New Techniques and Ideas in Quantum Measurement Theory
Reprinted from Vol. 480
ANNALS OF THE NEW YORK ACADEMY OF SCIENCES

Decay of Quantum Coherence due to the Presence of a Heat Bath

Markovian Master-Equation Approach[a]

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INTRODUCTION

Presently, there is much interest in models of the measurement process. We use the technique of Markovian master equations to describe the interaction of various systems with an environment of harmonic oscillators. As described by Zurek,[1] we find that certain couplings tend to diagonalize the system's reduced density operator in the so-called pointer basis.

In general terms, we are considering a quantum system, S, on which one carries out measurements via a quantum apparatus, A, which is coupled to an environment, E. In the first stage of the measurement (as shown by von Neumann[2]), unitary evolution introduces correlations between the state vectors of S and A. (This is not destroyed by the presence of the environment.) Also, after the correlation has been established, the interaction between S and A is essentially negligible. However, when the measurement is completed, there is an apparent ambiguity as to which states the quantum system will find itself in; this in turn stems from the ambiguity in the choice of the preferred basis for A (the variables of A being "traced out" in the second stage of the measurement). Zurek argues that a natural choice of what is actually observed emerges by consideration of an interaction, H_{AE}, between A and E that establishes correlations between A and E. In essence, H_{AE} determines the so-called "pointer observable," P, as being the observable for which $[P,H_{AE}] = 0$. In other words, taking the environment, E, into account determines the mixture into which the S-A-E collapsed.

There has been a recent surge of activity in many different areas that focuses on the crucial role of the environment. For example, in condensed matter physics, Caldeira and Leggett[3] and many others have considered the effect of dissipation due to the environment on quantum tunneling of a macroscopic variable. Zeh[4] and Wigner[5] pointed out that it is virtually impossible to isolate a macroscopic body, which, in turn,

[a]This research was partially supported by the U.S. Office of Naval Research under Contract No. N 00014-86-K-0002.

led Wigner to remark "... that quantum mechanics' validity has narrower limitation [in] that it is not applicable to the detailed behavior of macroscopic bodies...." The analysis of Wigner was based on the consideration of a cubic centimeter of tungsten at the temperature of 3 K of intergalactic space. An example from the realm of atomic physics is the recent demonstration[6] that the absorption spectrum of light by atoms is affected by the ambient temperature,[7] and Ford, Lewis, and O'Connell[8] have emphasized the necessity of analyzing this problem by treating the atom-laser system as being immersed in a heat bath of blackbody radiation. Furthermore, their exact analysis showed that, at least for this particular problem, non-Markovian effects play a crucial role. In addition, this calculation demonstrated that one can supplement traditional quantum mechanical techniques with stochastic methods for the purpose of calculating environmental effects.

We turn now to a detailed look at how the reduced density matrix tends to become diagonalized in the pointer basis; thus, from henceforth, we will simply use S to denote the combined correlated system-apparatus. In a recent series of papers, Walls, Milburn, and Savage (WMS)[9-11] discussed concrete models of the quantum measurement process using a Markovian master-equation approach, and they showed that any macroscopic quantum coherence will be rapidly destroyed by its interaction with the environment. Thus, this provides a possible mechanism for the reduction of the wave function. Their starting point is a Hamiltonian describing a quantum system coupled to an environment of harmonic oscillators, and they find that certain couplings tend to diagonalize the system's reduced density matrix, ρ, in the so-called pointer basis of Zurek. These papers considered specific system-environment interaction Hamiltonians, H_{SE}. Here, we consider a more general system observable, O_S, in the expression for H_{SE} (while recognizing that it does not encompass all the situations considered by WMS). In particular, if O_S is chosen to be a constant of the motion or, less restrictively, a quantum nondemolition variable, then we show that the off-diagonal elements, ρ_{mn}, decay to zero at an exponential rate proportional to $(m-n)^2$.

Consider a quantum system coupled to an environment of harmonic oscillators so that the total Hamiltonian is

$$H = H_S + H_E + H_{SE}, \tag{1.1}$$

where H_S and H_E are, respectively, the free system and environment Hamiltonians. We take the system-environment interaction Hamiltonian to be

$$H_{SE} = \hbar O_S(\Gamma_E + \Gamma_E^+), \tag{1.2}$$

where O_S is a system observable ($O_S = O_S^+$) and $\Gamma_E = \Sigma_i\, g_i a_i$ is the usual sum of environmental annihilation operators weighted by coupling strengths. In addition, $H_E = \Sigma_i \hbar\omega_i a_i^+ a_i$, as is usual.

If O_S is a constant of the system's free motion, $[O_S, H_S] = 0$, then the following interaction picture of the master equation for the system's reduced density operator may be derived (see also Louisell[12] and Lindblad[13]) as

$$\frac{\partial\rho}{\partial t} = \frac{\gamma}{2}(2O_S\rho O_S - O_S^2\rho - \rho O_S^2), \tag{1.3}$$

where $\gamma/2$ is the damping constant. Actually, in equation 4.3 of reference 13, Lindblad has written down a result for $(\partial\rho/\partial t)$ that he argues is the most general time-homogeneous quantum mechanical Markovian master equation with a bounded Liouville operator; replacing V by O in Lindblad's equation (to conform to our notation) and setting $O^+ = 0$ results in our equation 1.3. Taking matrix elements in the basis of O_S, $O_S|m\rangle_S = m|m\rangle_S$, we have

$$\frac{\partial \rho_{mn}}{\partial t} = -\frac{\gamma}{2}(m - n)^2 \rho_{mn}, \tag{1.4}$$

where $\rho_{mn} = \langle m|\rho|n\rangle$. The solution follows immediately:

$$\rho_{mn}(t) = \exp\left[-\frac{\gamma}{2}(m - n)^2 t\right]\rho_{mn}(o). \tag{1.5}$$

This solution has the remarkable property that the off-diagonal elements, ρ_{mn}, go to zero at an exponential rate given by $\gamma(m - n)^2/2$. In the case of nonzero temperatures, it may be verified that the corresponding rate is $\gamma(2\bar{n} + 1)(m - n)^2/2$, where $\bar{n}$ is the average number of photons.

In the less restrictive case, where O_S is not a constant of the motion, but, instead, is a quantum nondemolition (QND) observable, the eigenvalues, m and n, are functions of time. Consequently, equation 1.5 is replaced by

$$\rho_{mn}(t) = \exp\left[-\frac{\gamma}{2}\int_0^t \{m(t) - n(t)\}^2 dt\right]\rho_{mn}(o). \tag{1.6}$$

O_S is an example of Zurek's pointer observable. The diagonalization of the reduced density operator in the pointer basis is due to the interaction Hamiltonian, H_{SE}, correlating different pointer eigenstates with nearly orthogonal environmental states. This can be seen explicitly for the preceding system by considering the approximate unitary evolution of an initial superposition of pointer eigenstates correlated with the vacuum state of the environment:

$$|\psi, t = 0\rangle = (|m\rangle_S + |n\rangle_S)|\{o\}\rangle_E. \tag{1.7}$$

This is an eigenstate of the free Hamiltonian, so an approximate expression for the state at times, $t \ll \Omega^{-1}$, where Ω is the bath cutoff frequency, is

$$\begin{aligned}|\psi, t\rangle &= \exp\{iO_S(\Gamma_E + \Gamma_E{}^+)t\}|\psi, t = 0\rangle \\ &= |m\rangle_S|\{-ik_j mt\}\rangle_E + |n\rangle_S|\{-ik_j nt\}\rangle_E,\end{aligned} \tag{1.8}$$

where the environmental oscillators are in coherent states and we have used the result,

$$\exp[i(\alpha a^+ + \alpha^* a)]\,|O\rangle = |i\alpha\rangle. \tag{1.9}$$

Tracing the total density operator, $|\psi\rangle\langle\psi|$, over the environment yields the reduced density operator,

$$\rho(t) = |m\rangle\langle m| + |n\rangle\langle n| + \eta|m\rangle\langle n| + \eta^*|n\rangle\langle m|,$$

where

$$\eta = {}_E\langle\{-ik_jmt\}|\{-ik_jnt\}\rangle_E = \exp\left[-1/2\sum_j |k_j|^2(m-n)^2t^2\right]. \tag{1.10}$$

Because it diagonalizes the system's density matrix in the pointer basis, the environment can be regarded as performing a measurement on the pointer observable. Choosing it to be a constant of the motion, $[O_S, H_S] = 0$, results in the particularly simple behavior of equation 1.5. Choosing it to be a QND observable results in equation 1.6. Examples of systems for which O is a QND variable are the harmonic oscillator coupled to its environment by the number observable, a^+a, or by the quadrature phase observable, $X_\theta = a^+e^{i\theta} + ae^{-i\theta}$, where each of which is a QND observable of the harmonic oscillator. Phase damping of a spin in a magnetic field may be modeled by coupling the QND observable, J_z, to a bath. The QND coupling for a free particle is via momentum.

Many examples of non-QND pointer observables have been considered by Walls, Milburn, and Savage,[9-11] and thus it is of interest to review the salient points of their results for comparison with the situation of QND pointer observables. In all cases, the system(-apparatus) is taken to be an harmonic oscillator (except for the case considered in the section entitled FREE PARTICLE, where the frequency of the oscillator is taken to be zero), and the environment is treated as an ensemble of harmonic oscillators. Thus, the basic choice remaining concerns the form for H_{SE}, and particularly, the choice of O_S in equation 1.2 for H_{SE}. In the next section, we discuss the results for coordinate-coordinate interaction in the rotating-wave approximation[9] so that $\hbar O_S = a$, and hence, O_S is not a QND observable. A similar remark applies in the third section of this paper to the case where $\hbar O_S = x$, where x is the coordinate of the system oscillator, which again is not a QND observable quantity. This system displays residual correlations over lengths of the order of its thermal de Broglie wavelength.[10] The zero frequency (or high temperature) limit of the corresponding master equation[11] is considered in the final section.

HARMONIC OSCILLATOR: AMPLITUDE DAMPING, ZERO TEMPERATURES

Consider the following harmonic oscillator Hamiltonian,[9]

$$H = \hbar\omega a^+a + a\Gamma^+ + a^+\Gamma + H_E, \tag{2.1}$$

where Γ represents a reservoir operator. This reservoir coupling is equivalent to coordinate-coordinate coupling in the rotating-wave approximation in which high frequency terms are ignored. In the Born and Markov approximations (i.e., lowest order perturbation in the interaction with the bath is assumed and memory effects are neglected), the harmonic oscillator's reduced density operator satisfies the following Schrödinger picture master equation,[12]

$$\frac{\partial\rho}{\partial t} = -i\omega[a^+a,\rho] + \frac{\gamma}{2}(2a\rho a^+ - a^+a\rho - \rho a^+a), \tag{2.2}$$

where we have taken a zero temperature environment. In order to obtain a suitable solution for ρ, we introduce the quantum characteristic function,

$$\kappa(\lambda) = \mathrm{tr}(\rho e^{\lambda a^+} e^{-\lambda^* a}). \tag{2.3}$$

Because ρ satisfies equation 2.2, κ satisfies the equivalent equation,

$$\left\{\frac{\partial}{\partial t} + \left(\frac{\gamma}{2} - i\omega\right)\lambda\frac{\partial}{\partial\lambda} + \left(\frac{\gamma}{2} + i\omega\right)\lambda^*\frac{\partial}{\partial\lambda^*}\right\}\kappa = 0. \tag{2.4}$$

The general solution of this equation is

$$\kappa(\lambda e^{-(\gamma/2-i\omega)t}, \lambda^* e^{-(\gamma/2+i\omega t)}), \tag{2.5}$$

where κ is chosen to fit the initial condition of $\kappa(t = 0)$. The density operator and the characteristic function corresponding to an initial superposition of coherent states are, respectively,

$$\rho(o) = \sum_{\alpha,\beta} N_{\alpha\beta}|\alpha\rangle\langle\beta| \tag{2.6a}$$

and

$$\kappa(o) = \sum_{\alpha,\beta} N_{\alpha\beta}\,\langle\beta|\alpha\rangle\, e^{\lambda\beta^* - \lambda^*\alpha}. \tag{2.6b}$$

With this initial condition, the solution to equations 2.4 and 2.2 are[9]

$$\kappa(t) = \sum_{\alpha,\beta} N_{\alpha\beta}\,\langle\beta|\alpha\rangle \exp\left[(\beta^* e^{i\omega t}\lambda - \alpha e^{-i\omega t}\lambda^*)\, e^{-\gamma t/2}\right] \tag{2.7a}$$

and

$$\rho(t) = \sum_{\alpha,\beta} N_{\alpha\beta}\,\langle\beta|\alpha\rangle^{(1-e^{-\gamma t})}|\alpha e^{-(\gamma/2+i\omega)t}\rangle\langle\beta e^{-(\gamma/2+i\omega)t}|. \tag{2.7b}$$

Thus, the off-diagonal elements are reduced by the factor, $\langle\beta|\alpha\rangle^{(1-e^{-\gamma t})}$. The greater the separation of the initially superposed states, the more rapid and complete the reduction of coherence. After a few γ^{-1} of time, the initial coherent state superposition is reduced to a near diagonal mixture of coherent states.

HARMONIC OSCILLATOR COORDINATE OVERDAMPING

In this section, we consider the harmonic oscillator to be coupled via its coordinate, x, to the coordinates of the reservoir oscillators represented by X. The relevant Hamiltonian is[10]

$$H = \hbar\omega a^+ a + xX + H_E. \tag{3.1}$$

After taking the continuum limit for the number of bath oscillators and making the Born and Markov approximations, Agarwal[14] (see also Caldeira and Leggett,[15] and

Dekker[16]) obtained the Schrödinger picture master equation, which is

$$\frac{\partial \rho}{\partial t} = -i\omega[a^+ a,\rho] - \frac{i\gamma}{\hbar}[x,\mathrm{P}\rho + \rho\mathrm{P}] - \frac{2\gamma}{\hbar}(\bar{n} + \tfrac{1}{2})\, m\omega[x,[x,\rho]], \tag{3.2}$$

where P is the oscillator's momentum operator, m is its mass, and $\bar{n} = [\exp(\hbar\omega/kT) - 1]^{-1}$ is its mean occupation number when in equilibrium at temperature, T. Solving for the quantum characteristic function as in the previous section, Savage and Walls[10] obtained the Q function, $\langle z|\rho|z\rangle$, and from this, they also obtained an expression for the density operator. Taking coordinate basis matrix elements, they find in the heavily overdamped limit of $\gamma \gg \omega$,

$$\begin{aligned}\langle x - y|\rho|x + y\rangle = (2\pi\sigma_x^2)^{-1/2} \sum_{\alpha\beta} N_{\alpha\beta}\, \langle\beta|\alpha\rangle \times \exp\{-C^2(\beta^* - \alpha)^2 e^{-4\gamma t}\sigma_y^2\}\\ \times \exp\{-\tfrac{1}{2}\sigma_x^{-2}[x - C^{-1}(\beta^* + \alpha)\, e^{-(\omega^2/2\gamma)t}]^2\}\\ \times \exp\{-\tfrac{1}{2}\sigma_y^{-2}[y - C\sigma_y^2(\beta^* - \alpha)\, e^{-2\gamma t}]^2\}.\end{aligned} \tag{3.3}$$

The variances of the diagonal, σ_x^2, and off-diagonal, σ_y^2, parts are

$$\sigma_x^2 = C^{-2}[1 + 2\bar{n}(1 - e^{-(\omega^2/\gamma)t})],$$
$$\sigma_y^2 = C^{-2}[1 + 2\bar{n}(1 - e^{-4\gamma t})]^{-1},$$

with

$$C = (2m\omega/\hbar)^{1/2}. \tag{3.4}$$

As pointed out by Savage and Walls,[10] at high temperatures and after a time of a few γ^{-1}, the off-diagonal variance becomes $\hbar^2/(4mkT) = (\lambda_d/4\pi)^2$, where λ_d is the de Broglie wavelength associated with the oscillators mean kinetic energy at temperature, T. The spreading of the diagonal part occurs at a much slower rate determined by the quantity, ω^2/γ. Thus, for times, t, satisfying $\gamma/\omega^2 \gg t \gg 1/(4\gamma)$, the density operator has been substantially diagonalized in the coordinate basis without much thermal spreading of the diagonal matrix element. The off-diagonal variance can be made arbitrarily small by making the temperature or oscillator mass sufficiently high.

Thus, the overdamped oscillator described by the Hamiltonian in equation 3.1 provides a model of coordinate measurement on the harmonic oscillator. The diagonalization (otherwise referred to as collapse of the state vector) may occur without disturbing the initial diagonal coordinate distribution.

FREE PARTICLE

The zero frequency (or equivalently, the high temperature) limit of equation 3.2 yields the master equation for the coordinate damped free particle:[11]

$$\frac{\partial \rho}{\partial t} = \frac{i}{\hbar}\Big/[\vec{\mathrm{P}}^2/2m, \rho] - \frac{i\gamma}{\hbar}[\vec{x}, \vec{\mathrm{P}}\rho + \rho\vec{\mathrm{P}}] - 2m\gamma kT\hbar^{-2}[\vec{x}, [\vec{x}, \rho]]. \tag{4.1}$$

To solve this equation, Savage and Walls[11] introduced the quantum characteristic function,

$$\kappa(\vec{\lambda}, \vec{\mu}) = \mathrm{Tr}(\rho e^{i(\vec{\lambda}\cdot\vec{x} + \hbar^{-1}\vec{\mu}\cdot\vec{P})}). \tag{4.2}$$

An interesting initial condition is the superposition of plane waves with wave vectors, $\vec{K}_1$ and $\vec{K}_2$. The corresponding initial wave function and the coordinate basis density matrix elements are, respectively,

$$\psi(\vec{x}) = \frac{1}{\sqrt{2}}(e^{i\vec{K}_1\cdot\vec{x}} + e^{i\vec{K}_2\cdot\vec{x}}) \tag{4.3a}$$

and

$$\langle\vec{x}|\rho|\vec{x} - \vec{\mu}\rangle = \tfrac{1}{2}e^{i\vec{K}_1\cdot\vec{\mu}}(1 + e^{i(\vec{K}_2 - \vec{K}_1)\cdot\vec{x}}) + \tfrac{1}{2}e^{i\vec{K}_2\cdot\vec{\mu}}(1 + e^{i(\vec{K}_2 - \vec{K}_1)\cdot\vec{x}}). \tag{4.3b}$$

Solving for $x(t)$ and inverting, we find[6]

$$\begin{aligned}\langle\vec{x}|\rho|\vec{x} - \vec{\mu}\rangle &= \tfrac{1}{2}\exp\{-\tfrac{1}{2}mkT\hbar^{-2}(1 - e^{-4\gamma t})\vec{\mu}^2\} \\ &\times \{e^{ie^{-2\gamma t}\vec{K}_1\cdot\vec{\mu}} + e^{ie^{-2\gamma t}\vec{K}_2\cdot\vec{\mu}} + e^{-\eta}e^{(i/2)e^{-2\gamma t}(\vec{K}_1 + \vec{K}_2)\cdot\vec{\mu}}[e^{\vec{a}\cdot(\vec{K}_1 - \vec{K}_2)} + e^{-\vec{a}\cdot(\vec{K}_1 - \vec{K}_2)}]\},\end{aligned} \tag{4.4}$$

where we have introduced the quantities,

$$\begin{aligned}\eta &= \frac{kT}{2\gamma^2 m}\left[\gamma t - \frac{3}{4} + e^{-2\gamma t} - \frac{1}{4}e^{-4\gamma t}\right]|\vec{K}_1 - \vec{K}_2|^2, \\ \vec{a} &= \frac{kT}{2\gamma\hbar}(1 - 2e^{-2\gamma t} + e^{-4\gamma t})\vec{\mu} \\ &\quad + i\left\{-\vec{x} + \tfrac{1}{2}\vec{\mu} + \frac{\hbar}{4\gamma m}(1 - e^{-2\gamma t})(\vec{K}_1 + \vec{K}_2)\right\}.\end{aligned} \tag{4.5}$$

The first factor on the right-hand side of equation 4.4 tends to diagonalize the coordinate basis density matrix. For example, Savage and Walls[11] considered a neutron mass and environmental temperature of 100 K to enable this factor to be about $\exp[-(1 - e^{-4\gamma t})(10^{10}\vec{\mu})^2]$. Assuming a one percent decrease in particle momentum, we have $\gamma t \approx 0.05$ (see below) and the factor becomes $\exp[-(2 \times 10^9 \vec{N})^2]$. This factor is small for $|\vec{N}| \gg 10^{-9}$m, so quantum coordinate coherence extends over about 10^{-9} meters, which is, for example, much less than the width of a bubble-chamber track.

The first two terms in the second factor on the right-hand side of equation 4.4 describe a mixture of particles having momentum, $e^{-2\gamma t}\hbar\vec{K}_1$ and $e^{-2\gamma t}\hbar\vec{K}_2$. Thus, the environmental interaction damps the particle momentum.

The interference pattern of the initially superposed plane waves is revealed in the diagonal matrix elements ($\vec{\mu} = 0$ in equation 4.4),

$$\langle\vec{x}|\rho|\vec{x}\rangle = 1 + e^{-\eta}\cos\left[\vec{x}\cdot(\vec{K}_1 - \vec{K}_2) - \frac{\hbar}{4\gamma m}(1 - e^{-2\gamma t})(\vec{K}_1^2 - \vec{K}_2^2)\right], \tag{4.6}$$

where η is defined in equation 4.5. The factor, $e^{-\eta}$, reduces the visibility of the interference fringes.

The exponent, η, is always positive and increases with time. Being proportional to $|(\vec{K}_1 - \vec{K}_2)|^2$ means that the fringe visibility reduces rapidly with increasing separation in the wave-vector space of the initially superposed waves.

It is interesting to compare our expression for fringe visibility with an experiment in which electron interference fringes due to superimposed plane waves have been observed.[11] The electron wavelength was about 0.04 Å and the flight time was about 10^{-9} s. At room temperature, η is about

$$\eta \approx 10^{16}(\gamma t)^{-2}[\gamma t - 3/4 + e^{-2\gamma t} - 1/4\, e^{-4\gamma t}]. \tag{4.7}$$

Because the interference pattern was clearly visible, γt, which determines the momentum damping, would have to be exceedingly small ($\gamma t \approx 10^{-16}$) for the model to be consistent with experiment.

ACKNOWLEDGMENTS

R. F. O'Connell would like to thank G. W. Ford and J. T. Lewis, who originally brought to his attention the work of Lindblad (reference 13).

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