Zeitschrift für Astrophysik 65, 498—500 (1967)

Kurze Mitteilung

Evolution of the Radio Spectral Index of Supernova Remnants

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Received January 9, 1967

On the basis of observational evidence, HARRIS, 1962, proposed that the radio spectra of supernova remnants undergo an evolutionary flattening with time.

He plotted the diameter $2R$ of each remnant against its spectral index, and showed that the resultant curve could be fitted by a relation of the form

$$\gamma = 2\alpha + 1 = a\,R^{-b} \tag{1}$$

where α is the spectral index, and a and b are constants equal to 3.2 and 0.36 respectively. The quantity γ is the exponent which appears in the well-known formula (GINZBURG, 1957; GINZBURG and SYROVATSKII, 1966)

$$N(E, R) = K(R)\,E^{-\gamma} \tag{2}$$

for the energy spectrum of the electrons which are responsible for the synchrotron radiation.

SHKLOVSKY, 1960, deduced an explicit expression for $K(R)$ in the case of an adiabatic expansion of the supernova remnant. In the general case of a non-adiabatic expansion such an undertaking will be more difficult. Fortunately, for our purposes it will be sufficient to know only that $K(R)$ is independent of E and hence will not influence the spectral index.

Usually the assumption (HARRIS, 1962; KARDASHEV, 1962) is made that the remnant has a uniform rate of expansion v so that $t = R/v$.

It is the purpose of this communication to consider what physical process or processes are responsible for the decrease of γ with time.

The same problem was tackled by KARDASHEV, 1962, whose starting point was the Boltzmann kinetic equation in energy space discussed by KAPLAN, 1956, DAVIS, 1956, and GINZBURG, 1957. This equation describes how the distribution function $N(E, R)$ varies in time due to a variety of energy losses and gains suffered by the relativistic electrons.

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As it is evident from KAPLAN's work, a general solution of the kinetic equation is too complicated to be of any help, so KARDASHEV's procedure was to consider first the effect of each separate process of energy gain or loss, then two processes simultaneously, and then three. He concluded that "all of the processes discussed lead either to the retention of or to an increase of the spectral index with time . . .".

However, as we shall demonstrate, there is a model which leads to the desired decrease of γ with time, and it will also become clear why KARDASHEV's models were unable to achieve this effect.

Our procedure is opposite to that of KARDASHEV. We substitute the "observationally" determined $N(E, R)$ as given by Eq. (2) into the kinetic equation. We obtain

$$\begin{aligned}\frac{\partial N(E,R)}{\partial t} &= v\frac{\partial N(E,R)}{\partial R} = v\,N(E,R)\left(K^{-1}\frac{\partial K}{\partial R} + \frac{\gamma}{R}\,b\ln E\right)\\ &= N(E,R)\,[\gamma(\gamma-1)\,\alpha_1 + (\gamma-1)\,\alpha_2 - \alpha_3 + \beta(2-\gamma)\,E] + q(E,t)\end{aligned} \tag{3}$$

where the α_1 term arises from the random Fermi acceleration, the α_2 term from the variation of energy in the adiabatic expansion as well as the systematic Fermi acceleration, the β term from loss by synchrotron radiation, and the α_3 term from the disappearance of particles as a result of nuclear collisions or escape from the nebula.

The term $q(E, t)$ is a source term, representing the number of particles injected per cm^3 of energy E at time t. It is important to note that the coefficients α_1 and β are functions only of R and not of E (GINZBURG, 1957; KARDASHEV, 1962). Consequently the $\ln E$ term has to be identified with the source term $q(E, t)$, i.e.,

$$q(E\,,t) = \frac{N(E,R)}{t}\,\gamma b\ln E + f(t) \tag{4}$$

where $f(t)$ arises from part of the $K^{-1}\frac{\partial K}{\partial R}$ term.

Our main conclusion is that the "observed" decrease of γ with time can be due only to a continuous injection or "pumping" of relativistic electrons and cannot be caused by any of the energy gain or loss mechanisms discussed above[1]. The singularity at $t = 0$ is of no consequence since the observations under discussion do not include the very early stages of the supernova explosion. The source term q is clearly time-dependent and it is also apparent that KARDASHEV's models were unable to explain the observations due to the fact that he always considered a constant source term. The complicated form of $q(E, t)$, even when $f(t)$ equals zero, does not lend itself to an obvious physical inter-

[1] Another possibility, of course, is that the $\ln E$ term arises from some unusual and as yet unknown energy gain mechanism, as distinct from the Fermi accelerations.