $\epsilon_1(q,0) \gg 1$, i.e., strongly screened.

(ii) In the vicinity of $\omega = n\omega_c$ ($n = 1,2,\ldots$), $\epsilon_2(\mathbf{q},\omega) \neq 0$ and is symmetric. This reflects the fact that the Landau levels are broadened due to the presence of the dissipation term Dq^2 in the polarization function (1).

(iii) $\epsilon_1(\mathbf{q},\omega)$ is a periodic function, with respect to ω,

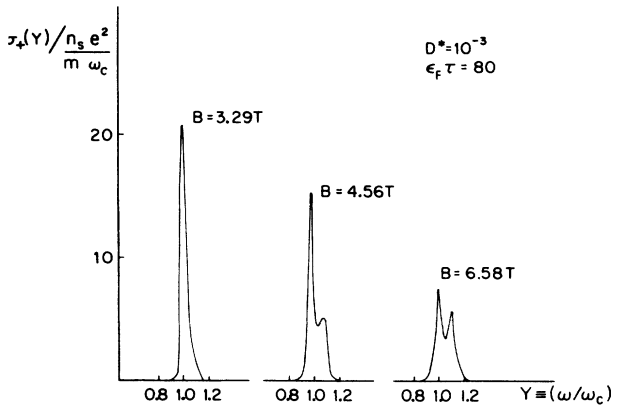

FIG. 3. The cyclotron resonance spectrum at three selected magnetic field values.

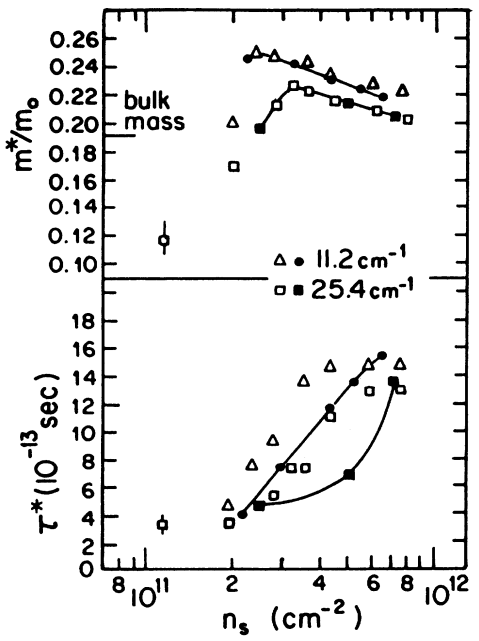

FIG. 4. The density dependence of the effective mass m^* and the scattering time τ^* at two different cyclotron frequencies, 11.2 and 25.4 cm^{-1}. The symbols ● and ■ denote theoretical values, which we have joined by solid lines. The corresponding experimental data of Kennedy *et al.* (Ref. 8) are denoted by △ and □, respectively.

with a period ω_c and a strongly decaying amplitude. Between each consecutive Landau level, there is a zero of ϵ_1, which corresponds to the magnetoplasma excitation. We note that a change of the value of the diffusion constant D will only change the above picture qualitatively. When D increases, the peak values of ϵ_1 and ϵ_2 will decrease and the half width will increase.

We now turn to the case of cyclotron resonance (CR) absorption which is proportional to the real part of $\sigma_+(\omega)$. We consider the case of a short-range impurity potential [$U(q) \equiv U$] and for definiteness we choose Si, for which $m = 0.19 m_0$ (m_0 is the free-electron mass), $\kappa_{sc} = 11.5$. According to (3)–(5), there are four parameters we need to choose: B, n_s, τ, and D^*. We fix $n_s = 0.95 \times 10^{12}$ cm^{-2} to study the influence of B, τ, and D^* on the CR absorption spectrum. Also, to simplify the discussion we choose $B = 3.29$, 4.56, 6.58 T, which corresponds to completely fill Landau levels $\nu = 6$, 4, 3. Our results are given in Figs. 2 and 3 where we plot σ_+ versus frequency. A striking feature of these results is the splitting of the cyclotron absorption line for a range of magnetic field values.

In the classical approximation, one usually takes the memory function in (4) as $M(\omega) = i/\tau$, i.e., the frequency dependence is neglected and the real part is taken to be zero, so that one simply obtains a single CR peak as is clearly obtained from (4). Our calculation shows that the real part of the memory function, $M_1(\omega)$, has strong frequency dependence near $\omega = \omega_c$, which is a direct consequence of the structure of $\epsilon(\omega)$ seen in Fig. 1. In general, $M_1(\omega)$ is a monotonically decreasing function near integral values of ω/ω_c and it changes sign at a value of ω/ω_c slightly larger than one, while $M_2(\omega)$ is a Gaussian-type positive function with peak value near $\omega = \omega_c$. The competition between $M_1(\omega)$ and $M_2(\omega)$ in (4) gives rise to the appearance of a double peak in the CR spectrum. Clearly, the peak behavior of σ_+ depends on the changing behavior of $M_1(\omega)$ and $M_2(\omega)$ near ω_c, which is determined by D^*, B, τ, n_s, and we describe their influence in turn in the following.

Figure 2 shows the influence of D^* on the CR spectrum, with $D^* = 10^{-2}$, 10^{-3}, and 10^{-4} ($B = 6.58$ T, $\tau = 1.1 \times 10^{-12}$ s). These values of D^* give a range of values corresponding to what one would expect in the experimental situation if we use the classical formula $D^* = mD/\hbar = k_B T\tau/N\hbar$ to make an estimate. For example, $D^* = 10^{-3}$, if we take $T \approx 2$ K, $\tau = 5 \times 10^{-12}$ s, $n_s = 0.5 \times 10^{12}$ cm^{-2} and assume that the concerned area of the sample is $A = (0.5\ \mu m)^2$. In all the cases, there are two absorption peaks on the two sides of $\omega = \omega_c$. As indicated by the figure, when D^* is increased, the values of the $\omega < \omega_c$ peak will increase, those of the $\omega > \omega_c$ peak will decrease, and the half width of both peaks will increase. We note that when $D^* = 0$, the peak on the $\omega < \omega_c$ side will vanish and the peak on the $\omega > \omega_c$ side will finally diverge at $\omega = \omega_c$, which is the well-known result of the regular RPA treatment.[7]

Figure 3 demonstrates the effect of B on the CR spectrum, where we choose $n_s = 0.95 \times 10^{12}$ cm^{-2}, $\tau = 4.4 \times 10^{-12}$ s, and $D^* = 10^{-3}$, to obtain three CR peaks at $B = 3.29$, 4.56, 6.58 T. As can be seen from Fig. 3, at low magnetic field (high number of filled levels and small ω_c) one can identify only one peak centered at $\omega = \omega_c$. When one increases the magnitude of B, the peak starts to split (as in the case of $B = 4.56$ T), and eventually two almost symmetrical peaks around $\omega = \omega_c$ (as in the case of $B = 6.58$) can be identified. Also, the basic physical behavior remains when we change the values of n_s, B, τ, and D^*. The theoretical finding here is very consistent with the experimental results,[8–10] and the values we used for the physical quantities n_s, τ, B, D^* fall in a reasonable range compared with the experimental situation. We note that this splitting phenomenon was previously explained by Schlesinger, Allen, Hwang, and Platzman[10] as being due to two-dimensional magnetoplasmon softening. Actually, we have verified that such a magnetoplasmon softening can arise by inclusion of the fluctuation effects discussed here. In addition, we have found that the real part of the memory function $M_1(\omega)$ also plays an important role through the renormalization of the cyclotron mass and scattering time. This latter point will be explored more when we compare our theory directly to the experimental results.

Finally, we compare our theory to the well-known experimental paper (for Si) by Kennedy, Wagner, McCombe, and Tsui,[8] where they found that the cyclotron resonance mass increases with decreasing n_s in certain regions (see Fig. 4), and the reason for which is still unknown.[3] For this purpose we rewrite (4) as[1]

$$\sigma_+ = \frac{in_s e^2}{m^*(\omega)} \left[\omega - \omega_c^* + \frac{i}{\tau^*(\omega)} \right]^{-1}, \qquad (6)$$

where

$$\omega_c^* = \frac{eB}{m^*(\omega)c} ,$$

$$m^*(\omega) = m\left[1+\frac{M_1(\omega)}{\omega}\right] ,$$

$$\frac{1}{\tau^*(\omega)} = M_2(\omega)\Big/\left[1+\frac{M_1(\omega)}{\omega}\right] ,$$

defines the effective CR frequency, mass, and scattering time, respectively. Our numerical results show that in the region where experiments were performed [2×10^{-11} (cm^{-2}) $< n_s < 8\times10^{-11}$ (cm^{-2}), and 11.2 $cm^{-1} < \omega < 61.3$ cm^{-1}], the cyclotron effective mass m^* decreases, while the effective scattering time τ^* increases, when the density n_s is increased. Also, this qualitative feature remains for all realistic values of D^* (of the order of 10^{-4} to 10^{-2}) which we calculated. In Fig. 4, we present our theoretical best-fitting curves for the m^* and the corresponding values of τ^*, together with the experimental results.[8] These curves are calculated at $D^* = 0.012(10^{12}$ $cm^{-2}/n_s)$ and integer filling Landau levels, with $n_s = 0.22$, 0.33, 0.44, 0.55, 0.66 ($\times10^{12}$ cm^{-2}), $\tau = 2\times10^{-13}$ s for $\omega_c = 11.2$ cm^{-1}, and $n_s = 0.25$, 0.5, 0.75 ($\times10^{12}$ cm^{-2}), $\tau = 0.4\times10^{-13}$ for $\omega_c = 25.4$ cm^{-1}. The fitting of m^* is surprisingly good, except we have to take slightly different values of τ for a best fit for both curves. We think this may be due to the fact we have used the simplest impurity model to perform this calculation. In the lower half of Fig. 4, we plot the calculated values of $\tau^*(\omega)$ at the corresponding best-fitting points of $m^*(\omega)$. The figure shows that $\tau^*(\omega)$ fall in a range close to the experimental data. Based on our numerical results, it is clear that the critical factor for this physical phenomenon is the positivity of $M_1(\omega)$ at $\omega \leq \omega_c$. Also, we note that no attempt is made in the present calculation to fit the low n_s part ($n_s \leq 2\times10^{11}$ cm^{-2}) of the experimental results. When n_s becomes smaller, the D^* in our model will be larger, and we expect that τ^* drops quickly. When $\omega_c^*\tau^*$ falls below unity, the Drude-type expression (6) is no longer a good description for the problem. Besides, when n_s is small enough we have a partially filled Landau-level case, which is not included in the present theory.

In summary, we have shown, to first order in impurity scattering, that the divergencies in the magnetoconductivity, associated with the sharpness of the Landau energy levels, can be removed by inclusion of fluctuation effects in the center-of-mass description. The corresponding numerical results display a splitting of the cyclotron absorption line, for a range of magnetic field values at low temperature, which was found experimentally by Schlesinger *et al.*[10] The comparison of the present theory with the experimental results of Kennedy *et al.*[8] for the Si 2D EG system, reveals that the increase of cyclotron effective mass m^* with the decrease of n_s in the range of $2 < n_s < 8$ ($\times10^{11}$ cm^{-2}) can be explained as due to the fluctuations of the center of mass of the electron system.

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