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THEORETICAL DETERMINATION OF THE ADMITTANCE OF THE METAL GATE IN A METAL–OXIDE–SEMICONDUCTOR SYSTEM AND EFFECT OF THE GATE ON FARADAY ROTATION AND ELLIPTICITY

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We calculate the admittance of the metal gate in a metal–oxide–semiconductor system in the Faraday geometry using the Boltzmann transport equation. We find that the value of the admittance calculated from this model agrees well with experimentally determined values. Using this calculated value of the admittance, we find that the contribution of the metal gate to the Faraday rotation and ellipticity to be negligible and hence we may ignore the metal gate altogether except for its contribution as a reflecting surface.

The inversion layer in a metal–oxide–semiconductor (MOS) system is a subject of much current interest. Following the usual practice of treating the inversion layer as a two-dimensional electron gas, we calculated its contribution to free-carrier Faraday rotation and ellipticity [1]. Here we wish to address the question of whether or not the free carriers in the metal will also make a contribution. As in ref. 1, we will ignore multiple reflections in the oxide layer, which were treated elsewhere [2].

Because the metal gate is very thin (20–50 Å) it also can be treated as a two-dimensional charge layer. Then, for the purpose of calculating its contribution to free-carrier Faraday rotation or ellipticity, we need to calculate the surface admittance (two-dimensional conductivity) of the metallic charge layer. The usual bulk metal refractive index is not useful in this context but, following Dingle [3], we may define a generalized complex refractive index by

$$N = (4\pi/c)Y, \tag{1}$$

where Y is the surface admittance of the gate defined by

$$Y = J/E, \tag{2}$$

J is the total current per unit area in the film and E is the field at the surface, having components in the x and y directions. Since the rotation is measured at liquid Helium temperatures the following condition holds for the metal:

$$a \ll \delta \ll l, \tag{3}$$

where l is the mean free path in the metal $\sim 10^{-2}$ cm, δ is the effective skin depth ~ 500 Å and a is the thickness of the metal film. δ is calculated using [4]

$$\delta^3 = c^2 l / 2\pi\gamma\sigma_0\omega, \tag{4}$$

where γ is a constant ~ 1, σ_0 is the static conductivity $\sim 10^{20}\ \mathrm{s}^{-1}$ and ω is the angular frequency of the incident ratiation $\sim 10^{12}\ \mathrm{s}^{-1}$. Since $\delta \ll l$ we are in the *anomalous skin effect regime*. This would ordinarily mean that the local instantaneous relationship between J and E would break down. The electric field may vary appreciably over a distance comparable to the mean free path, so that the current density at a point is defined by the electric field E in a region with dimensions of the order of l. The conductivity would no longer be a constant for the metal but depend on the spatial distribution of the electric field. However, even though we are in the anomalous skin effect regime the condition $a \ll \delta$ ensures a constant electric field across the thickness of the film along the z direction and we can neglect non-local effects [5].

We turn now to a calculation of Y. It may be mentioned here that we cannot use the 2-dimensional analog of the 3-dimensional a.c. Drude conductivity for the bulk metal in the presence of a static magnetic field B

$$Y_{\pm} = (\mathrm{i}Ne^2/m)a/(\omega \pm \omega_c + \mathrm{i}\nu)\,, \tag{5}$$

where $\omega_c = (eB/mc)$ is the cyclotron frequency and ν is the collision frequency of the electrons with the lattice. This is because ν does not include collisions of the electrons with the boundaries of the metal film. If one were to replace ν by an effective scattering frequency ν' which took both types of collisions into account, there would be no way to determine a priori the value of ν' and consequently the value of Y. However, it is experimentally possible to determine the value of Y, and since experimental conditions dictate $\nu' \gg \omega, \omega_c$,

$$Y \simeq \sigma' a\,, \tag{6}$$

where

$$\sigma' \equiv Ne^2\tau'/m \tag{7}$$

so that a determination of τ' is experimentally possible. We will see later, that using the Boltzmann transport equation and appropriate boundary conditions to calculate the value of Y, we obtain an expression which agrees well with the experimental value. In the case of a static electric field, Sondheimer [6] investigated transport phenomena in the metal film. For our calculation we generalize Sondheimer's method to an a.c. electric field $(E_x\,\mathrm{e}^{-\mathrm{i}\omega t}, E_y\,\mathrm{e}^{-\mathrm{i}\omega t}, 0)$ in the plane of the metal and a static external magnetic field $(0, 0, B)$, perpendicular to the plane of the metal. Since we are interested in the case of circularly polarized radiation, we consider E_x and E_y to be equal.

The distribution function f of the electrons may be written as [7]

$$f = f_0 + f_1(\boldsymbol{v}, z, t)\,, \tag{8}$$

where f_0 is the equilibrium Fermi function, $\boldsymbol{v}$ is the velocity of an electron, and f_1 is a function of $\boldsymbol{v}$, z, and t which must be determined; f_1 is a perturbation from the equilibrium value f_0 and produces the surface currents on the film. The distribution function is determined by the Boltzmann equation [7]

$$\frac{\partial f}{\partial t} + \frac{e}{\hbar}\left(\boldsymbol{E} + \frac{1}{c}\boldsymbol{v} \times \boldsymbol{B}\right) \cdot \nabla_k f + \boldsymbol{v} \cdot \nabla_r f = -(f - f_0)/\tau\,, \tag{9}$$

where we have omitted the negligible contribution due to the time-varying magnetic field of the electromagnetic wave. We use eq. (8) in eq. (9) and neglect the product of $\boldsymbol{E}$ with $\nabla_k f_1$ which would be of order E^2 and corresponds to deviations from Ohm's Law. It is found that f_1 satisfies

$$\frac{\partial f_1}{\partial t}+\frac{f_1}{\tau v_z}+\frac{eB}{mcv_z}\left(v_y\frac{\partial f_1}{\partial v_x}-v_x\frac{\partial f_1}{\partial v_y}\right)=\frac{-e}{mv_z}\left(E_x\frac{\partial f_0}{\partial v_x}+E_y\frac{\partial f_0}{\partial v_y}\right)\mathrm{e}^{-\mathrm{i}\omega t}\,, \tag{10}$$

where m is the electron mass. To solve eq. (10), we put

$$f_1=(v_xc_1+v_yc_2)\,\mathrm{e}^{-\mathrm{i}\omega t}\frac{\partial f_0}{\partial v}\,, \tag{11}$$

where v is the magnitude of $\boldsymbol{v}$, and c_1 and c_2 are functions of v, v_z and z. Combining eqs. (10) and (11), we obtain the following equations for c_1 and c_2:

$$\frac{\partial c_1}{\partial z}+\frac{c_1}{\tau v_z}-\frac{\omega_c c_2}{v_z}-\frac{\mathrm{i}\omega c_1}{v_z}=\frac{-e}{mv_zv}E_x\,, \tag{12}$$

$$\frac{\partial c_2}{\partial z}+\frac{c_2}{\tau v_z}+\frac{\omega_c c_1}{v_z}-\frac{\mathrm{i}\omega c_2}{v_z}=\frac{-e}{mv_zv}E_y\,, \tag{13}$$

where $\omega_c=(eB/mc)$ is the cyclotron frequency. If we multiply eq. (13) by i and subtract from eq. (12) we obtain

$$\frac{\partial g}{\partial z}+\frac{g}{\tau v_z}-\mathrm{i}(\omega+\omega_c)\frac{g}{v_z}=\frac{-e}{mv_zv}E\,, \tag{14}$$

where

$$g=c_1-\mathrm{i}c_2\,, \tag{15}$$

$$E=E_x-\mathrm{i}E_y\,. \tag{16}$$

The solution of eq. (14) is

$$g=\frac{-eE\tau}{mv[1-\mathrm{i}(\omega+\omega_c)\tau]}\left\{1+C\exp\left(-[1-\mathrm{i}(\omega+\omega_c)\tau]\frac{z}{\tau v_z}\right)\right\}, \tag{17}$$

where C is an arbitrary function of v and v_z. This corresponds to the case of right circularly polarized radiation. The case of left circularly polarized radiation may be obtained by replacing i in eqs. (15) and (16) by $-\mathrm{i}$ and replacing ω_c by $-\omega_c$ in eq. (17). The boundary conditions which are used to determine C depend on the nature of the scattering at the surface of the film. Gate metals in MOS structures are highly polycrystalline, the net effect of which is to average out the anisotropy associated with a preferred direction, so that the material is isotropic [8]. If the assumption is made that the electrons are scattered diffusely at the boundaries with complete loss of their drift velocities, the distribution function of the electrons leaving each surface of the film must be independent of their direction of motion. This is not an unreasonable assumption to make since the experimental data available in the literature fit the diffuse case for most metallic films that have a polycrystalline structure [9].

To satisfy this assumption, C must be chosen such that g is zero at $z=0$ for all $\boldsymbol{v}$ such that $v_z>0$, and g is zero at $z=a$ for all $\boldsymbol{v}$ such that $v_z<0$. This requirement is satisfied if we take

$$C=-1 \qquad v_z>0\,, \tag{18a}$$

$$C = -\exp\left([1 - \mathrm{i}(\omega + \omega_c)\tau]\frac{a}{v_z\tau}\right) \qquad v_z < 0\,. \tag{18b}$$

Eq. 18 ensures that g, and therefore f_1 as well, vanish for electrons at the boundaries of the metal, i.e. for electrons leaving the surface at $z = 0$ and $z = a$.

The current density $(j_x, j_y, 0)$ can now be calculated. Along the x direction the current density is

$$j_x = \frac{e}{4\pi^3}\int v_x f_1\,\mathrm{d}^3k\,. \tag{19}$$

Since the current is due only to the perturbation f_1 (representing the deviation from thermodynamic equilibrium), we may write eq. (19) in velocity space as

$$j_x = 2e\left(\frac{m}{h}\right)^3 \int\int\int v_x f_1\,\mathrm{d}^3v\,. \tag{20}$$

Substituting from eq. (11) into eq. (20), and using polar coordinates (v, θ, ϕ) (with $\mathrm{d}^3v = v^2\sin\theta\,\mathrm{d}v\,\mathrm{d}\theta\,\mathrm{d}\phi$)

$$j_x = 2\pi e\left(\frac{m}{h}\right)^3 \int\int v^4\sin^3\theta c_1\frac{\partial f_0}{\partial v}\,\mathrm{d}\theta\,\mathrm{d}v\,. \tag{21}$$

We next use the relation [6]

$$\bar{v}^4 = -\int v^4\frac{\partial f_0}{\partial v}\,\mathrm{d}v\,, \tag{22}$$

where the bar denotes the Fermi velocity, so that eq. (21) becomes

$$j_x = -2\pi e\left(\frac{m}{h}\right)^3 \bar{v}^4\int_0^\pi c_1\sin^3\theta\,\mathrm{d}\theta\,. \tag{23a}$$

Similarly

$$j_y = -2\pi e\left(\frac{m}{h}\right)^3 \bar{v}^4\int_0^\pi c_2\sin^3\theta\,\mathrm{d}\theta\,. \tag{23b}$$

Combining eqs. (23a) and (23b), we obtain

$$j = j_x - \mathrm{i}j_y = -2\pi e\left(\frac{m}{h}\right)^3 \bar{v}^4\int_0^\pi g\sin^3\theta\,\mathrm{d}\theta\,. \tag{24}$$

To find the total transverse current in the film of thickness a, we write

$$J = \int_0^a j\,\mathrm{d}z = \int_0^a \int_0^\pi \frac{2\pi e^2 E\tau \bar{v}^3}{m[1-\mathrm{i}(\omega+\omega_c)\tau]}\left(\frac{m}{h}\right)^3 \left\{1 + C(v, v_z)\exp\left(-[1-\mathrm{i}(\omega+\omega_c)\tau]\frac{z}{\tau v_z}\right)\right\}\sin^3\theta\,\mathrm{d}\theta\,\mathrm{d}z\,. \tag{25}$$

If we use the boundary conditions on C and define

$$\eta = [1-\mathrm{i}(\omega+\omega_c)\tau]\,, \tag{26}$$

eq. (25) reduces to

$$J = \frac{2\pi e^2}{m}\left(\frac{m}{h}\right)\bar{v}^3 E\tau \int_0^a \left[\int_0^{\pi/2} \frac{\sin^3\theta}{\eta}(1-\mathrm{e}^{-\eta z/\tau v\cos\theta})\,\mathrm{d}\theta + \int_{\pi/2}^{\pi} \frac{\sin^3\theta}{\eta}(1-\mathrm{e}^{-\eta a/v\tau\cos\theta-\eta z/v\tau\cos\theta})\,\mathrm{d}\theta\right]\mathrm{d}z\,. \tag{27}$$

In order to integrate eq. (27) we let

$$t = 1/\cos\theta\,, \qquad l = \bar{v}\tau \qquad \text{and} \qquad \kappa = a\eta/l\,,$$

where l is the mean free path. If we consider the first angular integral in eq. (27) we obtain

$$\int_0^{\pi/2} \frac{\sin^3\theta}{\eta}(1-\mathrm{e}^{-\eta z/\tau v\cos\theta})\,\mathrm{d}\theta = \int_1^\infty \frac{\mathrm{d}t}{\eta}\left(\frac{1}{t^2}-\frac{1}{t^4}\right)(1-\mathrm{e}^{-\eta z t/l})\,. \tag{28}$$

Also

$$\int_0^\infty \int_0^a \frac{\mathrm{d}t}{\eta}\left(\frac{1}{t^2}-\frac{1}{t^4}\right)(1-\mathrm{e}^{-\eta z t/l})\,\mathrm{d}z = \frac{a^2}{l}\int_1^\infty \frac{\mathrm{d}t}{\kappa}\left(\frac{1}{t^2}-\frac{1}{t^4}\right)\left(1+\frac{\mathrm{e}^{-\kappa t}}{\kappa t}-\frac{1}{\kappa t}\right)$$

$$= \frac{2}{3}\frac{a^2}{l}\left\{\frac{1}{\kappa}-\frac{3}{8\kappa^2}+\frac{3}{2\kappa^2}\int_1^\infty \mathrm{d}t\left(\frac{1}{t^3}-\frac{1}{t^5}\right)\mathrm{e}^{-\kappa t}\right\}. \tag{29}$$

The second angular integral in eq. (27) may be evaluated in a similar manner. Eq. (27) thus becomes

$$J = \frac{2\pi e^2}{m}\left(\frac{m}{h}\right)^3 \bar{v}^3 E\tau \frac{4}{3}\frac{a^2}{l}\left\{\frac{1}{\kappa}-\frac{3}{8\kappa^2}+\frac{3}{2\kappa^2}\int_1^\infty \mathrm{d}t\left(\frac{1}{t^3}-\frac{1}{t^5}\right)\mathrm{e}^{-\kappa t}\right\}. \tag{30}$$

If we now use

$$N = \frac{8\pi}{3}\left(\frac{m\bar{v}}{h}\right)^3\,, \tag{31}$$

where N is the number of electrons per unit volume, and we write the d.c. conductivity as

$$\sigma_0 = \frac{Ne^2 l}{m\bar{v}}, \tag{32}$$

eq. (27) becomes

$$J = \sigma_0 \frac{a^2}{l} \frac{E}{\psi(\kappa)}, \tag{33}$$

where

$$\frac{1}{\psi(\kappa)} = \frac{1}{\kappa} - \frac{3}{8\kappa^2} + \frac{3}{2\kappa^2} \int_1^\infty dt \left(\frac{1}{t^3} - \frac{1}{t^5}\right) e^{-\kappa t}. \tag{34}$$

Thus, from eq. (2),

$$Y = \sigma_0 \frac{a^2}{l} \frac{1}{\psi(\kappa)}. \tag{35}$$

Since $a \sim 10^{-7}$ cm and $l \sim 10^{-2}$ cm, $\kappa \ll 1$ and $1/\psi(\kappa)$ may be expanded by means of the power series [6]

$$\frac{1}{\psi(\kappa)} = \frac{3}{4}(1 - \gamma - \ln \kappa) + \frac{\kappa}{2} - \frac{\kappa^2}{192}(31 - 12\gamma - 12 \ln \kappa) + 3 \sum_{n=3}^{\infty} (-1)^n \frac{\kappa^n}{n(n-2)(n+2)!}. \tag{36}$$

If we retain only terms of the order of κ, eq. (34) may be written as

$$Y_\pm = \frac{3}{4} \sigma_0 \frac{a^2}{l} \left\{ \ln\left(\frac{1}{\kappa_\pm}\right) + 0.4228 + \frac{2}{3}\kappa_\pm \right\}, \tag{37}$$

where we have allowed for both polarizations and

$$\kappa_\pm = \frac{a}{l}\{1 - i(\omega \pm \omega_c)\tau\}. \tag{38}$$

These results constitute our starting-point for the calculation of the Faraday rotation θ and the ellipticity δ. Our procedure is exactly analogous to that used in the calculation of the corresponding quantities for the two-dimensional inversion layer [1] and so will not be repeated here.

For the input parameters, we take ω equal to $6.5 \times 10^{12}\,\mathrm{s}^{-1}$ (corresponding to a wavelength of $\lambda = 292.5\,\mu$m which was used in the experiment of Piller and Wagner [10]), and for the metal we take a range of values in the vicinity of $\tau = 2 \times 10^{-10}$ s, $\bar{v} = 10^8\,\mathrm{cm\,s^{-1}}$, $l = 10^{-2}$ cm and $\sigma_0 = 10^{20}\,\mathrm{s}^{-1}$, being representative of Ti (the gate metal used in the experiment of Piller and Wagner [10]) at very low temperatures [12, 13]. It is found for the above parameters that the value of $Y \sim 4 \times 10^8$ esu/□. This compares well with the experimental value of Y as quoted by Tsui et al. [11] ($Y \sim 5 \times 10^{-4}$ mho/□ $\sim 4.5 \times 10^8$ esu/□), who used a Ti gate. Even if parameters are chosen at 300°K ($\sigma_0 \sim 10^{17}\,\mathrm{s}^{-1}$, corresponding to the fact that the conductivity of Ti is measured [12, 13] to be about 10^{-3} smaller at room temperatures than at low temperatures, $\tau \sim 10^{-13}$ s, $l \sim 10^{-5}$ cm) about the same value of Y is obtained which shows that Y is fairly constant between 4–300°K.

In order to give a possible interpretation to eq. (37) in terms of experimental parameters we write

$$Y_\pm = \frac{Ne^2}{m}\left[\tau \frac{3}{4}\frac{a}{l}\left\{\ln\left(\frac{1}{\kappa_\pm}\right) + 0.4228 + \frac{2}{3}\kappa_\pm\right\}\right]a$$

$$= \frac{Ne^2}{m}\tau'_\pm a\,,$$

where

$$\tau'_\pm \equiv \tau \frac{3}{4}\frac{a}{l}\left\{\ln\left(\frac{1}{\kappa_\pm}\right) + 0.4228 + \frac{2}{3}\kappa_\pm\right\}, \tag{39}$$

where the expression for τ' may be thought of as the modifying effect of the boundary collisions on the electron lattice relaxation time τ. The experimentally determined value of τ' using eq. (6) is $\sim 10^{-15}$–10^{-16} s. Using the values for τ, a, l etc. quoted above, in eq. (39), the value for $\tau'_\pm$ agrees reasonably well with the experimental value.

In all cases we found that θ and δ were completely negligible compared to the corresponding quantities in the case of the inversion layer. The reason for this may be traced to two features of eq. (37). First of all, $Y_\pm$ contains an additional factor of (a/l) over what one might initially estimate and, since $a \sim 5 \times 10^{-7}$ cm and $l \sim 10^{-2}$ cm, this factor has the very small value of 5×10^{-5}. In addition, $\theta \sim (Y''_- - Y''_+)$ and $\delta \sim (Y'_- - Y'_+)$, where the primes and double primes refer to the real and imaginary parts, respectively. But it turns out that both of these quantities are relatively small, the reason being that κ_+ is not very different than κ_-, arising from the fact that, over the range of magnetic fields of interest, $\omega \gg \omega_c$ where ω_c is the cyclotron frequency of the metal (for $B = 1.3 \times 10^5$ G, which is the highest field used in the experiments of Piller and Wagner [10], we find that $(\omega/\omega_c) \sim 3$).

We conclude that the metal essentially does not contribute any free-carrier Faraday rotation or ellipticity. Thus, we may ignore the metal completely except for its contribution as a reflecting surface.

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