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HYDROGEN SPECTRUM IN MAGNETIC WHITE DWARFS: Hα, Hβ, AND Hγ TRANSITIONS*

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Using the results of an accurate variational calculation, we present a graphical display of the wavelengths of the Hα, Hβ, and Hγ lines of hydrogen, for magnetic-field values ranging from 0 to 560 megagauss, which is believed to cover the range of fields found in magnetic white dwarfs. This is the first *complete detailed* compilation of such results.

Key words: stars: white dwarfs/magnetic field—hydrogen atoms—Zeeman effects

The discovery of magnetic fields, of the order of 10^7 G to 10^8 G, in a subset of white dwarfs must surely rank high among the many exciting astronomical discoveries of the present time. Such fields are orders of magnitude larger than fields that can be produced in the laboratory. The magnitudes of the field strengths are consistent with the idea that magnetic flux is conserved during the evolution preceding the formation of white dwarfs. This lends support to the generally-accepted conclusion that even higher fields, of the order of 10^{12} G, exist in pulsars.

Rather than trace the history of the subject, we will simply refer to the reviews by Garstang (1977, 1982), which contain extensive references. In addition, the latter review discusses magnetic fields occurring in AM Herculis stars. The latest news on magnetic white dwarfs is given by Liebert et al. (1985).

Perhaps the most dramatic recent contribution is the work of Greenstein (1984) on the white dwarf Grw $+70°8247$; he identified an ultraviolet feature near 1347 Å as the σ_- transition of hydrogen $L\alpha, 1s0 - 2p-1$, produced in a magnetic field, B, between 200 and 400 megagauss (MG). Our own theoretical calculations (Henry and O'Connell 1984) predicted the maximum wavelength attainable by the σ_- transition as 1342.6 Å at $B \approx 500$ MG. The identification of the many other lines in this as well as in other white dwarfs underscores the urgency of obtaining the hydrogen spectrum over the complete range of magnetic fields of interest, and in fine detail.

Our initial work on this topic (Smith et al. 1972)—which was the first such calculation with white-dwarf applications in mind—did cover the range of interest, but not in the detail that is now required. At about the same time, motivated by experimental results on shallow-donor spectroscopy and exciton absorption in a magnetic field, Praddaude (1972) also produced a selection of very accurate results. The results of Garstang and Kemic (1974) were for magnetic fields less than about 10^7 G. More recent valuable contributions include Rosner et al. (1984) and Forster et al. (1984) but their results show notable "gaps" in some of the Hα and Hβ lines due to computational difficulties and, furthermore, they have obtained only six out of 25 of the Hγ lines. Since our own calculational procedures have been described in detail in our previous papers (Smith et al. 1972; Henry and O'Connell 1984) we will not repeat them here. However, we would like to emphasize that our use of Slater orbitals gives us the possibility of a complete basis set to describe our wave functions, in contrast to the hydrogenic basis used in some other calculations. For the present calculation, we use 17 Slater orbitals for each angular momentum, ℓ, and ℓ values up to 37.

Using the results of our accurate variational calculation, in which we have taken into account the small correction due to the finite proton mass (O'Connell 1979), we present in Figures 1, 2, and 3 the wavelengths of the Hα, Hβ, and Hγ lines of hydrogen, for magnetic field values ranging from 0 to 560 MG, which is believed to cover the range of fields found in magnetic white dwarfs. This is the first *complete detailed* compilation of such results. The identification of the lines in Figures 1, 2, and 3 are given in Tables I, II, and III, respectively, and follow the scheme of Garstang and Kemic (1974) and Garstang (1977). The only good quantum numbers are m, the z-component of the angular momentum, and π, the parity, but it is normal to label the energy levels with the familiar $n\ell m$ quantum numbers of the hydrogenic energy levels in absence of a magnetic field. In many cases we have used a mesh size as small as 10 MG, which of course is necessary for detailed analysis of certain spectral lines (Henry and O'Connell 1984).

Since the magnetic field in magnetic white dwarfs is inhomogeneous (the ratio between the maximum and minimum values generally thought to vary between 1 and 2), it is expected that the observable absorption features will coincide with transitions whose wavelength is stationary or nearly stationary with respect to variation

*Dedicated to Professor Jesse L. Greenstein, on the occasion of his seventy-fifth birthday.

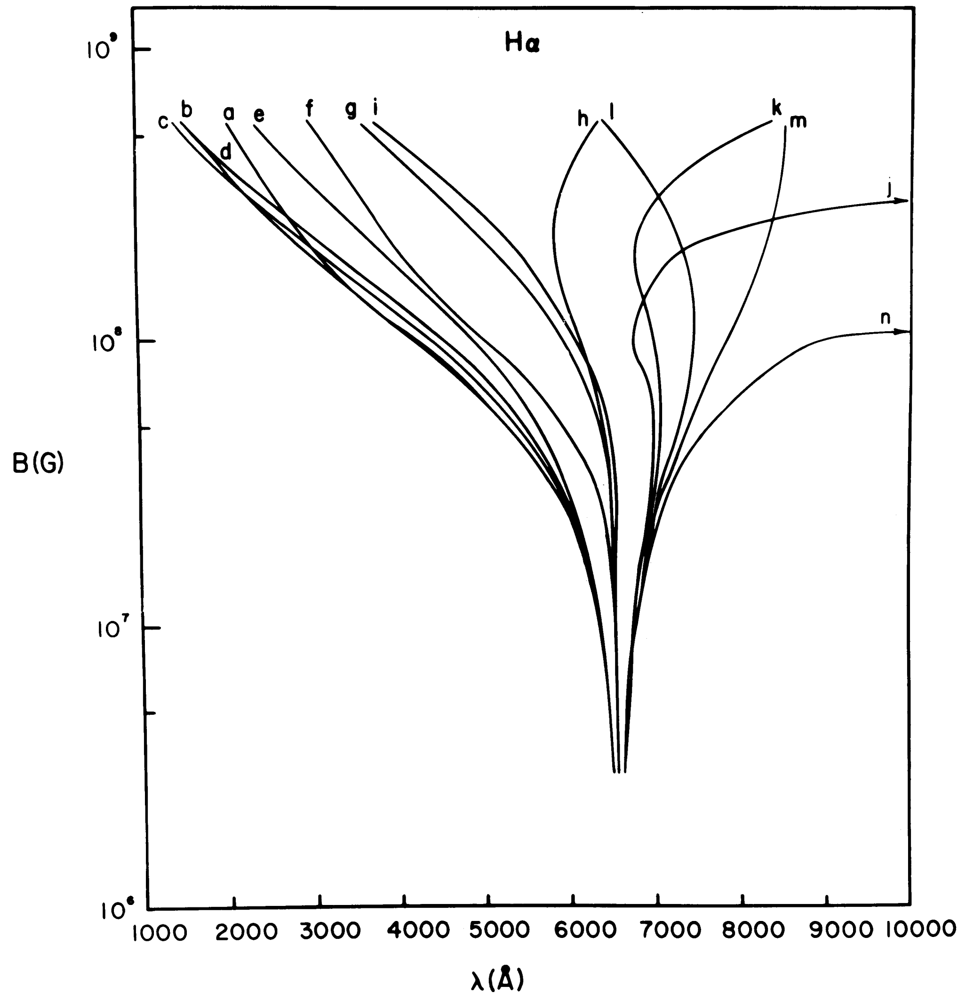

FIG. 1—Hydrogen Hα spectrum for magnetic field values ranging from 0 to 560 megagauss.

in magnetic field strength (Angel 1978, 1979). Thus, with a view toward making our results more useful in doing detailed identifications of spectra, we extract from Figures 1–3 the nearly stationary lines for more detailed study. This is achieved by concentrating our attention on the critical range of (100–500) MG and for this range of magnetic field values, we present in Figures 4, 5, and 6 the wavelengths of the Hα, Hβ, and Hγ nearly stationary lines. The usefulness of such results has been demonstrated by the recent work of Greenstein, Henry, and O'Connell (1985) and by Angel, Liebert, and Stockman (1985) on the identification of hydrogen lines, in a magnetic field ≈ 300 MG, in the white dwarf Grw $+70°8247$.

Using the scaling law given by Surmelian and O'Connell (1974), results immediately follow for the energy spectrum of a hydrogen-like atom of atomic number Z in a magnetic field equal to (B/Z^2).

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REFERENCES

Angel, J. R. P. 1978, *Ann. Rev. Astr. Ap.* 16, 487.
Angel, J. R. P. 1979, in *White Dwarfs and Variable Degenerate Stars, I.A.U. Colloquium No. 53*, H. M. Van Horn and V. Weidemann, eds. (Rochester: University of Rochester Press), p. 313.
Angel, J. R. P., Liebert, J., and Stockman, H. S. 1985, *Ap. J.* (in press).
Forster, H., Strupat, W., Rosner, W., Wunner, G., Ruder, H., and Herold, H. 1984, *J. Phys. B: At. Mol. Phys.* 17, 1301.
Garstang, R. H. 1977, *Rep. Prog. Phys.* 40, 105.
Garstang, R. H. 1982, *J. de Physique, Colloque C-2, supplement* 11, C2–19.
Garstang, R. H., and Kemic, S. B. 1974, *Ap. and Space Sci.* 31, 103.
Greenstein, J. L. 1984, *Ap. J.* (*Letters*) 281, L47.
Greenstein, J. L., Henry, R. J. W., and O'Connell, R. F. 1985, *Ap. J.*

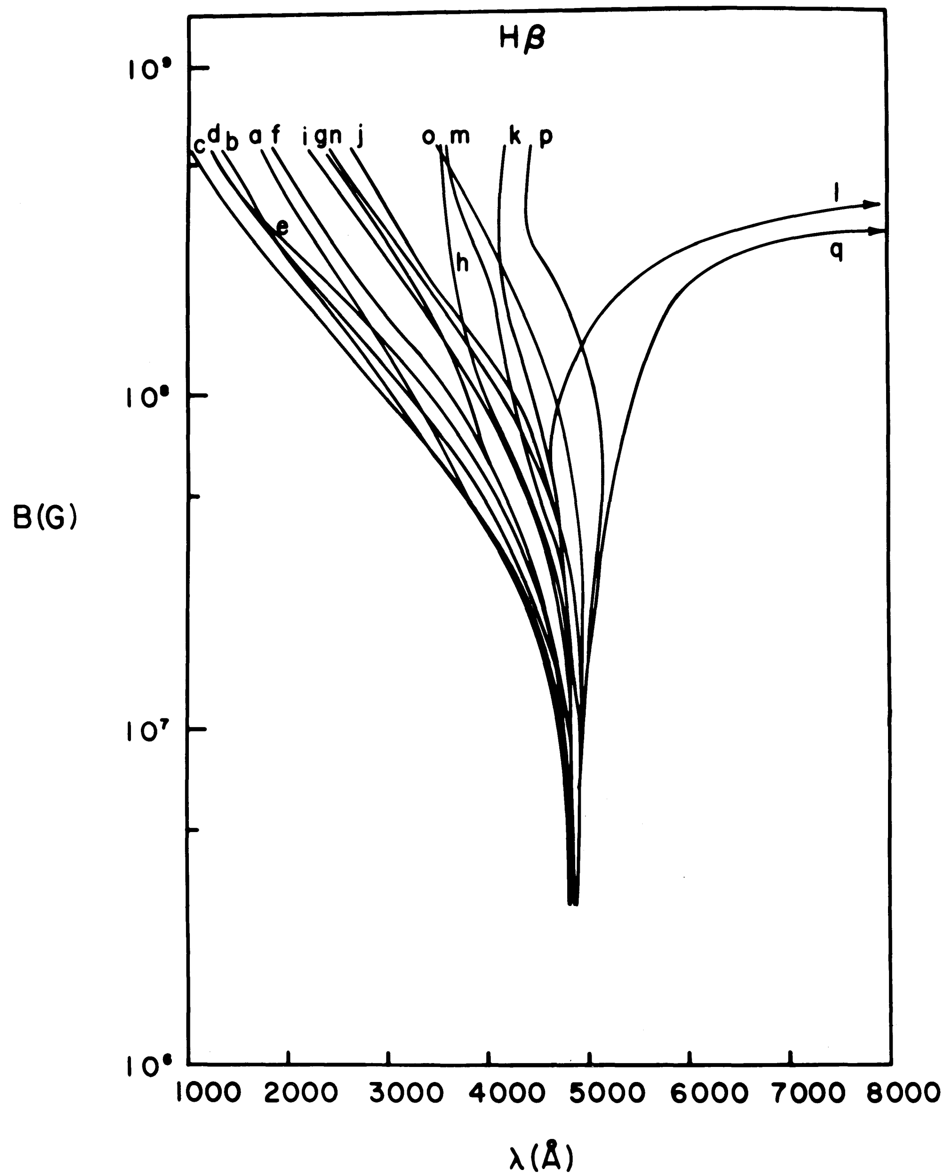

FIG. 2—Hydrogen Hβ spectrum for magnetic field values ranging from 0 to 560 megagauss.

(Letters) 289, L25.
Henry, R. J. W., and O'Connell, R. F. 1984, *Ap. J.* (*Letters*) 282, L97.
Liebert, J., Schmidt, G. D., Sion, E. M., Starrfield, S. G., Green, R. F., and Boroson, T. A. 1975, *Pub. A.S.P.* 97, 158.
O'Connell, R. F. 1979, *Physics Letters* 70A, 389.
Prauddaude, H. 1972, *Phys. Rev. A.* 6, 1321.
Rosner, W., Wunner, G., Herold, H., and Ruder, H. 1984, *J. Phys. B: At. Mol. Phys.* 17, 29.
Smith, E. R,. Henry, R. J. W., Surmelian, G. L., O'Connell, R. F., and Rajagopal, A. K. 1972, *Phys. Rev. D* 6, 3700. (Note that in Fig. 2 the labels $3s_0$ and $3d_0$ should be interchanged.)
Surmelian, G. L., and O'Connell, R. F. 1974, *Ap. J.* 190, 741.

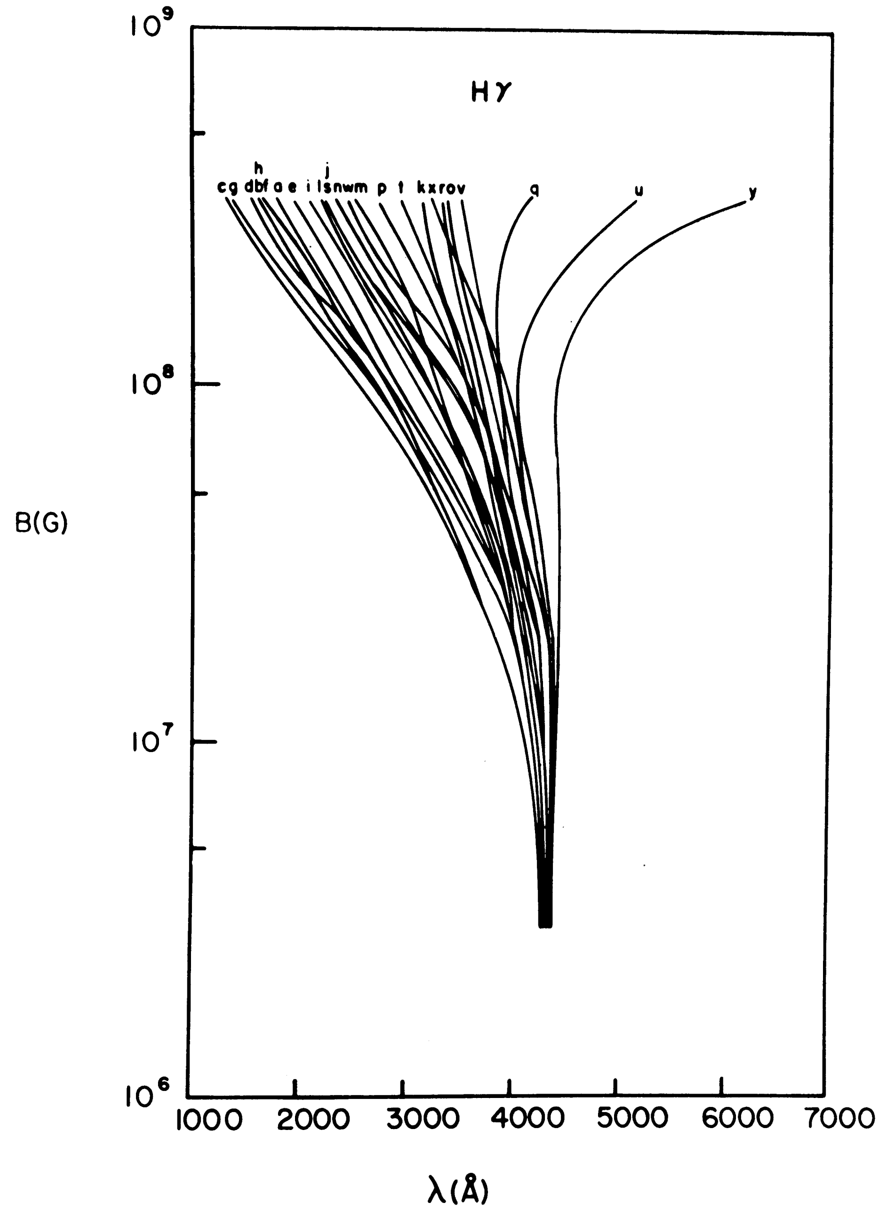

FIG. 3—Hydrogen Hγ spectrum for magnetic field values ranging from 0 to 330 megagauss.

TABLE I

Identification of Hα Transitions Shown in Figure 1

	Lower	Upper
a	2p-1	3s0
b	2s0	3p1
c	2p1	3d2
d	2p0	3d1
e	2p-1	3d0
f	2p0	3s0
g	2p-1 2p1	3d-1 3d1
h	2s0	3p0
i	2p0	3d0
j	2p1	3s0
k	2s0	3p-1
l	2p-1	3d-2
m	2p0	3d-1
n	2p1	3d0

TABLE II

Identification of Hβ Transitions Shown in Figure 2

	Lower	Upper
a	2p-1	4s0
b	2s0	4p1
c	2p1	4d2
d	2p0	4d1
e	2s0	4f1
f	2p-1	4d0
g	2p0	4s0
h	2s0	4p0
i	2p-1 2p1	4d-1 4d1
j	2p0	4d0
k	2s0	4f0
l	2p1	4s0
m	2s0	4p-1
n	2p-1	4d-2
o	2p0	4d-1
p	2s0	4f-1
q	2p1	4d0

TABLE III

Identification of Hγ Transitions Shown in Figure 3

	Lower	Upper
a	2p-1	5s0
b	2s0	5p1
c	2p+1	5d2
d	2p0	5d1
e	2p-1	5d0
f	2s0	5f1
g	2p1	5g2
h	2p0	5g1
i	2p-1	5g0
j	2p0	5s0
k	2s0	5p0
ℓ	2p-1 2p1	5d-1 5d1
m	2p0	5d0
n	2p-1 2p+1	5g-1 5g+1
o	2s0	5f0
p	2p0	5g0
q	2p1	5s0
r	2s0	5p-1
s	2p-1	5d-2
t	2p0	5d-1
u	2p1	5d0
v	2s0	5f-1
w	2p-1	5g-2
x	2p0	5g-1
y	2p1	5g0

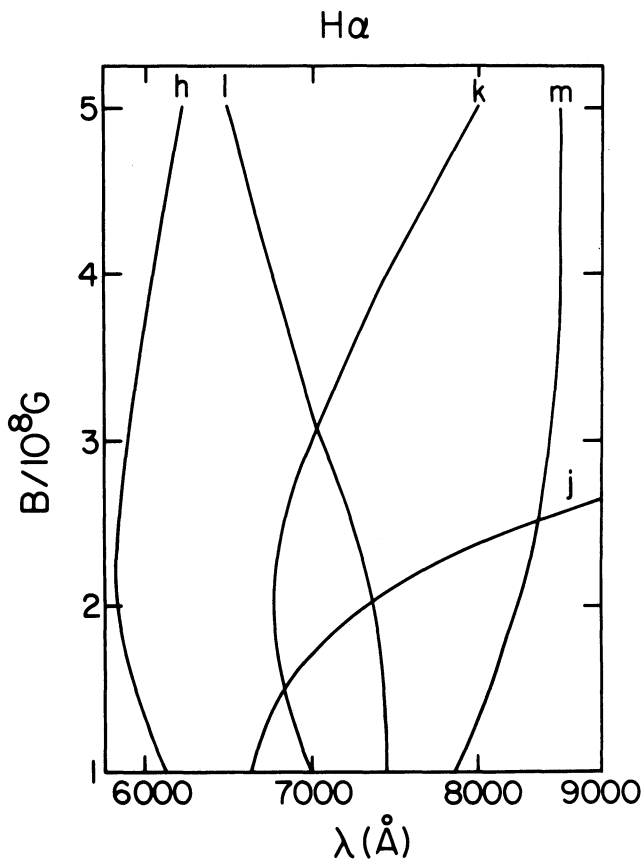

FIG. 4—Hydrogen Hα near-stationary line spectrum for magnetic field values ranging from 100 to 500 megagauss.

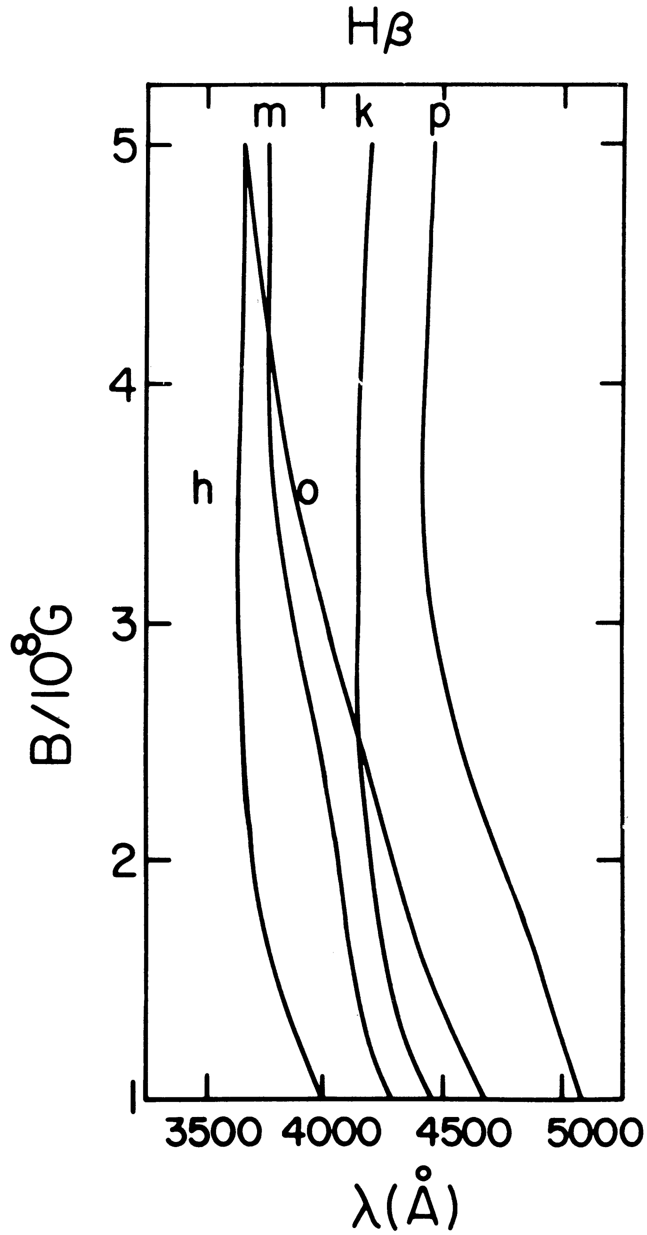

FIG. 5—Hydrogen Hβ near-stationary line spectrum for magnetic field values ranging from 100 to 500 megagauss.

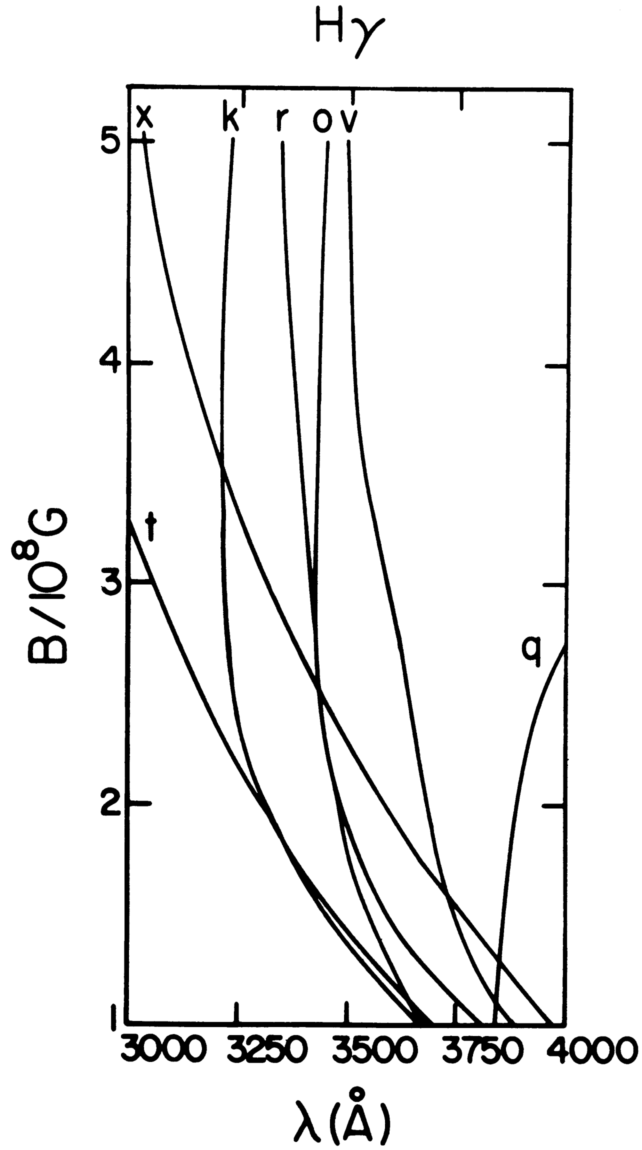

FIG. 6—Hydrogen Hγ near-stationary line spectrum for magnetic field values ranging from 100 to 500 megagauss.